Ion acoustic and electron acoustic solitons in multi-component space plasmas

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Abstract

Properties of ion- and electron- acoustic solitons are investigated in an unmagnetized multi-component plasma system consisting of cold and hot electrons and hot ions using Sagdeev pseudo-potential technique. The analysis is based on fluid equations and the Poisson equation. The solitary wave solutions are found when the Mach numbers exceed some critical values. The critical Mach numbers for the ion-acoustic solitons are found to be smaller than those for electron acoustic solitons for a given set of plasma parameters. The critical Mach numbers of ion acoustic solitons increase with the increase of hot electron temperature and the decrease of cold electron density. On the other hand, the critical Mach numbers of electron acoustic solitons increase with the increase of the cold electron density as well as hot electron temperature. The ion acoustic solitons have positive potential for the parameters considered. However, the electron acoustic solitons have positive or negative potentials depending whether the cold electron density is greater or less than the hot electron density. Further, the amplitudes of both the ion and electron acoustic solitons increase with the increase of hot electron temperature. Possible applications of this model to the electrostatic solitary structures (ESWs) observed in different regions of the Earth's magnetosphere by several spacecraft observed will be discussed.

1. Introduction

Electrostatic solitary waves (ESWs) have been observed throughout the Earth's magnetosphere at narrow boundaries, e.g., the plasma sheet boundary layer (Matsumoto et al, 1994), polar cap boundary layer (Tsurutani et al., 1998), the magnetosheath (Pickett et al., 2003), the bow shock (Bale et al., 1998), and in strong currents, such as those associated with the auroral acceleration region (Ergun et al., 1998). The electrostatic solitary structures are found in the electric field parallel to the background magnetic field, and are usually bipolar or tripolar. The solitary structures found in the ion beam regions of the auroral zone (Temerin et al., 1982) have usually negative potentials. These have been interpreted in terms of ion solitary waves or solitons (Reddy et al., 1992).

In other regions mentioned above, the solitary waves are usually positive potential structures moving at velocities comparable to the electron thermal velocity (~ 1000s of km s^{-1}) and are commonly interpreted to be either electron holes, such as BGK modes arising from the evolution of a bump on tail instability /or electron two stream instability or in terms of electron acoustic solitary waves (Dubouloz et al., 1991, Singh et al., 2001; Singh and Lakhina, 2001, 2004; Tagare et al., 2004). For a detailed discussion of various models, one can refer to Lakhina et al. (2003).

The earlier models based on electron acoustic solitons could explain the space observations (e.g. Viking) of solitary waves which had negative potentials in two/three temperature electron plasmas (Pottelette et al., 1990; Dubouloz et al., 1991, Singh et al., 2001; Singh and Lakhina, 2004; Tagare et al., 2004). To explain the positive structures, attempts have been made to study electron acoustic solitons in three electron (cold, hot, beam) component plasmas (Berthomier et al., 2000). These models show that depending on the beam density and temperature and below a critical velocity of the electron beam, nonlinear structures can have a positive potential signature. Recently, Verheest et al. (2005) have pointed out the possibility to obtain compressive electron acoustic solitons, those having positive potentials, even without the electron beam component, provided the hot electron inertia is retained in the analysis. More recently, Kakad et al. (2007) have studied electron acoustic solitons in a four-component unmagnetized plasma system consisting of cold background electrons, a cold electron beam, and two types of ion species, i.e., cold and hot ions having Boltzmann distribution. This model predicts the coexistence of rarefactive and compressive electron acoustic solitary modes for specific plasma parameters.

Observations indicate that both ion and electron beams can drive the broadband electrostatic. These solitary waves have amplitudes typically a few mV/m in the plasma sheet boundary layer, but they can be as large as 200 mV/m at Polar altitudes. Such nonlinear solitary structures observed in the plasma sheet boundary layer and on auroral field lines s may play a key role in supporting parallel electric fields in these regions. The spacecraft observations in the

Earth's plasma sheet boundary layer show the existence of cold and hot electrons (or some times electron beams) having energies of the order of a few eV to a few keV, respectively, and background cold ions and warm ions/ion beams with energies from a few keV to tens of keV. Here, we study a 3-component plasma system consisting of cold and hot electrons and hot ions. We develop a general formalism employing a multi-fluid approach for all the species. Thus, the restrictive assumption of treating the hot electrons and/ or hot ions as having Boltzmann distribution considered in many earlier studies (e.g., Singh et al., 2001; Kakad et al, 2007) is removed in the present analysis. Further, each species can have an arbitrary beam velocity.

2. Model

We consider an infinite, collisionless and unmagnetized plasma consisting of three components, namely, cold electrons (N_{ce}, T_{ce}, v_{ce}), hot electrons (N_{he}, T_{he}, v_{he}), and ions (N_i, T_i, v_i), where N_j, T_j, v_j represents the density, temperature and beam velocity (along the direction of wave propagation) of the species j, and j=ce, he and i for the cold electrons, hot electrons and the ions, respectively. We treat all the species as mobile. Then, their dynamics is governed by the multi-fluid equations of continuity and momentum of each species, and the Poisson equation.

