SPECTRAL ESTIMATION via ADAPTIVE TUNABLE NON-PARAMETRIC METHOD

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Abstract

We devise a new approach for non-parametric adaptive spectral analysis method, which is called the Adaptive Tuning Amplitude and Phase Estimation (ATAPES) method. The main advantage of the ATAPES algorithm is its elimination of biased estimation results in APES method, which is biased peak location and corresponding biased amplitude estimation problem. Therefore, ATAPES method provides more accurate peak location and amplitude estimation with higher resolution than APES method.

Index Terms **─ APES, Capon, nonparametric adaptive method.**

1. Introduction

Spectral estimation is important in a diverse range of applications, such as time series analysis, target range signature estimation, geophysics, synthetic aperture radar (SAR) imaging, and many others [1,2]. There are two broad main approaches to spectral analysis. One approach is parametric while the other is nonparametric. Recently, an emerging spectral analysis based application, which overcomes the resolution limit of the nonparametric approach and provides the tradeoff of better spectral resolution at the loss of some robustness and statistical stability is introduced, classified the nonparametric adaptive method [3]. This class of methods includes the Capon method [4] and recently APES method [5,6]. In the past three decades, and in particular for the Capon method, many papers have been proposed to overcome disadvantages of the Capon method, such as weak robustness and statistical instability. On the other hand, the APES method yields excellent spectral amplitude estimations at true peak locations with good spectral resolution, but lower spectral resolution than the Capon method. The APES method still has a problem with robustness and statistical stability, such as biased peak location problem and that corresponding peak amplitude biased estimation in many cases [7]. Moreover, no papers have been reported to solve the biased peak location problem of the APES method.

In this paper, we devise a new approach for the nonparametric adaptive spectral analysis method called here the Adaptive Tuning Amplitude and Phase Estimation (ATAPES) method using the APES principle but taking into account the small sample errors of sample covariance matrix. We introduce the use of a tuning factor for compensating smaller value of sample covariance compared with the true value. The ATAPES method solves the biased peak location estimation problem and its corresponding peak amplitude biased estimation problem with better spectral resolution compared with the APES method. To provide evidence of the performance gain of ATAPES approaches over other approaches, we will compare it with the Capon method, and the APES method.

2. Background of the APES method

Let $\{x(n);\ n=0,1,\ldots,N-1\}$ denote the available data including signal and noise, which is to be estimated, and we want to design filter that passes the frequency ω in $\{x(n)\}$ without any distortion and attenuation all the other frequencies as much as possible. If we let $h = [h_1\ \ h_2\ \ldots\ h_M]`$ denote the vector of the filter coefficients, then the filter output can be written as $h^H y(l)$, where $y(l) = [y(l)\ldots y(l+M-1)]`, l=0,\ldots,L-1,$ M denotes the filter's length, and the superscript ()` and $()^H$ stands for the transpose operator and conjugate transpose. The relations between data length and filter length can be defined as $L=N-M+1$.

In the APES approach, the $\{h(\omega)\}$ is designed such that the filtered sequence is as close to a signal as possible in a least square sense and the complex spectrum $\alpha(\omega)$ is not distorted by filtering. Mathematically, we obtain $h(\omega)$ along with the estimate of $\alpha(\omega)$ by minimizing the following LS criterion [5]

$$\min_{\alpha(\omega),h(\omega)} \frac{1}{L}\sum_{l=0}^{L-1}\left|h^H y(l)-\alpha(\omega)e^{j\omega l}\right|^2 \text{, subject to} \qquad h^H\alpha(\omega)=1 \tag{1}$$

The solution to the above quadratic minimization problem is given by

$$h_{APES}(\omega)=\frac{\hat{Q}^{-1}(\omega)a(\omega)}{a^H(\omega)\hat{Q}^{-1}(\omega)a(\omega)} \tag{2}$$

where $a(\omega) = [1 \quad e^{j\omega} \cdots e^{j(M-1)\omega}]'$

$Y(\omega)$ is the following normalized FFT vector such as

$$Y(\omega) = \frac{1}{L}\sum_{l=0}^{L-1} y(l)e^{-j\omega l} \tag{3}$$

and

$$Q(\omega) = R - Y(\omega)Y^H(\omega) \tag{4}$$

with $R = E[y(l)y^H(l)]$, here E denotes statistical expectation operator.

