COMPARISON OF THREE OMNIBUS NORMALITY TESTS FOR THE DETECTION OF PULSED SINUSOIDAL RFI SOURCES

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ABSTRACT

Normality tests have emerged as a viable option for the detection of Radio Frequency Interference (RFI) in microwave remote sensing. So far, only the kurtosis test for normality has been successfully implemented in an actual radiometric system. Shapiro-Francia, Shapiro-Wilk and QH tests of normality have been praised as powerful omnibus tests in the statistical literature. In this study, the performance of these three tests in detecting the pulsed sinusoidal RFI sources are compared. Results are presented for different duty cycles; effects of discretization on the detection performance are discussed.

1 INTRODUCTION

Recently, a radiometric system that uses the kurtosis test of normality for the detection and mitigation of RFI has been implemented [1]-[3]. This system, called Agile Digital Detector (ADD), operates on the principle that the fields received by a radiometer due to thermal emission sources have a Gaussian distribution, and RFI sources in almost all cases will disrupt the normality. ADD has been demonstrated to be effective against pulsed sinusoidal and continuous RFI sources among others. However, kurtosis statistic is not an "omnibus" test for normality; i.e a kurtosis value of 3, which is observed when the signal is Gaussian, can also be obtained for signals with other distributions. For example, kurtosis has a blind spot when detecting a pulsed sinusoidal RFI source with a 50% duty cycle [1].

The power of normality tests against different distributions have been studied in statistical literature [4]-[5]. Shapiro-Wilk and QH tests were recommended due to their power against different distributions. Therefore, these tests may be considered for RFI detection purposes in future remote sensing systems that will operate in an environment where a variety of RFI sources exist. In this paper, the performance of these two tests as well as the Shapiro-Francia test, which is a slightly modified version of the Shapiro-Wilk test, against pulsed sinusoidal RFI sources are analyzed. The pulsed sinusoidal RFI is studied due to its use in modelling pulsed radars and continuous systems (when the duty cycle is 100%); other RFI types may produce distinct conclusions regarding the relationship between the tests.

In the next Section, a brief summary of the Shapiro-Wilk, Shapiro-Francia and QH tests are given. Some of the results obtained are shown in Section 3. Conclusions are given in Section 4.

2 THEORETICAL BASIS

Shapiro-Wilk test measures the regression of the ordered samples of an experiment from the order statistics of a Gaussian distribution [6]. Therefore, the data needs to be sorted out to compute the Shapiro-Wilk test statistic which will be denoted by W. Let $\mathbf{y}' = (y_1, ..., y_N)$ be the samples of an experiment that are sorted in increasing values; here the prime $'$ is used to denote the transpose of a vector. Then Shapiro-Wilk test statistic is defined by:

$$W = \frac{\left(\sum_{i=1}^{N} a_i y_i\right)^2}{\sum_{i=1}^{N} (y_i - \bar{y})^2} \tag{1}$$

If $\mathbf{m}' = (m_1, ..., m_N)$ are the expected values of the standard normal order statistics and covariance matrix of these order statistics are given by **V**, the coefficients **a** in Equation 1 are given as:

$$\mathbf{a}' = (a_1, ..., a_N) = \frac{\mathbf{m}'\mathbf{V}^{-1}}{(\mathbf{m}'\mathbf{V}^{-1}\mathbf{V}^{-1}\mathbf{m})^{1/2}} \tag{2}$$

The calculation of the coefficients in this form is computationally inefficient, hence, the approximations given in [7] were used in this study.

Shapiro-Francia test has the same form with Shapiro-Wilk test. Shapiro-Francia test statistic W' is given by:

$$W' = \frac{\left(\sum_{i=1}^{N} b_i y_i\right)^2}{\sum_{i=1}^{N}(y_i - \bar{y})^2} \tag{3}$$

For this test, coefficients **b** are equal to:

$$\mathbf{b}' = (b_1, ..., b_N) = \frac{\mathbf{m}}{(\mathbf{m}'\mathbf{m})^{1/2}} \tag{4}$$

Shapiro-Francia test is relatively simpler to compute than the Shapiro-Wilk test. The tests have similar performance; for distributions with kurtosis greater than 3 Shapiro-Francia test is slightly better while opposite is true for distributions with kurtosis less than 3. It is possible to transform the test statistics W and W' to make them have a normal distribution which simplifies the interpretation of the results [7]. Such a transformation is not applied here however; because it affects the power of the tests negatively if the data is discretized. A detailed examination of the performance of Shapiro-Wilk test against pulsed sinusoidal interferers including discretization effects are given in [8].

