# Whistler Wave Mitigation of Energetic Electrons in the Magnetosphere

***S. P. Kuo***[1]***, Steven S. Kuo***[2]***, and James T. Huynh***[2]

[1]Department of Electrical & Computer Engineering, Polytechnic University, Brooklyn, NY USA
skuo@duke.poly.edu

[2]Northrop Grumman Corporation, One Space Park, Redondo Beach, CA USA
steven.kuo@ngc.com, James.huynh@ngc.com

## Abstract

An approach to reduce the population of energetic (MeV) electrons trapped in the magnetosphere is presented. Under a double resonance condition, a whistler wave is simultaneously in cyclotron resonance with the keV electrons as well as with the MeV electrons. The injected whistler waves are first amplified by the background keV electrons via a loss-cone negative mass instability. The numerical results show that whistler wave can be amplified by more than 25 dB. When the wave amplitude exceeds 0.08 % of the background magnetic field, it becomes effective to precipitate MeV electrons via chaotic scattering, demonstrated by the numerical results.

## 1. Introduction

Energetic electrons (those in the MeV range) in the radiation belts have a strong impact on passing satellite systems. Satellites are designed to survive a certain amount of radiation (ionizing) dose accumulated during their lifetimes. Unexpected enhancement of the radiation fluxes will significantly increase the total radiation dose to the satellites. Consequently, the radiation damage on active electronics and detectors of satellite systems will accumulate much faster than designed for. Therefore, mitigation of energetic electrons in the magnetosphere is an issue of great concern.

Whistler waves can be ducted in an L-shell of the magnetosphere to continuously interact with the energetic electrons trapped in the same L-shell. Induced electron precipitation [1-4] to both hemispheres simultaneously by whistler waves has been observed. Our early work [5-7] shows that the trajectories of trapped energetic electrons in the presence of a whistler wave can become chaotic and wander into the loss cones in the two hemispheres. By far chaotic scattering [8, 9] is the only viable process explaining the experimental observation that there is no apparent time delay between the precipitation events observed simultaneously in conjugate regions [2].

Doppler shifted electron cyclotron resonance enhances wave-electron interaction [10]. It can reduce the threshold wave field for the commencement of chaos in the electron trajectory [9]. It can also work in the reverse way to transfer electron energy to the wave for wave amplification [11]. In this work, we show that there exists a double resonance condition, by which a wave is simultaneously in cyclotron resonance with the keV electrons as well as with the MeV electrons. This suggests an optimal approach, which applies the chaotic scattering process under a double resonance condition, for the control of the population of energetic (MeV) electrons trapped in the magnetosphere. This approach is first to amplify the incident whistler waves by the keV electrons (having a loss-cone velocity distribution) in the radiation belts; the amplified whistler waves then scatter MeV electrons, which are also in cyclotron resonance with the wave, into the loss cones.

## 2. Double Cyclotron Resonances for Effective Precipitation of Very Energetic Electrons by Whistler Waves

Cyclotron resonance is an effective process to enhance the interaction between wave and charge particles and is essential to whistler wave amplification. To take the advantage of simultaneous whistler wave amplification and electron precipitation enhanced by cyclotron resonance, a double resonance condition has to be satisfied, namely, the wave is in cyclotron resonance with energetic (keV level) electrons for being amplified as well as with MeV electrons to instigate precipitation.

The Doppler shifted cyclotron resonance condition is given by

$$\omega = \Omega_0/\gamma + \mathbf{k}\cdot\mathbf{v} \tag{1}$$

where $\omega < \Omega_0$ for whistler waves; $\omega$ and $\Omega_0 = eB_0/mc$ are the wave frequency and the nonrelativistic electron cyclotron frequency, and $\gamma$ is the relativistic factor of the electron. For small $\gamma$, i.e., $\omega < \Omega_0/\gamma$, the resonant electrons are moving oppositely to the wave propagation direction.

