Thermal Corrections to the Refractive Index Surface of Whistler-Mode Waves

P. Kulkarni[1], U.S. Inan[2] and T.F. Bell[3]

[1] Space, Telecommunications and Radioscience Laboratory
Stanford University
350 Serra Mall, Room 308
Stanford, CA 94305 USA
pxk161@stanford.edu

[2] inan@stanford.edu
[3] bell@nova.stanford.edu

Abstract

We use numerical methods to calculate refractive index surfaces of VLF whistler mode waves at both equatorial and off-equatorial locations in the inner magnetosphere. We show refractive index surfaces for wave frequencies below, approximately equal to and above the local lower hybrid resonance frequency. We demonstrate the effect of including finite electron and ion temperatures for the background plasma. We make comparisons among the cold plasma theory, a fully adiabatic warm plasma theory where temperature effects are added to all elements of the dielectric tensor (not just the diagonal elements), and full kinetic solution. Finally, we analyze how thermal effects might impact whistler-mode raypaths.

Introduction

Resonant interactions between energetic electrons and very low frequency (VLF) waves are believed to be an important loss mechanism for trapped particles in the near-Earth space environment [*Kennel and Petschek*, 1966]. A recent study by *Abel and Thorne* [1998a, 1998b] documented the loss rates of radiation belt electrons in the 100 - 1500 keV energy range induced by Coulomb collisions and resonant interactions with plasma waves, including plasmaspheric hiss, lightning generated whistlers and VLF transmitter signals. The authors concluded that man-made VLF transmitters operating continuously in the 17 - 23 kHz range have a significant impact on 100 - 1500 keV electron lifetimes. As satellites become increasingly vulnerable to enhanced fluxes of energetic electrons, it becomes critical to evaluate and quantify the potential role of anthropogenic effects on the relativistic electron population.

Inan et al. [2003] discussed the feasibility of using an *in situ* source to inject whistler-mode waves into the inner belts in a scheme of controlled precipitation of radiation belt electrons. By power scaling the results from *Abel and Thorne* [1998a, 1998b], *Inan et al.* [2003] indicated that a space borne transmitter at operating frequencies of 1 - 10 kHz can drive diffusion rates that, compared to those from signals from the ground-based VLF transmitters, may be higher by up to a factor of ~30. The high diffusion rates can be further leveraged because whistler-mode waves at the frequencies considered often undergo multiple magnetospheric reflections. A single injection may endure for several seconds and thus can be much more efficiently stored in the magnetospheric cavity as compared to the higher wave frequencies from ground-based transmitters that make only a single traverse of the magnetosphere [*Inan et al.*, 2003].

Kulkarni et al. [2006] expanded on the results presented in *Inan et al.* [2003] to determine source locations in *L*-shell and geomagnetic latitude, operating frequencies and initial wave normal angles needed to fill the inner radiation belts with whistler-mode wave energy. The authors used the Stanford VLF raytracing code [*Inan and Bell*, 1977], coupled with path-integrated Landau damping and a realistic model of dipole antenna radiation in a magnetoplasma. *Kulkarni et al.* [2006] considered both equatorial and off-equatorial source locations, and concluded that three transmitters are sufficient to fill the inner magnetosphere with whistler-mode wave energy, although the authors ignored longitudinal spreading of the rays.

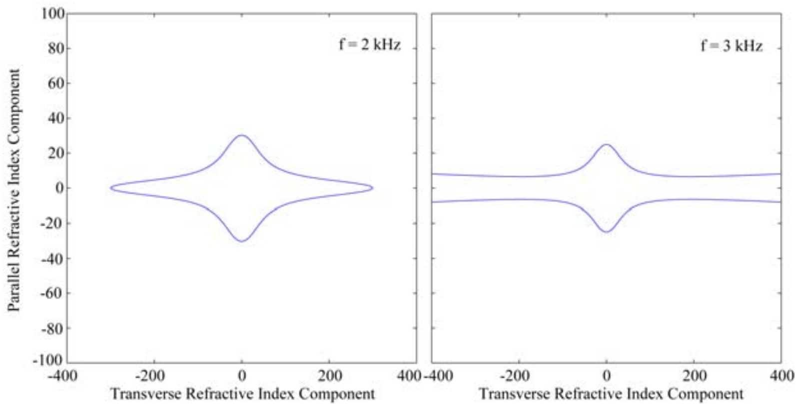
The results of both *Inan et al.* [2003] and *Kulkarni et al.* [2006] laid the groundwork to determine whether *in situ* injection is a feasible means of achieving controlled precipitation of energetic electrons. The former paper highlighted the higher diffusion coefficients and long lifetimes of low-frequency magnetospherically reflecting (MR) whistler-mode waves. The latter identified specific locations and operating frequencies that would be utilized if such a scheme were to be implemented.

In this connection, it is important to note that *Inan et al.* [2003] calculated diffusion coefficients for waves at a constant wave normal angle, ψ, of 45 degrees. Magnetospherically reflecting whistler mode waves, on the other hand, propagate obliquely with ψ near the resonance cone at an angle near 90 degrees. Therefore, the diffusion coefficients presented by *Inan et al.* [2003] may not apply to waves injected from an *in situ* source if these waves have reflected one or more times. If high wave normal angles result in small diffusion coefficients the benefit of multiple reflections may be lost.

