3D Numerical Modeling of the Ionospheric Electron Density Based on the FormoSat-3/COSMIC GPS radio occultation Data

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Abstract

Using the Abel inversion through compensated TEC values from FormoSat-3(FS3)/COSMIC, we have collected averagely eighteen hundreds of vertical electron density (\(n_e\)) profiles per day. Here we present a two-dimensional (2D) approach from the surface spherical harmonics analysis for mapping fitted Chapman-layer parameters including peak density, peak density height, and scale height. Another approach can be generalized to the 3D case (2D surface spherical harmonics and vertical empirical orthogonal functions, EOFs) for modeling the ionospheric \(n_e\). These results would be on investigation of near-Earth space plasma distribution in different seasons and under different solar-geophysical conditions.

1. Spherical harmonic analysis of ionospheric characteristics mapping

The construction of particular solution of the Laplace’s partial differential equation on the surface of a sphere (assumed independent of time) is known as the spherical surface harmonics. After least square error fitting of \(n_e\) profiles by Chapman layers, the numerical maps for fitted parameters, e.g. peak density (\(\text{critical plasma frequency}\)), peak density height, and scale height, can denote a spherical harmonic function, \(\Gamma(\theta, \phi)\), in spherical coordinates of \(0^\circ \leq \theta \leq 180^\circ\) and \(0^\circ \leq \phi \leq 360^\circ\), and the spherical surface Laplace equation is

\[
\frac{1}{\sin \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Gamma}{\partial \theta} \right) \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 \Gamma}{\partial \phi^2} = 0.
\]

where \(\theta\) is the geographic latitude added by 90°, \(\phi\) could be the geographic longitude (in the Earth coordination) or the local time angle (in the local time system coordination) measured westward from apparent noon. The resulting real functions obtained on separation of variables and the Gram-Schmidt orthogonalization process are defined by

\[
U_{nm}(\theta, \phi) = \sqrt{\frac{2n+1}{2(n+m)!}} P_n^m(\cos \theta) \cos m \phi, \text{ and}
\]

\[
V_{nm}(\theta, \phi) = \sqrt{\frac{2n+1}{2(n+m)!}} P_n^m(\cos \theta) \sin m \phi
\]

\((n = 0, 1, 2, \ldots; m = 0, 1, 2, \ldots, n),\)

where \(P_n^m()\) is the familiar associated Legendre polynomial of the first kind of degree \(n\) and order \(m\). The geographic variation of peak density (\(\text{critical plasma frequency}\)), peak density height, or scale height can thus be represented by a series of functions, \(U_{nm}(\theta, \phi)\) and \(V_{nm}(\theta, \phi)\), analogous to the Earth surface. And then, a three-dimensional \(n_e\) distribution can be represented by
 Profiles could not be obtained. For the remaining RO measurements the atmospheric excess phase calibration failed and the n_e profiles could not be obtained. The failed measurements were usually caused by the locking onto the GPS carrier signals either starting too late or ending too early.

As described, there are on the average about eighteen hundred n_e profiles and corresponding foF2 and hmF2 retrieved per day within the FS3/COSMIC mission. In the following, we have applied spherical harmonics analyses on monthly foF2 and hmF2 values at the geographic coordinate and within every local time (LT) hour. Example results of foF2 and hmF2 numerical maps are illustrated by the two-dimensional images in Figures 1 and 2, respectively, using the FS3/COSMIC data in July, 2007. From Figures 1 and 2, the solid points denote the line-of-viewing tangent point locations at the F2 peak of RO observations, and three curves in black present positions with magnetic dip latitudes of +20°, 0°, and -20° from top to down. The top-left, top-right, upper middle-left, upper middle-right, lower middle-left, lower middle-right, bottom-left, and bottom-right images in Figure 1 (Figure 2) represent the 2007 July foF2 (hmF2) numerical map at 2, 5, 8, 11, 14, 17, 20, and 23 LT, respectively. From Figure 1 the highest-latitude area at south is in continue darkness and has lower n_e than the highest-latitude area at north where is in continue daytime because of summer season. Features of the diurnal ionosphere variations are obtained and illustrated as follows. The foF2s are greater by day than by night; foF2 is increased promptly after sunrise and continues to increase for a few hours after noon and then after is decreased. There is a general tendency for hmF2 to fall at dawn and then to rise during the afternoon or evening. In low latitude and equator, hmF2 reaches a high level by about 14 LT and then falls such that in late night it is about 100 km lower than in afternoon, and the F2 layer is much thicker near the Equator than elsewhere particularly in daytime. Furthermore, we note that, in equatorial and low-latitude regions, features of the equatorial anomaly are observed in daytime, and two foF2 crests lie along the +20° and -20° magnetic dip latitudes. The features are developed after sunrise and become strongest at about 14 LT. It is well known that the equatorial anomaly results are something of equatorial fountain features and are caused by an eastward electric field at the magnetic equator creating a steady upward \( E \times B/B^2 \) plasma drift. The drift raises plasmas from the equator across magnetic field lines to higher altitudes. After losing momentum, the electrons slide down, assisted by gravity, along the field lines to either side of the equator to form two crests. This is called the fountain effect. One important factor is the \( E \times B \) ionization particularly matches to magnetic dip latitude and north-south asymmetry effects are incorporated and shown in Figures 1 and 2 too. Theoretically, in the absence of neutral winds, the upward \( E \times B/B^2 \) drift produces almost identical effects at conjugate points; however, neutral winds can cause

\[
n_e(\theta, \phi, h) = \sum_{i=1}^{n} n_{e_{\text{max}}}(\theta, \phi) \exp\left( \frac{\mu_i}{h} \right) \exp\left( \frac{\mu_i}{H} \right)
\]

where each \( i \) means a physical layer of F2-, F1-, E-, or D-layer, and the peak density \( (n_{e_{\text{max}}}) \), peak height \((h_m)\), and scale height \((H)\) are combinations of surface spherical harmonics.