Unlike the previous two tests, QH test compares the spacings between order statistics of the samples of an experiment with that of a Gaussian distribution [9]. The QH test statistic is given by:

$$QH = \frac{\sum_{i=1}^{N-1} \frac{y_{i+1} - y_i}{m_{i+1} - m_i}}{\sqrt{N-1}(\sum_{i=1}^{N}(y_i - \bar{y})^2)} \tag{5}$$

A transformation for the QH transformation also exists, which is not utilized here for the same reason.

3 RESULTS

In the results shown here, a pulsed sinusoid signal representing the RFI source is added on top of a Gaussian noise. The amplitude of the sinusoid is denoted as A and the sine wave duty cycle as d ($0 \leq d \leq 1$). Total radiometer integration period, shown by N_{IP}, is assumed to be divided into frames of N samples each. Modifying the equation given in [2] for the RFI-to-noise power ratio of the input signal, the ratio R of the RFI brightness temperature (T_{ps}) to NEDT of the system can be written as :

$$R = \frac{dA^2\sqrt{N_{IP}}}{2\sigma^2} \tag{6}$$

where σ is the standard deviation of the Gaussian noise.

The results presented here were obtained using a Monte Carlo simulation. N_{IP} was taken as 32768 samples, which corresponds to ≈ 500 μsec of integration time if the sampling period is $16n$sec. This integration interval was divided into 8 frames, therefore N is equal to 4096 samples. For all of the test statistics, the numerators and denominators in Equations 1, 3 and 5 are averaged separately over 8 frames, and the ratio of the averages is taken at the end. This method resulted in slightly improved detection performance in general compared to finding the test statistic for each frame and averaging them. Figure 1 shows the comparison of Receiver Operating Characteristics (ROC) curves for Shapiro-Wilk, Shapiro-Francia and QH tests when the pulsed RFI source is taken to be 32 samples long ($\approx 0.5\mu$sec under the assumption of a $16n$sec sampling period) and the total number of pulses in an integration period is 10. This gives a duty cycle of about 1%. The R ratios of 2.5 and 5 are shown. Out of the three tests, Shapiro-Francia has the best performance while QH test is the worst for this case.

The same comparison is made when there is a single pulse of 16384 samples long in Figure 2. Therefore, the duty cycle is 50%, which is a blind spot for the kurtosis test. Here, the performance of Shapiro-Wilk and Shapiro-Francia tests are very similar; this is consistent with the aforementioned observation that Shapiro-Francia test is better when kurtosis is greater than 3 (which is the case for the 1% duty cycle), while Shapiro-Wilk test give better results if kurtosis is less than 3. Again, the performance of the QH test is the worst. Although examples are not shown here, the pulse length (in comparison with the test frame sample size N) may have a significant effect on performance. For example, for the 50% duty cycle case a much higher R ratio is required for detection in all the tests if the RFI is simulated as a short duration pulse with a very high PRF instead of a one long pulse, but such a high PRF RFI source is not realistic. The effects of the variation of N on the Shapiro-Wilk test is studied in more detail in [8].

4 CONCLUSIONS

The performance of Shapiro-Wilk, Shapiro-Francia and QH tests are compared for the detection of pulsed sinusoidal RFI. Shapiro-Francia and Shapiro-Wilk tests are very similar in performance; Shapiro-Francia test performs slightly better in detecting sources with low duty cycles while Shapiro-Wilk test gets better as the duty cycle increases. QH test was the worst at detecting this particular kind of RFI among the three tests. All the tests are suitable for implementation in hardware; it will be beneficial to see the performance of these tests in a real life RFI environment if such an implementation is realized. The effect of discretization should be considered for a digital system. Although the results are not shown here, simulations show that discretization only causes serious degradation in performance for the QH test and will not be a serious issue for Shapiro-Wilk and Shapiro-Francia tests if the test statistics are directly used without any normalizing transformations as mentioned in Section 2.

