Calibration of Phased Array Feeds

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Abstract

The Australia Square Kilometre Array Pathfinder (ASKAP) is one of the key technology demonstrators on the way to the Square Kilometre Array (SKA). The main feature of the project is the development of the Phased Array Feed (PAF) technology to boost the instantaneous field of view. We consider a non-adaptive (weights are constant) beamforming and put emphasis on the calibration of the PAF. It is shown that for a typical sky brightness distribution, a small number of synthetic beams, perhaps an order of magnitude smaller than the number of physical feeds, contains the most of information. From the other side, only a small number of gains can be calibrated using the full-beam self-calibration approach.

1 Introduction

Wide instantaneous field of view is one of the key science requirements for the Square Kilometre Array (SKA) and the Australia Square Kilometre Array Pathfinder (ASKAP) as it directly affects the survey speed. One of the possible ways to increase the field of view at a given frequency for a given antenna size is to replace single-feed receivers, which are traditionally used for radio-interferometers, by multi-feed receivers. The elements of each feed array can also be phased together in the beamformer (hence, the term Phased Array Feed or PAF) before correlation. Although, it reduces the amount of measured information, different optimization strategies are possible and the correlation process becomes more manageable. The Australia Square Kilometre Array Pathfinder (ASKAP) is intended to explore this phased array option for the SKA.

Using an array of feeds instead of a single-feed receivers has serious implications for calibration. First, there is a significantly larger number of unknowns to calibrate. At least, there are separate gain and leakage terms for each element of the feed array and potentially parallel cross-talk and cross-leakage terms between elements because electrodynamical coupling will almost certainly be present in the closely packed system at some level. Second, the beamformer reduces the amount of available information, if the number of synthetic beams is less than the number of physical feeds (this is almost certainly the case, otherwise there is no need for a beamformer). Therefore, several measurements with different set of weights are likely to be required. And third, the traditional way of doing calibration by observing a strong point-like source periodically would take an impractically long time because it has to be done for each physical receiver. A more appealing approach is to use all sources available in the field of view and always do a self-calibration. It was demonstrated to work for wide-field Very Long Baseline Interferometry (VLBI) experiments at 1.4 GHz [1]. However, whether or not a random field on the sky has enough information at the observed frequency to determine all unknowns in the ASKAP case requires a further study.

2 Eigenbeams and optimization

A freedom to choose beamformer weights allows different optimization strategies. Following [2], one can try to maximize the total flux collected from a given region of the sky. The beamformer calculates a linear combination of inputs giving a synthetic voltage pattern

$$E(\mathbf{s}) = \sum_l w_l E_l(\mathbf{s}), \qquad (1)$$

where $E_l(\mathbf{s})$ is the voltage pattern of the lth element and w_l is a corresponding complex weight. The power beam is a quadratic form, which can be optimized using methods of linear algebra

$$A(\mathbf{w},\mathbf{s}) = E(\mathbf{s})E^*(\mathbf{s}) = \sum_{l,m} w_l^* E_l^* w_m E_m = \mathbf{w}^H \mathcal{E}(\mathbf{s})\mathbf{w}, \qquad (2)$$

where $\mathcal{E}(\mathbf{s})$ is a voltage pattern matrix $\mathcal{E}(\mathbf{s}) = \|E_l^*(\mathbf{s})E_m(\mathbf{s})\|_m^l$ for direction $\mathbf{s}$. One can maximize

$$F(\mathbf{w}) = \int A(\mathbf{w},\mathbf{s})K(\mathbf{s})\ d\mathbf{s} = \mathbf{w}^H \left[\int \mathcal{E}(\mathbf{s})K(\mathbf{s})\ d\mathbf{s}\right]\mathbf{w} = \mathbf{w}^H\mathcal{E}\mathbf{w}, \qquad (3)$$