The true covariance R is not realistic. Thus, we give below the commonly used substitution form for R, which is called sample covariance derived from the maximum likelihood (ML) sense.

$$\hat{R} = \frac{1}{L}\sum_{l=0}^{L-1} y(l)y^H(l) \tag{5}$$

However, problems occur if the filter length (M) is large, then the error, with which sample covariance is estimated, increases because less outer products are averaged in (5) [2]. The error further causes a statically unstable condition, and biased peak location problem in the APES method.

3. Adaptive Tunable Amplitude and Phase Estimation method

In the previous section, we mentioned the problem of the sample covariance matrix. Mathematically the difference between the value of sample covariance and true value is increased because of the value of sample covariance is decreased compared with true value if L is not big enough. Our proposed method is that we increase overall values of sample covariance proportionally to compensate smaller values of sample covariance.

$$\hat{R}_{tuning} = T\hat{R} \tag{6}$$

where T called tuning factor.

Lower bound of the tuning factor is slightly bigger than one, since we need to increase the value of the sample covariance to reduce the gap between sample covariance and true value. Upper bound of the tuning factor is designed independent on a characteristic of the particular collection of data but dependent on the assumption, which is an estimate of a universe that the sample represents. It guarantees tuning sample covariance is in the proper range between ML sample covariance and unbiased sample covariance [8].

Therefore, we define the range of tuning factor as follows

$$1 < T \le \frac{L}{L-1} \tag{7}$$

In the ATAPES approach, the *{h(ω)}* is designed such that the filtered sequence is much closer to a signal as possible by filtering, since sample covariance is increased proportionally to the almost same as the true value by tuning factor. Mathematically, it can be obtained by

$$\min_{\alpha(\omega),h(\omega)} \frac{1}{L}\sum_{l=0}^{L-1}\left|h^H y(l) - \alpha(\omega)e^{j\omega l}\right|^2 \text{, subject to } \quad h^H\alpha(\omega) = 1 \tag{8}$$

$$= \min_{\alpha(\omega),h(\omega)}\left(\left|\alpha - h^H Y(\omega)\right|^2 + h^H T\hat{R}h - \left|h^H Y(\omega)\right|^2\right) \tag{9}$$

To minimize (10) with respect to α, we get

$$\hat{\alpha}(\omega) = h^H(\omega)Y(\omega) \tag{10}$$

Thus, the following minimization problem leads to the determination of the Adaptive Tuning Amplitude and Phase Estimation (ATAPES) filter

$$\min_{h(\omega)}\left(h^H \hat{Q}_{tuning}(\omega)h\right) \text{ ,subject to } h^H\alpha(\omega) = 1 \tag{11}$$

Therefore, the solution of the ATAPES method is given by

$$h_{ATAPES}(\omega) = \frac{\hat{Q}_{tuning}{}^{-1}(\omega)a(\omega)}{a^H(\omega)\hat{Q}_{tuning}{}^{-1}(\omega)a(\omega)} \tag{12}$$

Where $\hat{Q}_{tuning}(\omega) = T\hat{R} - Y(\omega)Y^H(\omega)$

The tuning factor does not cause robust estimation, but eliminates biased estimation results in a given condition. That is, if the filter length (M) is decreased, which corresponds to the larger number of snap shot (L) in a given data length (N), the gap between lower bound and upper bound sample covariance is small. The larger tuning factor can be applied to get higher spectral resolution with accurate peak location and good amplitude estimation. On the other hand, if the filter length (M) is increased, corresponding to the smaller number of snap shot (L) in given data length (N), the gap between lower bound and upper bound sample covariance is large. The smaller tuning factor can be applied to get higher amplitude estimation with accurate peak location, because even though tuning factor is small, the corresponding changing values of tuning covariance are large enough. Therefore, as tuning factor is increased, the accuracy of amplitude estimation rises and falls with increasing spectral resolution. Furthermore, amplitude estimation errors at upper bound tuning are still acceptable with highest spectral resolution.