To show that a double resonance situation is possible, we first express the resonance condition in a general form $\omega = \Omega_0/\gamma + kP_z/\gamma m$, where $\gamma = (1 + P_\perp^2/m^2c^2 + P_z^2/m^2c^2)^{1/2}$, $P_\perp = \gamma m v_\perp$, $P_z = \gamma m v_z$, and $\mathbf{k} = \hat{\mathbf{z}}\, k$ is assumed. Because of the $\gamma$ dependence, this condition leads to a quadratic equation for $P_z$ as $AP_z^2 + 2BP_z + C = 0$, where $A = (1 - \omega^2/k^2c^2)$, $B = m\Omega_0/k$, and $C = (m/k)^2[\Omega_0^2 - \omega^2(1 + P_\perp^2/m^2c^2)]$. This quadratic equation has two real solutions $P_z = [-B \pm (B^2 - AC)^{1/2}]/A$, subject to the condition $B^2 \geq AC$. The double solutions suggest that the wave can be simultaneously resonant with two different groups of electrons. The coefficients $A$ and $C$ of the quadratic equation are positive because $\omega/kc < 1$ and $\Omega_0/\gamma_{1,2} > \omega$ are considered; thus both $P_z$ are negative, i.e., the two groups of electrons, which can resonantly interact with the wave, move opposite to the propagation direction of the wave.

## 3. Amplification of Whistler Waves

There exists substantial amount of keV electrons in the Van Allen Radiation Belts. These electrons have a loss-cone velocity distribution. Only a small portion of electrons is near cyclotron resonance with the wave. The wave is experiencing cyclotron damping to the electrons which are initially at exact cyclotron resonance with the wave. However, the wave can exchange energy with other electrons having a mismatch frequency slightly different from zero. Depending on the initial phases of those electrons, the interaction can cause them either to gain energy from, or lose energy to, the wave, initially. Their phase will be bunched together through the relativistic effect. Due to the slow wave nature of the whistler wave and the loss-cone distribution of electrons, such a phase bunching leads to a positive feedback to the energy transfer from electrons to the wave. A normalized phase average equation characterizing the temporal evolution of the whistler wave field amplitude X is derived to be [8, 9]

$$[d_t^3 - 48Xf_c\, d_t^2 + (1 + bX^2 + 48f_sX)\, d_t]X = 16[1 - g(X^2 - X_0^2)]f_c \tag{2}$$

where $X_0 = X(0)$; $f_c = \int_0^t X(\xi)\cos\varphi(t, \xi)d\xi$ and $f_s = -\int_0^t X(\xi)\sin\varphi(t, \xi)d\xi$; $\varphi(t, \xi) = \int_\xi^t \{1 - 2g[X^2(\xi') - X_0^2]\}d\xi'$; b and g are constant coefficients; (1) is subjected to the initial conditions: $X(0) = X_0$, $d_\xi X(0) = \sqrt{3}\, X_0$, $d_\xi^2 X(0) = 3X_0$.

Eq. (2) is analyzed numerically with the background parameters, b = 1440 and g = $2\times10^7$, and with the initial field amplitude $X_0 = 3.58\times10^{-4}$; The result is presented in Fig. 1, showing the temporal evolution of the field amplitude $E_0(t)$. The numerical result shows that whistler wave amplitude can be amplified more than 25 dB by trapped relativistic electrons through the loss-cone negative mass instability and oscillates in time, agreeing well with the Siple experimental results [12].