To study the properties of stationary arbitrary amplitude EASWs, we transform the equations to a stationary frame moving with velocity V, the phase velocity of the wave, i.e., $\xi = (x - M\,t)$, where $M=V/C_i$ is the Mach number with respect to the ion thermal speed, $C_i = (T_i/m_i)^{1/2}$ (here, m_j represent the mass of j^{th} species). Then, solving for perturbed densities, putting these expression in the Poisson equation, and assuming appropriate boundary conditions for the localized disturbances along with the conditions that $\phi = 0$, and $d\phi/d\xi=0$ at $\xi \to \pm\infty$, we get the following energy integral

$$\frac{1}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2 + \psi(\phi) = 0 \qquad (1)$$

where,

$$\psi(\phi) = \mu_{ei} n_{ce}^0 \left\{ (V - v_{ce})^2 - \frac{(V - v_{ce})}{\sqrt{2}} B_{ce}^{\;1/2} \right\} + n_{ce}^0 T_{ce} \left\{ 1 - 2\sqrt{2}\,(V - v_{ce})^3 B_{ce}^{\;-3/2} \right\} + \mu_{ei} n_{he}^0 \left\{ (V - v_{he})^2 - \frac{(V - v_{he})}{\sqrt{2}} B_{he}^{\;1/2} \right\}$$
$$+ n_{he}^0 T_{he} \left\{ 1 - 2\sqrt{2}\,(V - v_{he})^3 B_{he}^{\;-3/2} \right\} + n_i^0 \left\{ (V - v_i^0)^2 - \frac{(V - v_i^0)}{\sqrt{2}} B_i^{1/2} \right\} + n_i^0 \left\{ 1 - 2\sqrt{2}\,(V - v_i^0)^3 B_i^{-3/2} \right\} \qquad (2),$$

is the Sagdeev potential. Here,

$$B_{ce} = A_{ce} + \sqrt{A_{ce}^2 - \frac{12 T_{ce} (V - v_{ce})^2}{\mu_{ei}}}, B_{he} = A_{he} + \sqrt{A_{he}^2 - \frac{12 T_{he} (V - v_{he})^2}{\mu_{ei}}}, B_i = A_{hi} + \sqrt{A_i^2 - 12 (V - v_i)^2},$$
$$A_{ce} = (V - v_{ce}^0)^2 + \frac{3T_{ce}}{\mu_{ei}} + \frac{2\phi}{\mu_{ei}}, \quad A_{he} = (V - v_{he}^0)^2 + \frac{3T_{he}}{\mu_{ei}} + \frac{2\phi}{\mu_{ei}}, \quad A_i = (V - v_{hi}^0)^2 + 3 - 2\phi, \quad \mu_{ei} = \frac{m_e}{m_i}.$$

Further, in eq. (2), the densities are normalized with respect to the equilibrium ion densities, i.e., $n_j^0 = N_j/N_i$ such that $n_{ce}^0 + n_{he}^0 = n_i^0 = 1$, and the velocities and temperatures of the species are normalized with ion thermal velocity and temperature, respectively. In order to have soliton solutions, the pseudo potential $\Psi(\phi)$ must satisfy the following conditions:

$$\psi(\phi) = 0, \quad \left(\frac{\partial\psi(\phi)}{\partial\phi}\right) = 0, \quad \left(\frac{\partial^2\psi(\phi)}{\partial\phi^2}\right) < 0 \text{ at } \phi = 0,\ \psi(\phi) = 0 \text{ at } \phi = \phi_0 \text{ and } \psi(\phi) < 0 \text{ for } 0 < |\phi| < \phi_0.$$

3. Results and discussion

To study the ion and electron acoustic soliton, we have numerically solved Eq. (2) for the Sagdeev potential $\Psi(\phi)$, for the typical parameters, which are shown in Figure1 and Figure 2. Here we are presenting a simple case where all beam velocities considered to be zero. In both the category, the solitary wave solutions are found when the Mach numbers exceed some critical values. Figure 1(a) shows the variation of Sagdeev potential associated with compressive