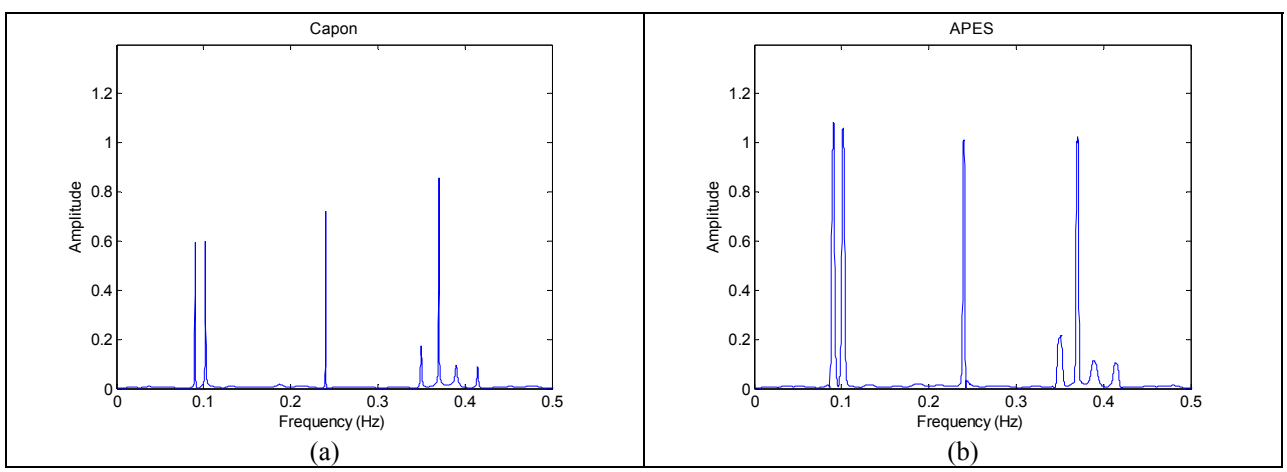
4. Numerical Example

In this section, numerical simulations are used to study the advantage of the ATAPES method compared with the Fourier method, Capon method, and APES method. The data sequence used is N = 96, while the filter length is M = N/2 = 48, which is known as optimum filter length for the APES method [6], with white Gaussian noise of a 0.01 variance. Four dominant spectrums are located at the frequencies: 0.09, 0.102, 0.24, and 0.37 Hz, with amplitudes one, where Hz means cycle-per-sample.

Although the spectral resolution given by the Capon method is very high, the corresponding estimated amplitude spectrums are not accurate, such as 23.38% of RMSEs of spectral amplitude estimates in the fig 1(a). Fig 1(b) shows the spectral estimation via APES method, which has good spectrum amplitude estimation with good spectral resolution. The RMSEs of spectral amplitude estimates for the APES method is 5.2%. In addition, it is well known that the APES method yields excellent amplitude estimates at the true frequency locations but suffers from biased peak location estimation [6]. RMSEs of spectral amplitude estimates for the APES method at true peak locations is 1.62%. However, the true peak location is not available known value, because it is to be estimated values. From the fig 1(c), the spectral resolution given by the ATAPES method is higher than those given by the APES method and has better spectral amplitude estimation. The RMSEs of spectral amplitude estimates for the TAPES method is from 0.2043% to 1.5973% in the tuning range. Fig 1(d) shows the RMSEs of spectral amplitude estimates via tuning points via ATAPES method and APES method (T=1). In a tuning range from little larger than 1 to 1.0208, RMSEs of spectral amplitude estimates are small with better resolution as compared with the APES method. Fig 1(e) zooms in on third dominant signal spectrum for the APES method and TAPES method, and corresponds to the errors of the peak location estimates in the tuning range in fig 1(f). The APES method shows the large biased peak location error. On the other hand, the ATAPES method mends the biased peak location error with higher resolution. The biased peak location error by the ATAPES is comparable with the biased peak location error using Capon method.

5. Conclusion

We presented a new nonparametric adaptive approach called ATAPES method. Smaller value of sample covariance compared with the true value was compensated by overall increasing factor called tuning factor. We compared the ATAPES method with other methods including capon, and APES method.



