Study on Reflection of Dust Plasma in Polar Mesosphere to Electromagnetic Wave

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Abstract

Based on Boltzmann function and the generalized Ohm’s law, the influence of charging on dielectric constant and electric conductivity are analyzed. Formulas of the dielectric constant and electric conductivity of dust plasma, which are applied to high frequency electromagnetic(EM) wave, are obtained. With the relative formulas and propagation characteristics in isotropic medium, the reflection coefficient of polar mesosphere where PMSE phenomena is observed is presented by using ECT02 experimental data. The relation between the characteristics of dust plasma in mesosphere and altitude is discussed, and the reflection profile changing with altitude is given.

1. Introduction

Dust plasma is defined as electron-ion-dust particle plasma. Owing to charged dust particles, dust plasma has many characteristics different from general plasma[1]. In 1979, Polar Mesosphere Summer Echoes(PMSE) were first observed using VHF radar at Poler Flat, Alaska(65.13ºN, 147.46ºW) by Ecklund and Balsley[2]. Meanwhile, Noctilucent Clouds(NLC) are accompanied with the phenomena. At present, with the rocket and radar detection[3, 4], many experiments have been done to investigate these phenomena. It is revealed that dust particles are the necessary condition of PMSE. So, as to study the mechanics of PMSE, it is of great importance to start with dust plasma. The characteristics of dust plasma[5-8] and the relation between dust plasma and PMSE have been studied by many scholars. Research fellow Li F. discussed the absorption effect of dust particles in plasma to electromagnetic wave[5]. Doc. Shi analyzed the influence of charging on dielectric constant and electric conductivity of dust plasma[8], but his results have some locations for high frequency incident EM wave. Some discussions about the relation of charged particles and PMSE have been done by Prof. O. Havnes[9,10]. But, it is a pity that the characteristics of dust particles in Mesosphere have not been presented with wave propagation theory. In the paper, with Boltzmann function and the charging equation of dust particles, dielectric constant and electric conductivity are deduced, and the reflection characteristics of dust plasma to electromagnetic wave are discussed. Our conclusions can also be used to rocket puff, chip production in integrated circuit. With our results, we can further analyze the origin of PMSE with dust plasma theory.

2. Relative dielectric constant and electric conductivity of dust plasma

We consider dust plasma as isotropic and homogeneous. When plasmas are in a weak electromagnetic field, Boltzmann equation of electrons[11] can be written as
\[
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{r}} f + \frac{e}{m_e} \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{H} \right) \cdot \nabla_{\mathbf{v}} f + S = 0,
\]

where \( f \) is the distributed function, \( e \) and \( m_e \) are the quantity of electricity and the mass of electrons respectively, \( \mathbf{v} \) is the velocity of electrons, and \( S \) is a collision factor, which is related to \( f \). In the same way, when the number density of charging dust particles is relatively small, Eq.(1) can also be used to electrons in dust plasma. In the weak field, the distribution function of electrons has been shown in Ref.[11].

With relation of distribution function and current density vector, we can obtain

\[
\mathbf{j}' = e \int \mathbf{v} f d\mathbf{v} = \frac{4\pi e}{3} \int_0^\infty f_i v^3 d\mathbf{v} = \frac{8e^2 N_e E}{3\sqrt{\pi} m_e} \left\{ \int_0^\infty \frac{v(u)u^4 e^{-u^2}}{\omega^2 + v^2(u)} \, du - i \omega \int_0^\infty \frac{u^4 e^{-u^2}}{\omega^2 + v^2(u)} \, du \right\}.
\]

In the other hand, current density vector and electric field are also related by the general ohm law[11].

\[
\mathbf{j}' = \sigma_{\text{complex}} \mathbf{E} = \left( \sigma_e + i \omega \epsilon_0 (\varepsilon - 1) \right) \mathbf{E}.
\]

Factors \( \sigma_{\text{complex}} = \sigma_e + i \omega \epsilon_0 (\varepsilon - 1) \), \( \sigma_e, \epsilon_e = \varepsilon - 1 \) denote complex electric conductivity, relative dielectric constant respectively. They are greatly influenced by collision of electrons. Clearly, Eq.(2) is equal to Eq.(3). Let effective collision frequency \( \nu_{\text{eff}} \) instead of \( \nu(u) \), the expressions of relative dielectric constant and electric conductivity can be obtained.

\[
\sigma_e = \epsilon_0 \frac{\omega_{pe}^2 \nu_{\text{eff}}}{\omega^2 + \nu_{\text{eff}}^2}, \quad \epsilon_e = -\frac{\omega_{pe}^2}{\omega^2 + \nu_{\text{eff}}^2}.
\]

Here, \( \omega_{pe} = \sqrt{e^2 N_e / \epsilon_0 m_e} \) is the plasma frequency of electrons, \( \nu_{\text{eff}} = \nu_{e,m} + \nu_{e,j} + \nu_{e,d} \) is the sum of effective collision frequency of electrons with other kinds of particles. With the same process, the contribution of ions experiencing ionization only once to relative dielectric constant and electric conductivity can be deduced.

