# Influence of Charge Dynamics on the Internal Field and Current Distribution in a Conductive Particle

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***Abstract***---Coupling of the charge transport equation to the Maxwell's equations provides a realistic description of the long-wavelength polarization within a conductive spherical particle, allowing the boundary condition for charge motion at the surface to be accounted for. While the electric field and local charge density can be obtained with quasi-static analysis, a full-wave formulation is found to be necessary to describe the current distribution within the spherical particle.

## 1. Introduction

The interaction of a particle with an electromagnetic wave with large wavelength can be accounted for by considering the dipole polarizations induced on the particle, from which the scattering amplitude can be calculated [1] [2]. The electric dipole moment for a spherical particle can be written in terms of the complex permittivity of the material, which can account for dispersion and dissipation effects of the particle arising from the dynamics of its constituent polar molecules that exhibit displacement or orientation polarizations [3]. The formulation provides a realistic description of the polarization for bound charges, including the case when the frequency is lowered to the static limit. In the presence of mobile charges, an extension of the complex permittivity to include the conductivity can be employed to account for phenomena not influenced by material boundaries, an example being the absorption peak near the bulk plasma frequency exhibited by metallic particles in the optical region. On the other hand, the use of dielectric functions based on bulk properties will show its limitation when surface effects are significant. This can be seen by considering the dipole moment of a particle with finite conductivity at very low frequencies. A complex permittivity based on bulk conductivity as given by the Drude model would lead to a dipole moment the same as a perfect electric conductor sphere when a sufficiently low frequency is reached, even though the concentration of mobile charge is low. This difficulty can be overcome by employing a transport formulation to account for the dynamics of the mobile charges, for which the equations of motion of the charges are employed in stead of the bulk conductivity [4]. It was observed in results of calculations based on transport formulation that the low frequency polarization for a spherical particle with moderate charge concentration could be realistically accounted for, while tending to the limit of an ideal dielectric sphere as the charge concentration was made vanishingly small.

Besides providing a realistic description of the lower frequency polarization in conductive particles, a transport formulation is also important in accounting for the internal field of the conductive particle. In this paper, the results of an investigation on the internal electric field and current distribution in a spherical particle with mobile charges are presented. It is also the purpose of this discussion to emphasize the need for a full-wave solution of the Maxwell's equations when the current distribution in a highly conductive body is of interest.

## 2. Mathematical Model

The spherical particle containing mobile charges is assumed to be placed inside a dynamic electric field with wavelength large compared to the radius of the sphere. In this study, the mobile charges are taken to be electrons, although the presence of holes and other conducting species such as ions can be incorporated when additional equations are introduced. The current density for the electrons can be written as

$$\frac{\partial(n\boldsymbol{v_n})}{\partial t} = -\frac{kT}{m_n}\nabla n - \frac{qn}{m_n}\boldsymbol{E} - \frac{n\boldsymbol{v_n}}{\tau_n} \tag{1}$$

$$\boldsymbol{J_n} = -qn\boldsymbol{v_n} \tag{2}$$

which represents a drift-diffusion model following from the first moment of the Boltzmann transport equation. In (1), $n$ is the local concentration of the electron, $v_n$ is the velocity, and $\tau_n$ is its relaxation time . For particle conservation, the continuity equation

$$\nabla \cdot \boldsymbol{J_n} = q\frac{\partial n}{\partial t} + qR_n \tag{3}$$

is introduced, for which $R_n = \frac{n}{t_n}$ , and $t_n$ is the electron lifetime. These two equations are coupled with the inhomogeneous Helmholtz equation

$$\nabla^2\boldsymbol{E} + k^2\boldsymbol{E} = \frac{\nabla\rho}{\varepsilon} + j\omega\mu\boldsymbol{J} \tag{4}$$

which follows from the Maxwell's equations with harmonic time dependence. This system of vector equations were solved with material parameters typical for semiconductor material, under the condition that the electron cannot escape the surface of the sphere, along with the usual boundary conditions for the electric field components at material boundary.

## 3. Numerical Results

The numerical results are obtained for a semiconductor sphere of 0.5 μm radius, with a relative permittivity $\varepsilon_r$ = 11.9, electron mobility $\mu_n = 1500 cm^2/V-s$ , electron momentum relaxation time $\tau_n = 2.156\times10^{-13} s$, and electron lifetime $t_n = 0.0025 s$. The applied electrical field is 1.0 $V/cm$ at $f = 100 GHz$ . The rapid decrease in the field intensity as one moves towards the center of the sphere is mainly caused by the screening effect of the mobile electron, which is also observed in the static case. If the mobile charges were absent, the internal field would only be influenced by the attenuation of the field due to dielectric loss, which is a dynamic effect and accounted for by the skin depth.