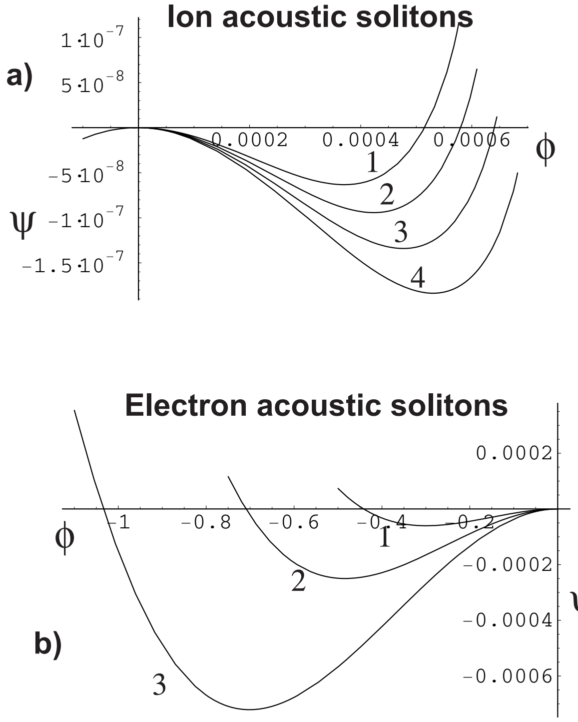

Figure 1: Ion acoustic (panel a) and electron acoustic (panel b) solitons for plasma parameters: N_{ce}/ N_i =0.3, T_{ce}/ T_i =0.01, and T_{he}/ T_i =5.0. For panel a, the Mach number M= 1.7660, 1.7670, 1.7680, 1.7690 for the curves 1, 2, 3, and 4, respectively. For panel b, the Mach number M= 93.0, 94.0, and 95.0 for the curves 1, 2, and 3, respectively.

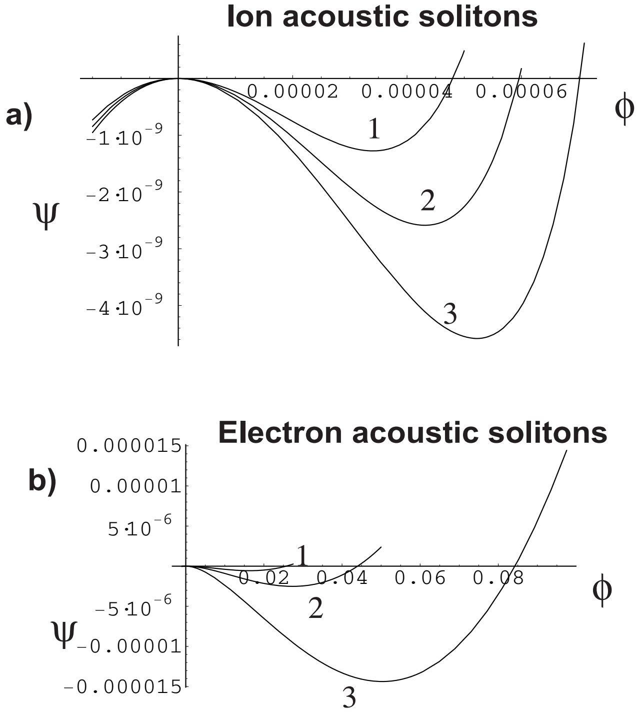

Figure 2: Ion acoustic (panel a) and electron acoustic (panel b) solitons for plasma parameters: N_{ce}/ N_i =0.9, T_{ce}/ T_i =0.01, and T_{he}/ T_i =5.0. For panel a, the Mach number M= 1.7430, 1.7435, 1.7440, for the curves 1, 2, and 3, respectively. For panel b, the Mach number M= 159.0, 160.0, 162.0, for the curves 1, 2, and 3, respectively.

ion acoustic solitons for different Mach number, M. It is seen that the maximum electrostatic potential ϕ increases with the increase of the Mach number, M. Variation of Sagdeev potential associated with rarefactive electron acoustic solitons for different Mach number, M are shown in figure 1(a). In this case it is found that the maximum electrostatic potential ϕ increases with the increase of the Mach number, M. Also, the critical Mach numbers for the ion-acoustic solitons are found to be smaller than those for electron acoustic solitons for a given set of plasma parameters. For the considered set of parameters in the numerical computation, this model supports only compressive ion acoustic solitons. However, it is interesting to note that, for the existence of compressive and rarefactive electron acoustic solitons the cold electron to ion density ratio (N_{ce}/ N_i) plays an important role. It is found that for the small value of (N_{ce}/ N_i) this model supports rarefactive electron acoustic solitons (shown in figure 1b), whereas it supports compressive electron acoustic solitons for the large value of (N_{ce}/ N_i) which is shown in figure 2(b).

The plasma sheet boundary layer of the Earth's magnetotail is found to contain multi plasma species, e.g. cold electrons, electron beams, cold ions and ion beams. This is, therefore, one of the situations where the model developed in this paper may be applicable. Considering the typical parameters, namely T_{he}=2.5 keV, T_i = 500 eV, n_o=0.1 cm^{-3}, we have calculated the associated electric field with the ion and electron acoustic solitons. For the typical electric field associated with compressive ion acoustic solitons is found to be in the range of (0.05 – 0.6) mV/m. Also the typical electric field associated with compressive and rarefactive electron acoustic solitons is found to be in the range of (13–36) mV/m and (1–6) mV/m respectively.

In this paper, we have theoretically investigated the nonlinear propagation of ion- and electron- acoustic solitons in an unmagnetized multi-component plasma system consisting of cold and hot electrons and hot ions using

Sagdeev pseudo-potential technique. To bring the model closer to the observations, in this new model, we have considered the dynamics for both the ions and electrons. Using particular boundary conditions, we have restricted our study for electron and ion acoustic solitons, which supports the bipolar electric field structures. The application of our model might be particularly interesting to provide interpretation of recent observations of solitary waves in the Earth's plasma sheet boundary layer region.

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