

# Minimum Receiver noise in Phased Array Feeds and Aperture Arrays

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## Abstract

Recent results showing 14 K noise temperature obtained with a CMOS transistor at 1 GHz suggest application in Aperture Arrays (AA) and Phase Array Feeds (PAF) for the Square Kilometer Array without invoking cooling and still reaching system temperatures currently only obtained with cooled GaAs transistors. We present the formula for the effective receiver noise temperature at the input of an antenna system as function of transistor and of antenna characteristics. Application to a single antenna element, an infinite AA and a PAF shows increased effective receiver temperature as function of bandwidth, scanning and tracking respectively. Especially the PAF suffers from additional noise power emanated by array elements that are not significantly contributing to a synthesized dish illumination pattern. We conclude that a dedicated CMOS transistor development is needed to obtain adequate wide band performance over the required scanning and tracking ranges.

## 1. Introduction

Technology road mapping in 2002 [1] compared the potential performance of low noise transistors in CMOS, SiGe, GaAs and InP technologies with increasing cutoff frequency enabled by advanced lithographic technology and indicated adequate low noise performance for CMOS transistors at room temperature for the important 0.5-1.5 GHz frequency band of the Square Kilometer Array (SKA) after 2010. Recent demonstration [2] of ~14 K noise temperature of a Low Noise Amplifier (LNA) using a CMOS transistor at 1 GHz has now opened the way for large scale application in Aperture Arrays (AA) and Phased Array Feeds (PAF) for the important 0.5-1.5 GHz SKA observing band without using expensive cooling systems. Figure 1 gives the trend for amplifier noise temperatures and is based on the road mapping report [1], but has been updated with the most recent result and shows even a faster decrease for CMOS than anticipated in 2002.

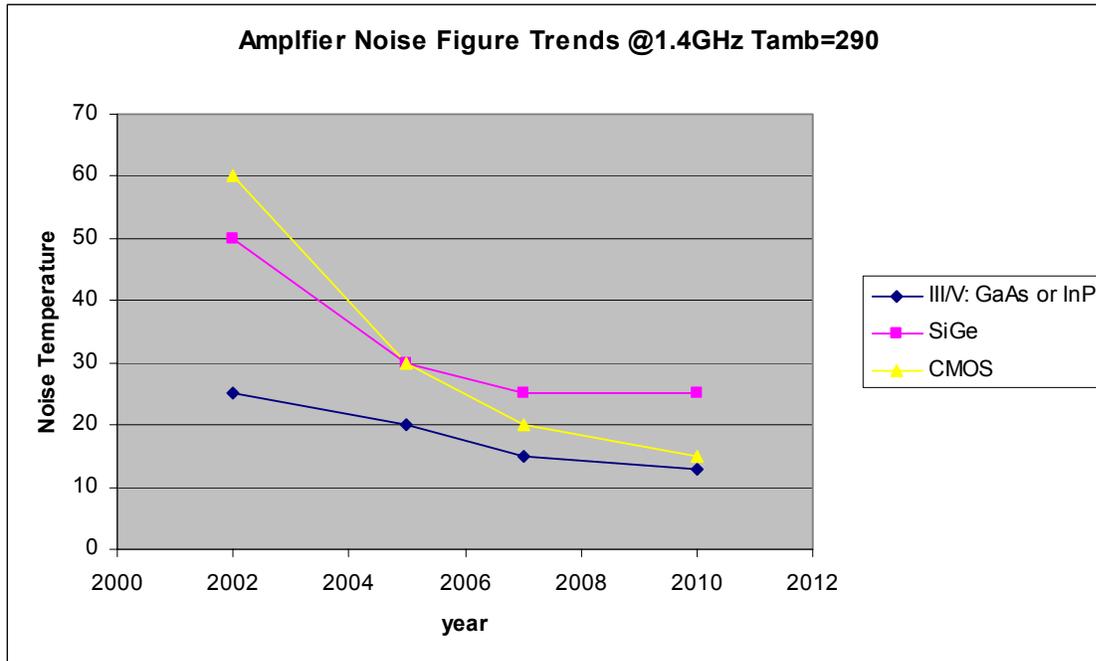


Figure 1. Amplifier Noise Temperature Trends

The demonstrated performance of a narrow band LNA will degrade in wider band application and even more when the LNA has to be optimized for AA and PAF application where the coupling between the antenna elements and their associated LNA has to be taken into account. These issues have been addressed by various authors [3-6] and this paper will attempt to summarize the results for a non-specialist audience. A full treatment of the effective obtainable system sensitivity including not only the effective receiver temperature but also the temperature contributions of source, sky, environment and telescope structure as well as the effective area used of an AA or of a telescope dish when illuminated by a PAF is outside the scope of this paper. The purpose is to explain the basic mechanism of obtaining low noise performance by a CMOS transistor by appropriate noise matching to an antenna and to show that the often quoted minimum noise temperature  $T_{\min}$  of a transistor is not the only parameter that determines the effective receiver temperature. A critical transistor parameter is the noise resistance  $R_n$ , which has to be multiplied with the physical temperature of the transistor and with a mismatch factor. The latter is the important factor in the design of a LNA that has to show low noise performance over a wide bandwidth, over the angular scanning range of an AA and over the spatial tracking range of a synthesized feed pattern over a PAF. We show that the optimum low noise design for a CMOS transistor has not only to focus on  $T_{\min}$  but especially for AA and PAF applications  $R_n$  has to be minimized as well and a trade-off has to be made.