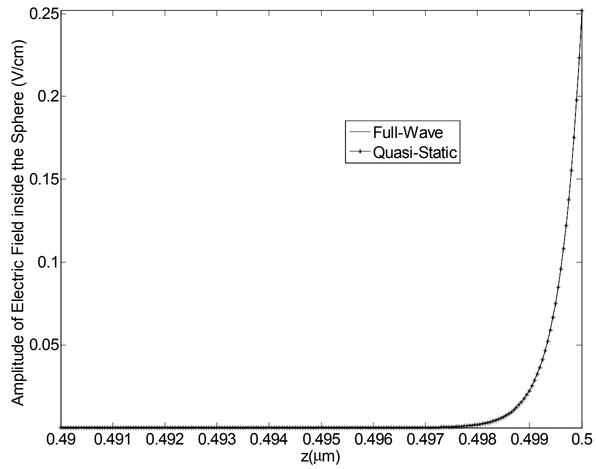


Figure 1. Electric field inside the semiconductor sphere with $N_D = 10^{20}\,cm^{-3}$ along the $\hat{r}$ Axis at $\theta = 0$ .

It was found that when the wavelength is large compared to the radius of the sphere, the quasi-static solution can give accurate accounts of both the external and internal fields of the spherical particle. As the charge concentration in the sphere is increased, the wavelength within the particle is further decreased, possibly making it less than the radius of the sphere. One would tend to expect that the quasi-static formulation can no longer account for the internal field. However, results of calculations indicated that as far as the field intensity (and hence local charge concentration) is concerned, the screening effect of the charge has a characteristic length that decreases much faster than the wavelength and the skin depth in the sphere as it is made to be more conductive by increased doping. Hence there is good agreement between the full-wave solution and the quasi-static formulation.

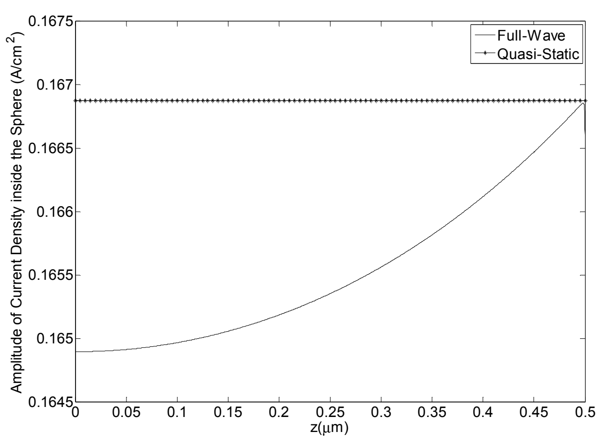


Figure 2. Current density inside the semiconductor sphere with $N_D = 10^{20}\,cm^{-3}$ along the $\hat{r}$ axis at $\theta = \pi / 2$

The same is not true for the current density, which showed different distributions for full-wave and quasi-static formulations. The calculated current distributions within the sphere are shown in Fig. 2. It can be seen that the quasi-static formulation gives rise to an essentially constant current that flows through the center of the sphere, while the full-wave solution leads to a current distribution that has less presence in at the center of the sphere. A sectional view of the current flow within the sphere is illustrated in Fig. 3.

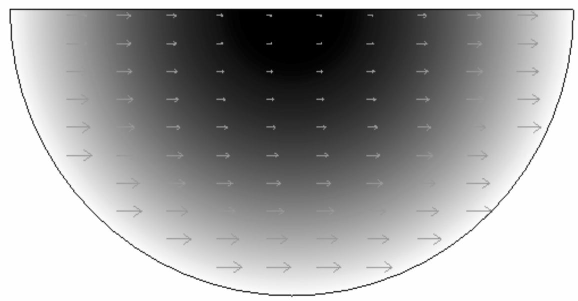

Figure 3. Sectional view of current distribution with the spherical particle as calculated by a full-wave formulation.

## 4. Summary

Employing a transport-based formulation to describe the motion of electrons within a spherical particle immersed in a dynamic electric field with a large wavelength, the internal field and current distribution were obtained by seeking numerical solution to the system of coupled vector equations involving the electric field, charge density and current density, subject to boundary conditions on the particle surface. It was found that the full-wave solution and the quasi-static solution gave good agreement on the field intensity and charge distribution for all practical levels of impurity concentration (doping) in a semiconductor particle. The screening effect of the electron cause the field to decay at a faster rate than that resulted from the attenuation effect due to energy dissipation. On the other hand, a quasi-static formulation led to a current density that essentially stayed constant across the interior of the particle. To realistically describe the current density for particle with high electron concentration (doping level), the full set of Maxwell's equations need to be employed. As the electron concentration was increased, numerical results obtained by full-wave formulation showed that the current density would progressively crowd towards the surface of the particle, eventually revealing itself as a surface current in the highly conductive case as found in metals.

**References:**


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