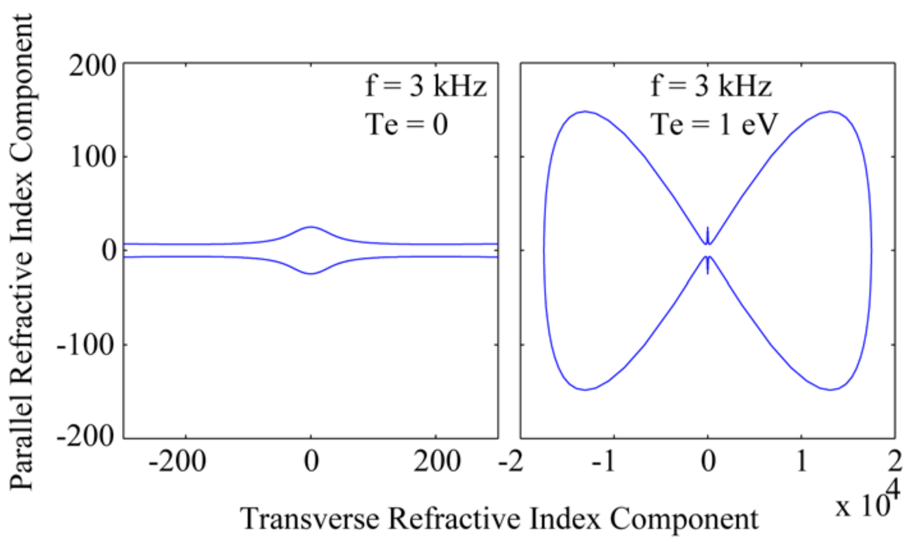
To determine whether magnetospheric reflecting whistler-mode waves injected from an *in situ* source will precipitate energetic electrons, it is necessary to accurately determine the refractive index in an inhomogeneous magnetoplasma. The standard Appleton-Hartree formulation neglects both the electron and ion temperatures, which may have an important effect for waves close to the resonance cone. Below we briefly introduce the effects of including a finite electron temperature.

The Refractive Index Surface

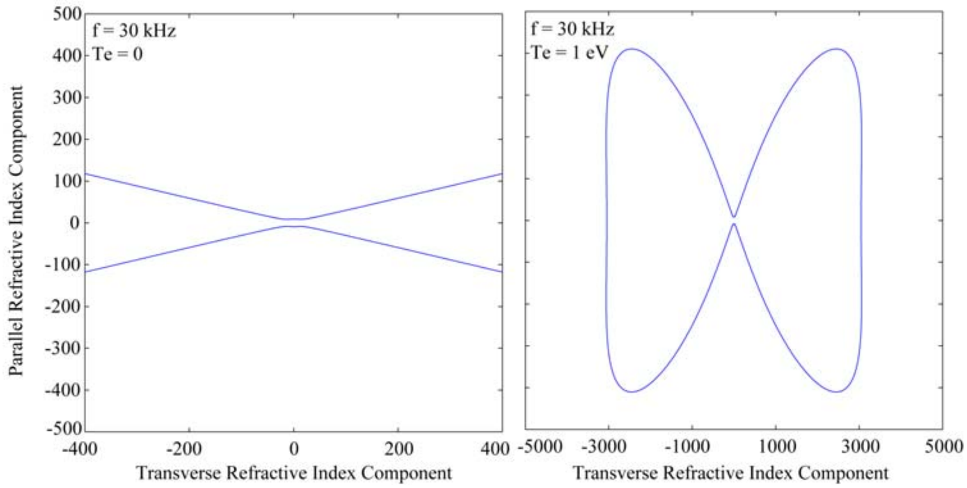
The properties of the magnetoplasma enter the ratracing equations only through the refractive index, μ, and its components. Determining the refractive index is thus critical to analyzing wave propagation in the inner magnetosphere. If the wave frequency is above the local lower hybrid resonance frequency (f_{LHR}), wave propagation cannot occur for wave normal angles beyond the resonance cone angle. Figure 1 below shows the difference between open and closed refractive index surfaces for an equatorial source at L = 2. The f_{LHR} at L= 2 is ~2.5 kHz. Note that the left panel, where the wave frequency is 2 kHz, shows a closed refractive index surface while the right panel, f = 3 kHz, shows an open one.



If the wave is below the f_{LHR}, propagation is possible at all wave normal angles. If, however, we include a finite electron temperature of 1 eV (~11,600 K) while calculating the refractive index, propagation is now allowed for all wave normal angles even when the wave frequency is above the local $f_{LHR.}$ For the 3 kHz wave shown in the right panel above, incorporating Te results in:



Note that the refractive index is now closed and propagation is possible at all wave normal angles. We see a somewhat more pronounced effect when the wave frequency is 30 kHz, substantially higher than the local f_{LHR}:



For 30 kHz waves, there is a more pronounced resonance cone when Te = 0. Including an electron temperature of 1 eV again closes the refractive index. Additional work includes incorporating a finite ion, along with electron, temperature and determining the impact on wave propagation and energetic electron precipitation. For all figures shown here, we have used the formulation specified in *Aubry et al.* [1970].

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