We start in section 2 with a description of the effective noise temperature of a CMOS transistor in a microwave circuit and present a simple formula. This shows how the noise at the output of a low noise transistor is minimized by reflecting part of the noise emanated by the input circuit of the transistor with appropriate amplitude and phase as determined by the optimum mismatch between antenna and transistor input. We show the impact of non-optimum mismatch and the role of the transistor parameter  $R_n$  and its physical temperature  $T_0$  when we want to realize wide band performance.

In section 3 we introduce the array antenna with the sparse, dense and infinite AA conditions that lead to simplified approaches and expressions valid for those regimes. In addition to the bandwidth effect we have to deal with the pointing of the array into a specific direction by the beam former that adds all the element signals with a certain phase. This leads effectively to matching to the active array impedance that varies with scan angle and introduces an extra noise contribution comparable to the wide band contribution.

In section 4 we introduce the PAF, which uses only a small fraction of a finite dense array. This system suffers in addition to the AA scanning mismatch term from noise power emanated by elements that will not be cancelled in the beam former. This requires special attention for minimizing the emanated noise power from the input of the LNA.

Section 5 summarizes the conclusions and stresses the importance of further research on minimizing  $R_n$  in the transistor design.

## 2. Matching a CMOS Transistor for Low Noise to a single Antenna

The noise behavior of an LNA or, more generally, a two-port may be described with its four noise parameters, which can be represented in various forms, e.g. using impedances, admittances or reflection coefficients. The four noise parameters and the properties of the source connected to the two-port input fully determine the effective input noise temperature of the two-port. For our application we prefer the use of wave parameters and choose the description using reflection coefficients to express the effective input noise temperature  $T_N$  as

$$T_N = T_{\min} + T_R |\Gamma_s - \Gamma_{\text{opt}}|^2 / (1 - |\Gamma_s|^2)$$

$$T_R = 4 T_0 (R_n / Z_0) / |1 + \Gamma_{\text{opt}}|^2$$

with, reference temperature  $T_0 = 290$  K, normalizing impedance  $Z = 50 \Omega$ , source reflection coefficient  $\Gamma_s$ , and the two-port noise parameters  $T_{\min} \sim 14$  K,  $R_n \sim 8 \Omega$  and  $\Gamma_{\text{opt}} < 0.5$ . It is obvious that  $T_N = T_{\min}$  for  $\Gamma_s = \Gamma_{\text{opt}}$  (noise matching). The noise equations together with the two-port S-parameters completely describe the small signal behavior of the two-port in terms of scattering waves and noise temperature. Awareness of the existence or knowledge of the properties of noise waves is not required in this case. However, for phased arrays a description in terms of noise waves, emanating from the two-port input and output, will appear to be very convenient.

In practical systems all the parameters are frequency dependent, which means that for a wide band system the optimum noise matching cannot be realized for the whole band simultaneously and an additional noise contribution has to be accepted which is proportional with the average mismatch factor ( $|\Gamma_s - \Gamma_{\text{opt}}|^2 < 0.1$ ) and with  $T_R \sim 100$  K. This makes clear that reduction of  $T_R$  is important, for instance by reducing  $R_n$  by transistor design or cooling.

### 3. Aperture Array

The simplest description of a multi-port array antenna assumes a set of radiators with equal beam pattern and impedance without mutual coupling. The far field radiation pattern is then a simple summation of the individual contributions. The phases in the summation are determined by the geometry of the array and the amplitudes are the product of the individual excitations and the radiator beam pattern. In reality when element  $i$  is excited with a current  $I_i$  a voltage  $V_j$  develops over the open port terminal  $j$  then  $Z_{ij} = V_j / I_i$  is called the coupling impedance between port  $i$  and port  $j$  and  $Z_{ii}$  the impedance of port  $i$ . The consequence is that when each antenna port is connected to its own two port device, part of the noise wave emanating from the input is transferred to other receiver inputs via coupling between the individual antenna elements. We assume all two port devices identical and a beam former that adds all the two port output signals with an appropriate phase factor. We have now formed a so called phased array and we could apply phase corrections such that the appropriate geometrical path correction is obtained for a specific direction, the so called look direction.