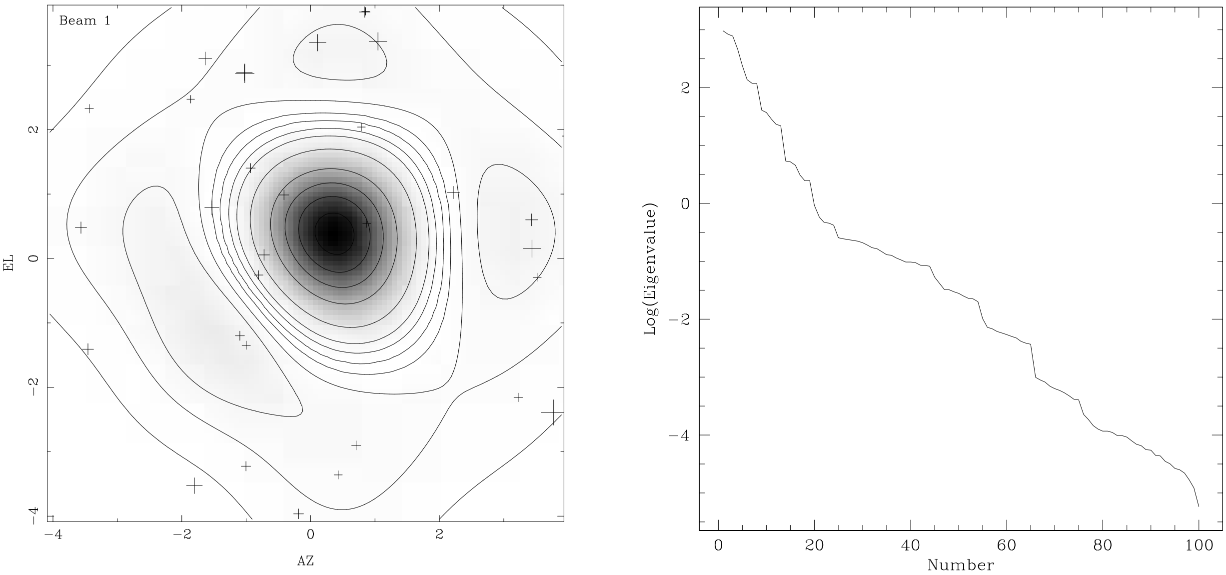

Figure 1: Beams corresponding to the first eigenvector obtained using an NVSS field as a sky model (left) and the corresponding eigenvalue spectrum in the logarithmic scale (right). Crosses on the left-hand image mark the positions of sources (the cross size represent the flux).

where the matrix $\mathcal{E} = [\cdots]$ in (3) is direction independent, and $K(\mathbf{s})$ is an arbitrary kernel, which essentially defines the desired optimization. This quadratic form attains its maximum under condition of $\mathbf{w}^H\mathbf{w} = 1$, if $\mathbf{w}$ is an eigenvector of $\mathbf{E}$ corresponding to the largest eigenvalue. If the beamformer has multiple outputs (i.e. the observations can be performed with a number of different weight vectors simultaneously), eigenvectors corresponding to the appropriate number of the largest eigenvalues can be used. Each eigenvector corresponds to a synthetic primary beam, called eigenbeam. It can be calculated using (2). Maximization of the total collected flux can be achieved by using the sky brightness distribution as $K(\mathbf{s})$. Fig. 2 illustrates the first eigenbeam and eigenspectrum generated for some NVSS field using this approach. The eigenvalues decrease in magnitude very steeply. It is possible to show that the number of non-zero eigenvalues is the number of sources in the field, if the sky model is a collection of point sources. Indeed, for each $\mathbf{s}$, $\mathbf{E}(\mathbf{s})$ is a matrix of rank 1 because it is composed from the vector of voltage pattern values at the source location for each feed. Assuming that there is no degeneracy in the voltage pattern or sampling, a sum over N sources produces a matrix of rank N, therefore giving N non-zero eigenvalues. The element voltage patterns ($E_l(\mathbf{s})$) used for optimization simulations producing Fig. 2 were defined for 40×40 degree region. As a result, a large number of sources contribute to $\mathcal{E}$ through the high order sidelobes. Therefore, there are no zeros in the eigenspectrum (Fig. 2), but there is a break of the slope at some number. This break most likely indicates the number of sources observed through the main lobe by one of the feeds. The shape of the eigenspectrum means that a small number of eigenbeams contains the majority of information. From another point of view, the amount of information may not be enough to constrain the calibration solution for a large number of element gains by observing the same region on the sky (fixed configuration of sources).

It is worth noting, that the current analysis does not take into account the spill-over and the noise properties of eigenbeams. A generalization can be made by analogy to [3], although the appropriate simulations would require a more elaborated model of the PAF installation mounted on the dish. A generalized problem allows to optimize a signal-to-noise ratio instead of just the total flux.

3 Conclusions

1. A relatively small number of synthetic beams (linear combinations in the beamformer) contains most of the information.

2. Only a small number of linear combinations of gains can be determined in the full-beam self-calibration proce-

dure (using all known sources in the field of view). Therefore, one needs to track the relative gains somehow (e.g. by a noise source in the dish vertex illuminating the whole focal plane array installation).

References

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