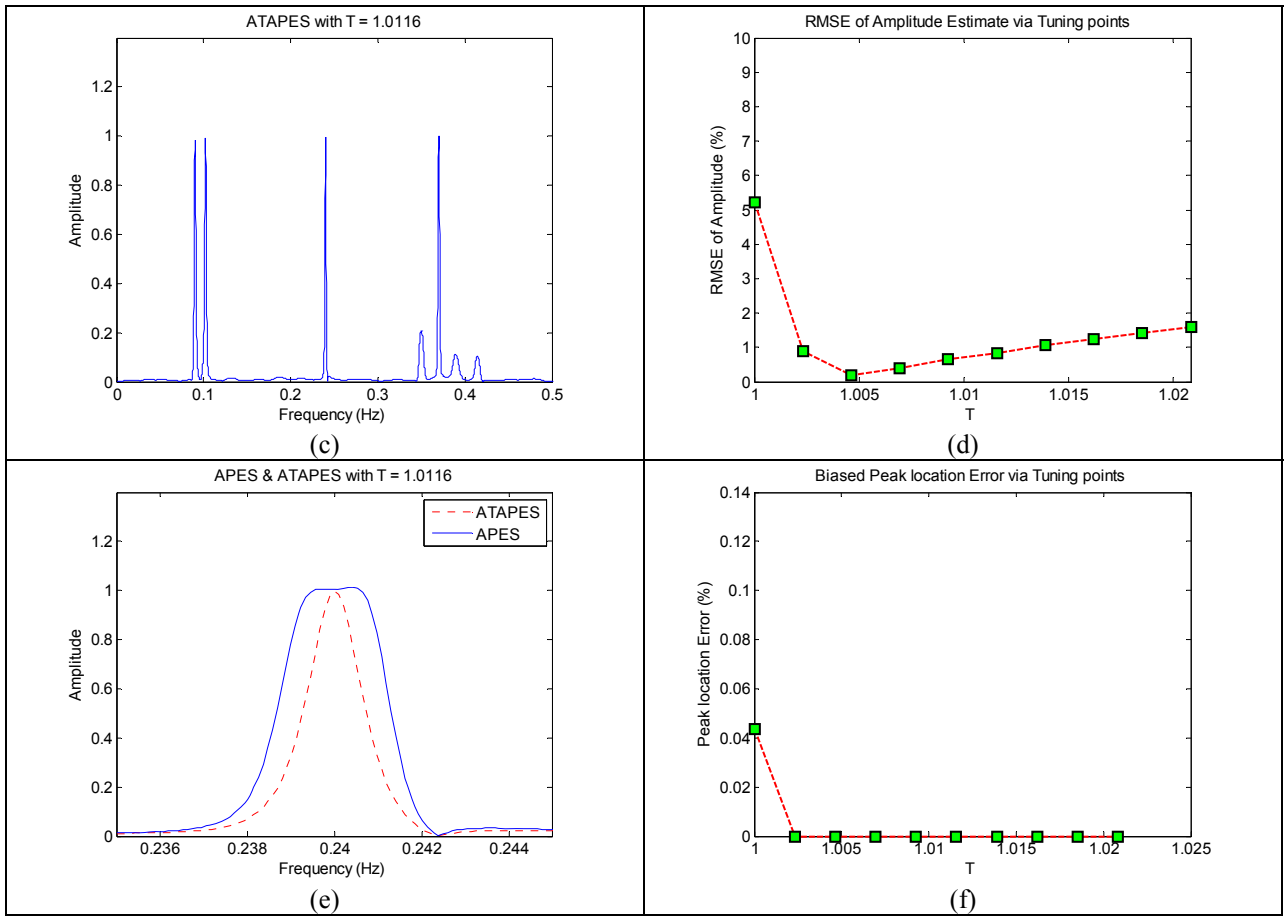


Fig 1. (a) Spectrum estimation via Capon method. (b) Spectrum estimation via APES method. (c) Spectrum estimation via ATAPES method. (d) RMSEs of amplitude spectrum via tuning points for ATAPES method and APES method. (e) Zoom in third dominant signal for ATAPES method and APES method. (f) Corresponding errors of the peak location via tuning points.

We showed, by means of numerical examples, that most remarkable points of ATAPES method eliminate the biased estimation problem of the APES method, which is biased peak location problem and its corresponding biased amplitude estimation problem. Peak location of the ATAPES method is comparable to the Capon method, which is known as best peak location method. The amplitude estimation of the ATAPES method is comparable to the APES method at true peak location. Therefore, the ATAPES method gave more accurate amplitude and peak location estimation with higher resolution than the APES method.

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