To properly analyze noise matching in phased arrays it is essential to be able to interpret noise matching as the (partly) cancellation of two-port noise waves by the wave emanating from the input, which is reflected back at the source into the two-port. Obviously the reflected wave is fully correlated with the one emanating from the input but is also correlated with the output noise wave. Maximum cancellation will result in  $T_{\min}$  for  $\Gamma_s = \Gamma_{\text{opt}}$ .

In the phased array we have the term that reflects back from the mismatch between the two port input impedance and  $Z_{ii}$  but also the terms that are coupled to the other ports and amplified with the same gain, but now added in the beam former with some phase depending on the actual settings for a specific look direction. These noise contributions are all correlated with the output noise of the two port connected to antenna port  $i$  and lead to some cancellation. For an infinite array with all identical antenna ports and identical two port devices to the beam former the same happens to all the independent noises of all the two ports that just average at the output of the beam former to the output level of a single two port. A full analysis [5] shows that optimum noise matching for the individual two ports is obtained when they are not optimally matched to the so called passive element impedance  $Z_{ii}$  but to active array impedance observed at port  $i$  when all antenna port are excited simultaneously with a current of unity strength but a phase appropriate for a certain look direction. This means that optimum noise matching is only possible for a specific look direction and that the two ports are no longer optimally matched for another direction that has a different active array impedance. Such a noise mismatch will result in a contribution from the second term in the expression for  $T_N$ . In an actual design the average of  $T_N$  over the relevant range of look directions is optimized, which is again higher than when optimized over a frequency range as for a single port antenna.

In a finite array we have apart from the phase control in the beam former for each antenna element also a gain control that allows tapering down the signals from elements near the edge of the array. Applying such a taper also influences the effective two port LNA receiver mismatch and thus the effective receiver temperature apart from the effective collecting aperture. This effect will be reduced when the transistor design could be done such that  $T_R$  is small compared with  $T_N$ .

### 4. Phased Array Feed

The Phased Array Feed is a small part of a finite array placed in the focus of a reflecting collector antenna. Such a finite array has typically hundred antenna elements in a dense configuration. At the highest frequency the focal spot is covered by at least four adjacent elements, while the twelve surrounding elements collect the signal from the first Airy ring. Optimum SNR is obtained with amplitude weights equal to the signal strength, while the phase is typically  $0$  or  $\pi$  to allow adding and subtracting. In contrast to the AA, which could be well operated as a phased array, the PAF is an amplitude array that uses order sixteen elements from a larger array antenna to synthesize a feed illumination pattern at a specific position in the focal plane of a reflector.

It is clear that the LNA design has not to be matched to the active reflection coefficient of an infinite array but to the passive impedance of a single element with corrections [7] for the coupling to the surrounding elements based on the expected weight factors needed to synthesize an appropriate feed pattern. The lowest frequency of a PAF could be a factor three lower than the highest one typically invoking 12 elements with a high weight and the rest of the array with low weights. This shows that the design of a LNA for a wide band PAF covers a larger range of impedances than for a wide band AA, with as a consequence a higher average receiver noise temperature.

A third effect specific for amplitude tapering and a serious one for a PAF is that LNA noise emanated from elements that are not combined in a specific beam former cannot be cancelled, but add incoherently to the output noise of that beam former. We identified two effects that complicate the design of a wide band LNA for a PAF and

that will increase the effective receiver noise temperature above that for an AA in the same antenna and the same transistor technology. To reduce this effect transistors are needed that have a low  $R_n$  and LNA designs that emanate less noise power from their input. To this end a more complicated LNA design can be afforded for a PAF than for an AA to mitigate the additional noise effect.

## 5. Conclusions

Recent demonstration of a LNA based on a CMOS transistor showing a  $T_{\min} \sim 14$  K paves the way for large scale application in AA and PAF for the SKA, since it avoids cooling to obtain adequate system temperatures.

However, wide band application will boost the effective receiver noise temperature in single antenna application with a term  $T_R \sim 100$  K that has to be multiplied with an average mismatch term  $< 0.1$  over a multi octave frequency band.

For Aperture Arrays this average mismatch term is further increased by the necessity to cover an adequate scanning range over which the active array impedance, to which needs to be matched, varies considerably.

For Phased Array Feeds the impedance, to which effective noise matching has to be obtained, strongly depends on the number of array elements that has a high weight in the synthesized feed pattern. This number of elements is also strongly frequency dependent, which complicates the LNA design with respect to the AA and will lead to an even higher receiver noise temperature assuming the same antenna, transistor and LNA technology. More over the PAF suffers from incoherently coupled noise powers emanated by all the individual LNAs that will not be cancelled in the beam former that synthesized the feed pattern.

A work package has been defined in the European SKA Design Studies program to investigate the design options for an LNA with CMOS transistors in forthcoming advanced lithographic technology that not only minimizes  $T_{\min}$  but also comes to grips with  $R_n$ .

Especially for PAF application a LNA design is needed that minimizes the emanated noise from its input.

## 8. References

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