Cascade of Langmuir Waves in HAARP Heating Experiments

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Abstract

The feasibility of cascading Langmuir waves via lower hybrid decay modes in HAARP heating experiments is studied. This parametric instability process cascades Langmuir waves in a narrow layer located slightly below the O-mode HF reflection height, to produce a broad spectrum of high-frequency heating wave enhanced plasma lines (HFPLs).

1. Introduction

In HF heating experiments, Langmuir waves and ion waves can be excited parametrically by the HF heater [1-8]; Langmuir waves can grow to large amplitudes to excite additional parametric instabilities which cascade Langmuir waves to broaden the frequency spectrum.

Backscatter radars detect some of the excited plasma lines, which have their wave numbers twice the wave number of the probing radar signal and propagate parallel or anti-parallel to the radar pointing direction. These spectral lines in radar returns are termed "HF wave enhanced plasma lines (HFPLs)", which usually reveal discrete cascade feature and originate from spread layers of altitude [9-14]

A major facility for conducting ionospheric heating experiments is available in Gakona, Alaska, as part of the High Frequency Active Auroral Research Program (HAARP) [15]. HAARP HF transmitting system is capable to deliver effective radiated power (ERP) of 90 dBw, and various parametric instabilities may be excited. In the present work, parametric excitation of a Langmuir sideband and a lower hybrid decay mode by a Langmuir pump wave is considered as a cascade process of Langmuir waves. This process produces a discrete spectrum of HFPLs originating from a narrow layer of altitude located slightly below the O-mode HF reflection height.

2. Parametric Coupling

The first (mother) Langmuir pump $\phi_{10}(\omega_{10}, k_{10})$ is generated by PDI in a layer region located at a height $h = h_{10}$, where the plasma density is determined by the relation $\omega_p^2(h_{10}) = \omega_{10}(\omega_{10} + \nu_e) - 3k_{10}^2 v_{te}^2 - \Omega_e^2 \sin^2\theta_0$ (i.e., h_{10} is the matching height of the Langmuir pump having a propagation angle θ_0 with respect to the geomagnetic field). A process that decays a Langmuir pump $\phi_1(\omega_1, k_1)$ to a Langmuir sideband $\phi_2(\omega_2, k_2)$ and a nearly field-aligned lower hybrid resonance mode $n_{s1}(\omega_{s1}, k_{s1})$ in each cascade step is analyzed, where $k_{1\perp} \cdot k_{2\perp} = 0$. The frequency and wave vector matching conditions in parametric couplings are $\omega_1 = \omega_2 + \omega_{s1}^*$ and $k_1 = k_2 + k_{s1}$.