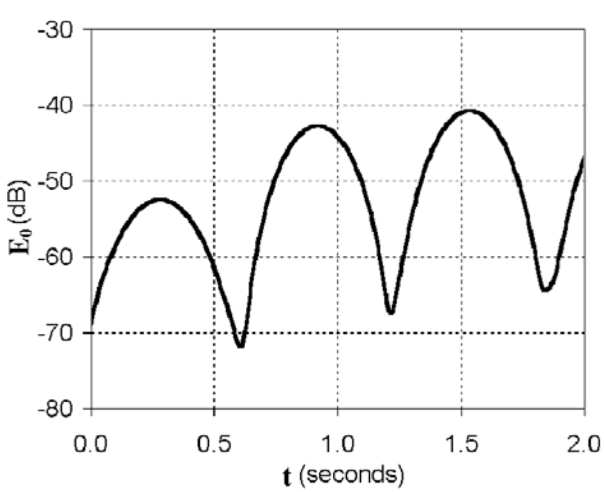


Figure 1. Temporal evolution of the amplitude of a whistler wave traversing the magnetosphere.

## 4. Chaotic Scattering

We now consider the $\gamma = 4$ case that precipitates 1.5 MeV electrons. Equations of motion of electrons in the whistler wave field are now integrated numerically to evaluate the pitch angle scattering, resulting from the wave-electron resonance interaction. Presented in Fig. 2 is a result that double resonance condition is satisfied. As shown, when the wave magnetic field $B_1$ increases to $7 \times (1.16 \times 10^{-4} B_0)$, large pitch angle scattering occurs. In other words, with wave magnetic field amplitude $B_1 = 0.08$ % of the background magnetic filed, electron is scattered to a pitch angle < $50^0$, less than the loss cone angle. This is an example that a whistler wave, with proper parameters, can be amplified by the keV electrons and simultaneously scatters MeV electrons, both processes via the cyclotron resonance interaction. The results show that cyclotron resonant interaction reduces the required field amplitude, for achieving effective chaotic scattering, by a factor of about 10. However, the required interaction time is increased by more than two orders of magnitude.

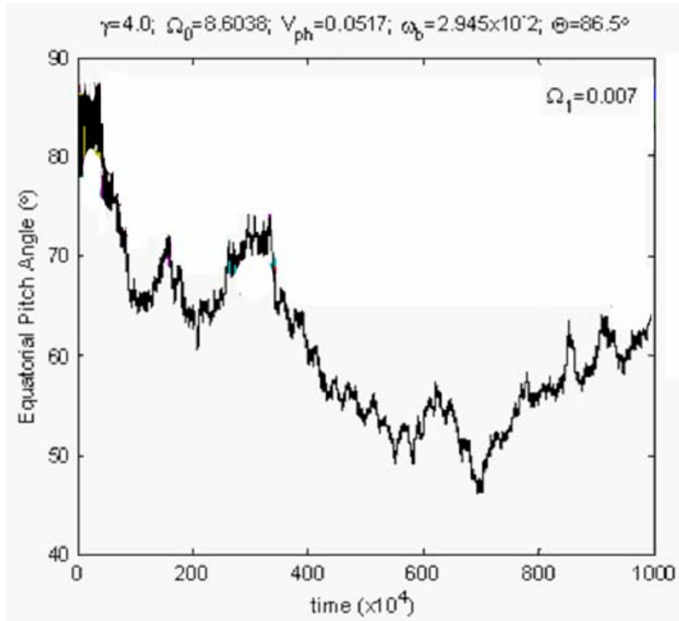


Figure 2. Temporal evolution of the pitch angle of a 1.5 MeV electron, which is interacting with a whistler wave at Doppler shifted cyclotron resonance.

## 5. Summary and Conclusion

It is shown that energetic electrons in the radiation belts can amplify a whistler wave through a loss-cone negative mass instability. The results show that this amplification process can enhance the field intensity of incident whistler wave by about 25 dB. Such an amplification reduces considerably the required field intensity of the incident whistler wave for the purpose of precipitating energetic electrons in the MeV range.

The numerical results demonstrate that a 1.5 MeV electron can be scattered from an initial pitch angle of $86.5^0$ to a pitch angle < $50^0$ by a whistler wave with the magnetic field amplitude of 0.08 % of the background magnetic field. It converts to about 3100 pT at L=2, and 200 pT at L=5. Thus wave amplification is indeed needed and double resonances facilitates the process. Finally, it should be pointed out that this optimal approach, relying on electron cyclotron resonance interaction, requires that the wave have a broad frequency spectrum, so that a considerable fraction of energetic electrons can be precipitated simultaneously.