2. Example results of \( foF2 \) and \( hmF2 \) mapping

In the FS3/COSMIC mission, all six spacecraft (FM1 to FM6) were integrated and launched together into a parking orbit of 515 km altitude; whereupon each spacecraft was separated and then transferred from the parking orbits to their final orbits at ~300 km. The final spacecraft orbits will be separated into 30 degrees and the daily radio occultation (RO) measurements on board FS3/COSMIC can be distributed uniformly onto the entire atmosphere and/or ionosphere after 2007. Generally FS3/COSMIC can perform over 2500 RO measurements per day, and more than 70% of the RO measurements can be successfully retrieved into \( n_e \) profiles. For the remaining RO measurements the atmospheric excess phase calibration failed and the \( n_e \) profiles could not be obtained. The failed measurements were usually caused by the locking onto the GPS carrier signals either starting too late or ending too early.

As described, there are on the average about eighteen hundred \( n_e \) profiles and corresponding \( foF2 \) and \( hmF2 \) retrieved per day within the FS3/COSMIC mission. In the following, we have applied spherical harmonics analyses on monthly \( foF2 \) and \( hmF2 \) values at the geographic coordinate and within every local time (LT) hour. Example results of \( foF2 \) and \( hmF2 \) numerical maps are illustrated by the two-dimensional images in Figures 1 and 2, respectively, using the FS3/COSMIC data in July, 2007. From Figures 1 and 2, the solid points denote the line-of-viewing tangent point locations at the F2 peak of RO observations, and three curves in black present positions with magnetic dip latitudes of +20°, 0°, and -20° from top to down. The top-left, top-right, upper middle-left, upper middle-right, lower middle-left, lower middle-right, bottom-left, and bottom-right images in Figure 1 (Figure 2) represent the 2007 July \( foF2 \) (\( hmF2 \)) numerical map at 2, 5, 8, 11, 14, 17, 20, and 23 LT, respectively. From Figure 1 the highest-latitude area at south is in continue darkness and has lower \( n_e \) than the highest-latitude area at north where is in continue daytime because of summer season. Features of the diurnal ionosphere variations are obtained and illustrated as follows. The \( foF2 \)s are greater by day than by night; \( foF2 \) is increased promptly after sunrise and continues to increase for a few hours after noon and then after is decreased. There is a general tendency for \( hmF2 \) to fall at dawn and then to rise during the afternoon or evening. In low latitude and equator, \( hmF2 \) reaches a high level by about 14 LT and then falls such that in late night it is about 100 km lower than in afternoon, and the F2 layer is much thicker near the Equator than elsewhere particularly in daytime. Furthermore, we note that, in equatorial and low-latitude regions, features of the equatorial anomaly are observed in daytime, and two \( foF2 \) crests lie along the +20° and -20° magnetic dip latitudes. The features are developed after sunrise and become strongest at about 14 LT. It is well known that the equatorial anomaly results are something of equatorial fountain features and are caused by an eastward electric field at the magnetic equator creating a steady upward \( E \times B/B^2 \) plasma drift. The drift raises plasmas from the equator across magnetic field lines to higher altitudes. After losing momentum, the electrons slide down, assisted by gravity, along the field lines to either side of the equator to form two crests. This is called the fountain effect. One important factor is the \( E \times B \) ionization particularly matches to magnetic dip latitude and north-south asymmetry effects are incorporated and shown in Figures 1 and 2 too. Theoretically, in the absence of neutral winds, the upward \( E \times B/B^2 \) drift produces almost identical effects at conjugate points; however, neutral winds can cause
conjugate-hemisphere differences by modulating the fountain and moving ionospheric electrons at the conjugate hemispheres to different altitudes.

Figure 1. The 2007 July $f_{o}F2$ numerical maps at 2, 5, 8, 11, 14, 17, 20, and 23 LT are respected to the top-left, top-right, upper middle-left, upper middle-right, lower middle-left, lower middle-right, bottom-left, and bottom-right images, respectively. The solid points denote the line-of-viewing tangent point locations at the F2 peak of RO observations, and three curves in black present magnetic dip latitudes of +20°, 0°, and -20° from top to down.
Figure 2. The 2007 July hmF2 numerical maps at 2, 5, 8, 11, 14, 17, 20, and 23 LT are respected to the top-left, top-right, upper middle-left, upper middle-right, lower middle-left, lower middle-right, bottom-left, and bottom-right images, respectively.