The wavevectors of the modes involved in this parametric coupling process are matched in three-dimensional arrangements [16, 17]. For example, for an oblique propagation angle $\theta = \tan^{-1}(k_\perp/k_0)$, the Langmuir sidebands of OTSI or PDI are represented by the four lines having $k_{1\pm} = \hat{z}\, k_0 \pm \hat{x}\, k_\perp$ and $k_{4\pm} = \hat{z}\, k_0 \pm \hat{y}\, k_\perp$. The cascade of these lines is described in the following. The two lines $k_{1\pm}$ as the Langmuir pumps decay to the pairs of Langmuir sidebands $k_{2\pm} = \hat{z}\,(k_0 - k_z) \pm \hat{y}\, k_\perp$ and lower hybrid decay modes $k_{s1\pm} = \pm(\hat{x} - \hat{y})\, k_\perp + \hat{z}\, k_z$, where $|k_z/k_\perp| << 1$. Similar cascade process occurs simultaneously on the $k_{4\pm}$—lines as the pumps. When the $k_{2\pm}$—lines grow to a level exceeding the threshold of the subsequent cascading, the wave vectors of the sidebands and decay modes of the proceeding cascade will be $k'_{1\pm} = \hat{z}\, k_0 \mp \hat{x}\, k_\perp$ and $k'_{s1\pm} = \pm(\hat{x} + \hat{y})\, k_\perp - \hat{z}\, k_z$, which are separated from the $k_{1\pm}$ and $k_{s1\pm}$ —lines. This cascade process continues when the $k'_{1\pm}$—lines become strong enough to be pumps, which decay to sidebands with $k'_{2\pm} = \hat{z}(k_0 + k_z') \mp \hat{y} k_\perp$ and decay modes with $k''_{s1\pm} = \mp(\hat{x} - \hat{y}) k_\perp - \hat{z} k_z'$; again, $|k_z'/k_\perp| << 1$. $k'_{2\pm}$—lines then cascade to $k''_{1\pm} = \hat{z} k_0 \pm \hat{x} k_\perp$ with $k'''_{s1\pm} = \mp(\hat{x} + \hat{y}) k_\perp + \hat{z} k_z'$ as the decay modes. This completes the description of a cycle of cascade process. $k''_{1\pm}$—lines can repeat the cycle for continuing cascade. If, for example, the wavevector of a HFPL is given by $k = \hat{z} k_0 + \hat{x} k_\perp$, then k'_{1-}—line with a frequency of $\omega_1 - 2\omega_{s1}$ and k''_{1+}—line with a frequency of $\omega_1 - 4\omega_{s1}$ in this cascade cycle will be the second and fourth cascade lines, respectively, in the HFPLs. A similar cascade cycle with $k_{4\pm}$ -lines as the pumps will then produce the first and third cascade lines in the HFPLs. Therefore, the combined cycle produces the first four cascade lines in the HFPLs. Depending on the intensity of the HF heating wave, this combined cycle can repeat to produce higher cascade lines.

With the aid of Poisson's equation, the continuity and momentum equations of the electron fluid are combined to obtain a coupled mode equation, for the potential function ϕ_2 of the Langmuir sideband [4], to be

$$\{[(\partial_t + \nu_e)^2 + \Omega_e^2](\partial_t^2 + \nu_e\partial_t + \omega_p^2 - 3v_{te}^2\nabla^2)\nabla^2 - \Omega_e^2(\omega_p^2 - 3v_{te}^2\nabla^2)\nabla_\perp^2\}\phi_2$$

$$= -\omega_p^2\{[(\partial_t + \nu_e)^2\nabla + \Omega_e^2\nabla_z]\cdot<\nabla\phi_1(n_{s1}^*/n_0)> - \Omega_e(\partial_t + \nu_e)\,\hat{z}\cdot\langle\nabla(n_{s1}^*/n_0)\times\nabla\phi_1\rangle\} \tag{1}$$

where $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency; $\langle\ \rangle$ stands for a filter, which keeps only terms having the same phase function as that of the potential function ϕ_2 on the left hand side of the equation; $\nu_e = \nu_{ei} + \nu_{eL}$, ν_{ei} is the electron-ion collision frequency and $\nu_{eL}/2 = (\pi/8)^{1/2}(\omega_0^2\omega_p^2/k_z k^2 v_{te}^3)\exp(-\omega_0^2/2k_z^2v_{te}^2)$ is the electron Landau damping rate, accounting for the lowest-order kinetic effect.

The coupled mode equation for the low frequency decay mode $n_{s1}(\omega_{s1}, k_{s1})$ is derived [6] from the fluid equations of electrons and ions, with the aid of quasi-neutral assumption (i.e., $n_{se1} \approx n_{si1} = n_{s1}$), to be

$$\{[\nabla_\perp^2\partial_t(\partial_t + \nu_e) + \Omega_e^2\nabla_z^2]\,[\partial_t(\partial_t + \nu_i) - C_s^2\nabla^2] + \Omega_e\Omega_i\nabla_\perp^2\partial_t^2\}(n_{s1}/n_0)$$