## Acknowledgments


One of the authors (SPK) is grateful to Dr. Paul Kossey for valuable discussions. This work was supported by the High Frequency Active Auroral Research Program (HAARP), Air Force Research Laboratory at Hanscom Air Force Base, Massachusetts, and by the Office of Naval Research, Grant No. ONR-N00014-05-1-0109. Part of the financial support was arranged through NorthWest Research Associates, Inc.


## References


1. R. L. Arnoldy and P. M. Kintner, “Rocket Observations of the Precipitation of Electrons by Ground VLF Transmitters,” *J. Geophys. Res*., **94**, 1989, pp. 6825-6832.

2. W. C. Burgess and U. S. Inan, “Simultaneous Disturbance of Conjugate Ionospheric Regions in Association with Individual Lightning Flashes,” *Geophys. Res. Lett*., **17**, 1990, pp. 259-262.

3. W. L. Imhof, R. M. Robinson, H. L. Colin, J. R. Wygant, and R. R. Anderson, “Simultaneous Measurements of Waves and Precipitating Electrons Near the Equator in the Outer Radiation Belt,” *J. Geophys. Res.*, **99**, 1994, pp. 2415-2427.

4. Pradipta Rezy, A. Labno, M. C. Lee, W. J. Burke, J. A. Cohen, L. M. Burton, S. P. Kuo, D. Rokusek, “Electron Precipitation from the Inner Radiation Belt above Arecibo”, *Geophys. Res. Lett.*, **34**, 2007, pp. L08101, doi:10.1029/2007GL029807.

5. J. Faith, S. P. Kuo, and J. Huang, “Chaotic Electron Motion Driven by Whistler Waves in the Magnetosphere,” *Comments Plasma Phys. Controlled Fusion*, **17**, 1996, pp. 173-182.

6. J. Faith, S. P. Kuo, and J. Huang, “Electron Precipition Caused by Chaotic Motion in the Magnetosphere due to Large Amplitude Whistler Waves,” *J. Geophys. Res.*, **102**, 1997, pp. 2233-2241.

7. J. Faith, S. P. Kuo, J. Huang and G. Schmidt, “Precipitation of Magnetospheric Electrons Caused by Relativistic Effect Enhanced Chaotic Motion in the Whistler Wave Fields,” *J. Geophy. Res.*, **102**, 1997, pp. 9631-9638.

8. S. P. Kuo, Paul Kossey, James T. Huynh, and Steven S. Kuo, “Amplification of Whistler Waves for the Precipitation of Trapped Relativistic Electrons in the Magnetosphere”, *IEEE Trans. Plasma Sci.*, **32**, 2004, pp. 362-369.

9. S. P. Kuo, Steven, S. Kuo, James T. Huynh, and Paul Kossey, “Precipitation of Trapped Relativistic Electrons by Amplified Whistler Waves in the Magnetosphere”, *Phys. Plasmas*, **14**, 2007, pp. 009706 (1-7).

10. J. M. Albert, “Gyroresonant Interactions of Radiation Belt Particles with a Monochromatic Electromagnetic Wave,” *J. Geophys. Res*., **105**, 2000, pp. 21191-21205.

11. S. P. Kuo and M. C. Lee, “Saturation of Cyclotron Maser Instability Driven by an Electron Loss-Cone Distribution,” *Int'l Journal of Infrared and Millimeter Waves*, **7**, 1986, pp. 623-633.

12. R. A. Helliwell, D. L. Carpenter, and T. R. Miller, “Power Threshold for Growth of Coherent VLF Signals in the Magnetosphere,” *J. Geophys. Res.*, **85**, 1980, pp. 3360.