When EM wave, whose frequency of incident wave is far greater than relaxation charging frequency, is incident upon dust plasma, the shift of quantity of electric charge can be neglected[6]. Therefore, influence of the charge fluctuation of dust particles can also be discarded, rendering the difference from Ref.8. In electric field, dust particles are charged by electrons and ions. Charging current is related to the number density of electrons and ions, temperature, and quantity of charge. In the OLM model, Charging current[7] can be expressed as

\[
I_e = -e \int_{-\infty}^{\infty} v f e \, dv = -\frac{8e^2 N_e E \omega_{pe}^2}{3\sqrt{\pi} m_e} \int_{-\infty}^{\infty} \frac{\sigma_e^d u^4 e^{-u^2}}{i \omega + v_e(u)} \, du,
\]

\[
I_i = e \int_{0}^{\infty} v f_i \, dv = -\frac{8e^2 N_e E \omega_{pe}^2}{3\sqrt{\pi} m_e} \int_{0}^{\infty} \frac{\sigma_i^d u^4 e^{-u^2}}{i \omega + v_i(u)} \, du.
\]

The total current is \( I_d = I_e + I_i \). \( \sigma_{\alpha}^d (\alpha = e, i) \) is the collecting cross section[1] of an electron or a ion to a dust particle. \( f_{\alpha} (\alpha = e, i) \) is distribution function of electrons or ions. Charging current and electric field are related.
by the following equation

$$\mathbf{j}_d = \sigma_{\text{complex}} \mathbf{E} = (\sigma_d + i \omega \varepsilon_0 \varepsilon_d) \mathbf{E}$$

(7)

Substituting Eqs. (5) and (6) into Eq. (7), we can obtain

$$\left(\sigma_d + i \omega \varepsilon_0 \varepsilon_d\right) \mathbf{E} = N_d \left(I_e + I_j\right)$$

(8)

Eq. (8) can be simplified as

$$\sigma_d + i \omega \varepsilon_0 \varepsilon_d = -\frac{8\varepsilon_0^2 N_e N_d}{3\sqrt{\pi} m_e} \int_{-\infty}^{\infty} \frac{\sigma_d u}{i \omega + \nu_e(u)} e^{-\nu_e^2} \, du - \frac{8\varepsilon_0^2 N_e N_d}{3\sqrt{\pi} m_i} \int_{-\infty}^{\infty} \frac{\sigma_i u}{i \omega + \nu_i(u)} e^{-\nu_i^2} \, du$$

(9)

The influence component by charging can be gained from Eq. (9).

$$\sigma_d = \pi r_d^2 \left(\frac{\varepsilon_0^2 N_e N_d}{\omega^2 + \nu_e^2} + \frac{\varepsilon_0^2 N_e N_d}{\omega^2 + \nu_e^2}\right), \quad \varepsilon_d = -\pi r_d^2 N_d \left(\frac{\omega_p^2}{\omega^2 + \nu_e^2} + \frac{\omega_p^2}{\omega^2 + \nu_i^2}\right)$$

(10)

### 3. Numerical analysis

In order to study NLC and PMSE, in 1974, the ECT02 experiment was made at Andøya rocket launching site. In the following, 61 data of the experiment are used to analyze electromagnetic characteristics of dust plasma. Experimental data lead us to a conclusion that dust plasma in polar mesosphere can be considered as weakly ionized plasma. Based on ECT02 experimental data, it can be calculated that radius of a dust particle is mainly between 30 and 80nm. Substituting radius of a dust particle into Eq. (4), relative dielectric constant and electric conductivity are calculated, and are respectively shown in Figs. 1 and 2. It is indicated that an inverse correlation between relative dielectric constant and electric conductivity is available. There are two special film structures for dielectric constant and electric conductivity. The results are corresponding with the distributions of the number density of electrons and charged dust particles.

![Figure 1](image1.png)  ![Figure 2](image2.png)

**Figure 1** Influence of collision on variation of dielectric constant with altitude  
**Figure 2** Influence of collision on variation of electric conductivity with altitude

Based on wave propagation theory in layered medium, the reflection coefficient of dust plasma to electromagnetic wave is calculated. The polar mesosphere between 82 and 90km is divided into 61 layers. Considering vertical incident wave, the reflection coefficient of each layer is calculated. The radar frequency (53.5MHz) that was used in ECT02 experiment is also adopted here.
The variation of the reflection coefficients with altitude is described in Fig.3. Because the assumption of the reflection coefficient is 0 in 90km above the earth, the result that we calculated is smaller than measured values. The distribution of reflection coefficients is similar to that of the number density of charged dust particles.

4. Conclusion

Based on charging theory of dust plasma, taking charging and collision into account, expressions of the relative dielectric constant and electric conductivity of dust plasma are deduced. With ECT02 experimental data, the electromagnetic characteristics of polar mesosphere are discussed. The reflection characteristics of dust plasma are analyzed by wave propagation theory. Our result is of great importance in the analysis of mechanics of PMSE.

5. References