$$= (m/M)\nabla^2[(\partial_t + \nu_e)\partial_t\nabla_\perp\cdot\mathbf{a}_p + \Omega_e^2\partial_z a_{pz} - \Omega_e^2\partial_t\nabla\cdot\mathbf{J}_B/n_0 - \Omega_e\partial_t\nabla\cdot\mathbf{a}_p\times\hat{z}\,] \tag{2}$$

where $\mathbf{a}_p = \langle\mathbf{v}_e\cdot\nabla\mathbf{v}_e\rangle$ and $\mathbf{J}_B = \langle n_e\mathbf{v}_e\rangle$; $\Omega_i^2 << |\partial_t^2|$ and $|\Omega_i^2\nabla_z^2| << |\partial_t^2\nabla_\perp^2|$ are assumed; $\nu_e = \nu_{ei}$ and $\nu_i = \nu_{in;}$ ν_{in} is the ion-neutral particle collision frequency. The differential thermal force term is neglected because the decay modes to be considered have relatively large real frequencies. The third and fourth coupling terms on the RHS of (2) cancel each other; thus only the first two terms on the RHS of (2), which are attributed to the transverse convective force and parallel ponderomotive force, contribute to the parametric coupling.

3. Analysis

Eqs. (1) and (2) are analyzed in the spectral domain by assuming a general expression $p = p\exp[i(\kappa\cdot r - \varpi t)]$ as the spatial and temporal variation of the perturbation quantities, where κ and ϖ are the appropriate wave vector and frequency of each perturbation. The resultant algebraic equations are then combined to obtain a dispersion equation.

We now consider that cascade occurs in the matching height layer of the mother Langmuir pump (i.e., the sideband of PDI/OTSI) at $h_{10} = h_1$, where $\omega_p^2(h_1) = \omega_1(\omega_1 + \nu_e) - 3k_1^2v_{te}^2 - \Omega_e^2\sin^2\theta$. Substitute $\omega_s = \omega_{s0} + i\gamma_N$ into the dispersion relation, where γ_N is the growth rate of the instability for the Nth cascade, reduces the dispersion relation for the Nth cascade to

$$[i\omega_1(2\gamma_N + \nu_e) - \Delta\omega_N^2]\,[-i\omega_{s0}(2\gamma_N + \nu_3) - \Delta\omega_s^2] =$$

$$2\xi^{-1}(e^2/mM)\cos^2\theta(1 + \xi\Omega_e^2/\omega_1^2 + i\xi\Omega_e/\omega_1)[1 + i\Omega_e\omega_1\tan^2\theta/(\omega_1^2 - \Omega_e^2)]k_0^2k_\perp^2|\phi_N|^2 \tag{3}$$

where $\Delta\omega_N^2 = 2\omega_1(N\omega_{s0} + \nu_e/2)$, and $\Delta\omega_s^2 = 2k_\perp^2C_s^2 + \Omega_e\Omega_i\xi + \nu_e\nu_i - \omega_{s0}^2$; $\nu_3 = \nu_i + \nu_e(1 - 2k_\perp^2C_s^2/\omega_{s0}^2)$; $\xi = 1 + Mk_z^2/2mk_\perp^2$.

Eq. (3) leads to two real equations for determining the real frequency ω_{s0} of the decay mode and the growth rate γ_N of the instability. We first determine the threshold field by setting $\gamma_N = 0$ in (3). The resulting equation is then solved to obtain

$$\omega_{s0} \cong [2k_\perp^2 C_s^2 + \Omega_e\Omega_i\xi - \xi\nu_e\omega_0\Omega_i\cos^2\theta/(2k_\perp^2C_s^2+\Omega_e\Omega_i\xi)^{1/2}]^{1/2} \tag{4}$$

and

$$|\phi_N|_{th} = (m/e)\xi[\Delta\omega_N^2\nu_e\Omega_e\omega_1/2\omega_{s0}(1 + \xi\Omega_e^2\sin^2\theta/\omega_0^2)k_0^2k_\perp^2]^{1/2}. \tag{5}$$

The growth rate is found to be

$$2\gamma_N = [\nu_e^2 + (\nu_e^2 + \Delta\omega_N^2\nu_e\Omega_i/\omega_{30}^2)(|\phi_N|^2/|\phi_N|_{th}^2 - 1)]^{1/2} - \nu_e \tag{6}$$

where the slight frequency shift of the decay mode owing to the pump field is neglected.