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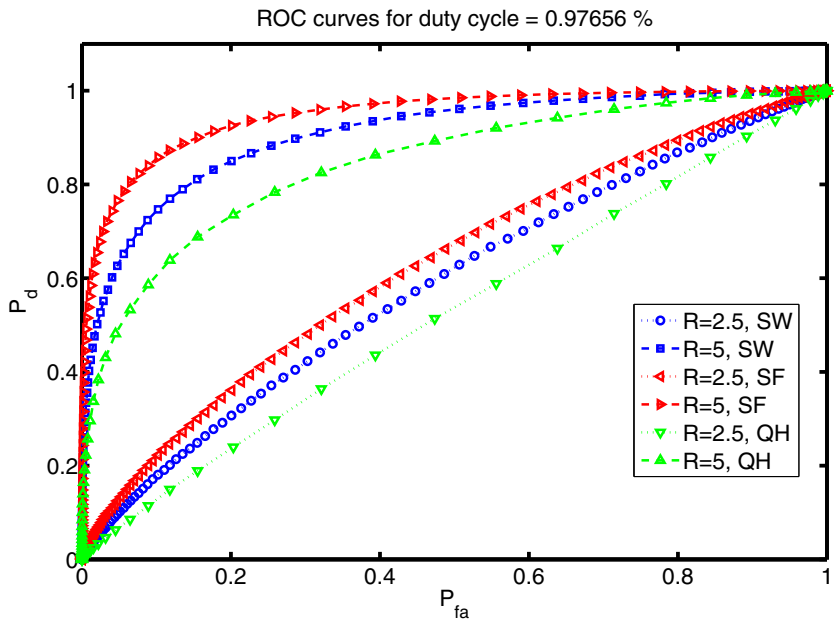

Figure 1: Comparison of ROC curves for Shapiro-Wilk, Shapiro-Francia and QH tests for 10 pulses of 32 samples long, $N = 4096$ samples and $N_{IP} = 32768$. Results for R ratios of 2.5 and 5 are shown.

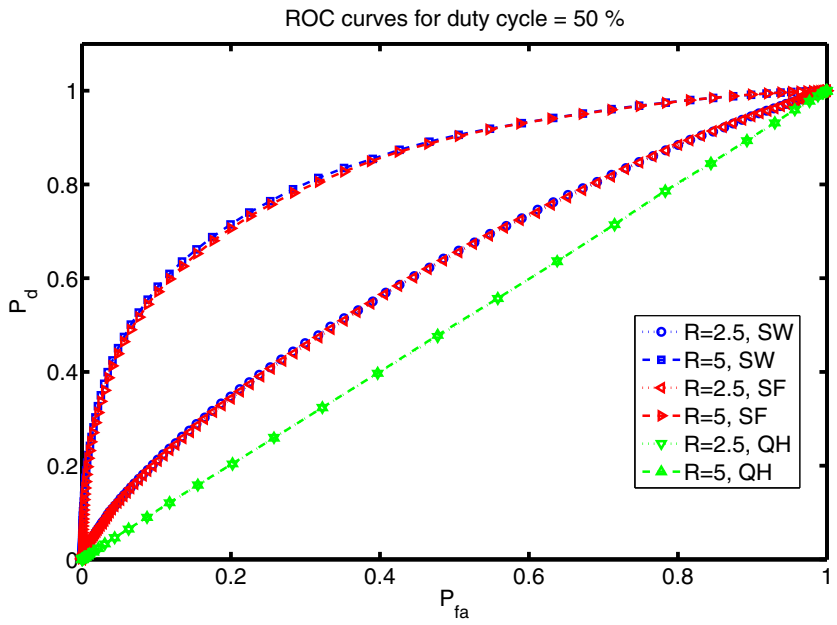

Figure 2: Comparison of ROC curves for Shapiro-Wilk, Shapiro-Francia and QH tests for a pulse of 16384 samples long, $N = 4096$ samples and $N_{IP} = 32768$. Results for R ratios of 2.5 and 5 are shown.