The threshold field determined by (5) increases with $\Delta\omega_N^2$, and $\Delta\omega_N^2$ increases with the number of the cascade process. Cascade of Langmuir waves eventually stops when the threshold field becomes too high.

Consider the experimental conditions, O-mode HF heating wave at frequency $\omega_0/2\pi$ = 4.53 MHz, $\Omega_e/2\pi$ = 1.43 MHz, T_i = 1000 K, $T_e/T_i = \eta$, $\nu_e = 0.82\eta^{-3/2}$ kHz, $v_{te} = 1.23\times10^5\ \eta^{1/2}$ m/s, $v_{ti} = 7.17\times10^2$ m/s, $C_s = (3 + \eta)^{1/2}v_{ti}$, and L = 35 km, where T_e, expected to depend on the power and the operating mode of the HF heater, and thus a modal electron heating function $\eta(E_p) = 1 + 1.5\times[(E_p - 0.5)/4.5]^2$ is introduced. The threshold field for ($k_1=6\pi,\theta=15^0$) cascade line, determined by (5), is expressed numerically as

$$|E_N|_{th} \cong 2.12\ \eta^{-3/4}\ \xi N^{1/2}\quad \text{V/m}. \tag{7}$$

It is noted that the lower hybrid frequency is proportional to $\xi^{1/2}$, thus a larger ξ will need fewer number of cascades to obtain the same bandwidth in the cascade spectrum, i.e., $N \propto \xi^{-1/2}$. Include this fact the threshold field in (7) is still proportional to $\xi^{3/4}$, thus the decay instability prefers to excite the field-aligned lower hybrid resonance mode having ξ = 1. The lower hybrid wave frequency given by (4) is about 8 kHz, which will be the frequency separation between two adjacent cascade lines in the HFPLs.

We now exemplify a case producing a broad frequency downshifted HFPLs of about 100 kHz bandwidth near the HF reflection height. Since the lower hybrid wave frequency $f_{s0} \cong 8$ kHz, thus N = 12 is needed in ξ = 1 case. The threshold field evaluated from (7) for N = 12 is 5.7 V/m for HF heater operated at cw mode, and is 7.34 V/m for HF heater operated at low duty cycle pulsed mode. These lines are all located in a narrow layer region near the HF reflection height and thus prefer to be detected by radar operating at high spatial resolution mode.

4. Conclusion

The process considered in the present work starts with PDI generating 15^0 Langmuir waves in their matching heights to contribute to the second spectral peak of the HFPLs, which is frequency downshifted by about 4.3 to 5 kHz (depending on the electron heating; the first spectral peak, having zero frequency shift and lower spectral intensity than that of the second peak, is contributed by a sideband of OTSI). Through parametric coupling with lower hybrid waves, the 15^0 Langmuir waves then cascade continuously to broaden the spectral width of the HFPLs. The frequency downshift of the cascade lines is determined by the lower hybrid resonance frequency, which is about 8 kHz. The threshold field of the instability is proportional to $N^{1/2}$, the square root of the number N of the cascade step. Thus it increases as cascade proceeds. Cascade stops when the threshold field becomes too large. The threshold field decreases as the electron temperature increases. Thus the number of cascade lines in the spectrum of the HFPLs for HF heater operated at cw mode will be more than that for HF heater operated at low duty cycle pulsed mode. The cascade location does not move and thus all cascade events occur in a narrow layer slightly below the HF reflection height. Consequently, these cascade lines prefer to be detected by radar operating at high spatial resolution mode.

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