Theoretical Model of Hydrodynamic and Electromagnetic Disturbances Caused by Explosions in the Ionosphere

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Abstract

A new analytical solution of the snowplow model of explosion in the rarefied gas has been obtained. This solution has been compared with Stuart's and Holway's numerical models and with experimental data obtained by the SPOLOH experiments. The comparison shows the solution corresponds to these models with good accuracy and allows calculate the main hydrodynamic characteristics. These characteristics were used for evaluating of electric current and electromagnetic field occurring by explosion in the ionosphere. Our calculations show the electromagnetic disturbance propagates mainly along magnetic field. Such result agrees with the data obtained by experiments TOR, Trigger and Ariel. Presented model allows calculating hydrodynamic and electromagnetic disturbances caused by active impacts and rocket engines disasters.

1. Introduction

Low-frequency electromagnetic pulsations have been observed during ionospheric explosion experiments such as Ariel, TOR, Trigger, BUARO etc. [1-4]. Electromagnetic oscillations in 1-100 Hz frequency range have been registered in these experiments at the distances of the order of tenths km from the explosion points in the ionosphere. Large magnitude electric field disturbances have been observed in the vicinity of geomagnetic field line corresponding to the blasting area only. Photographies of explosions of barium containers have been made in the SPOLOH and Firefly series [5, 6]. They show at the initial moment explosion mass expands with velocity of a few km/s during the first 0.5-2 s. Then the velocity is getting significantly less. The diameter of the explosion mass stays about a few kilometers during tens seconds (Fig. 2).

The hydrodynamic and electromagnetic effects caused by explosion were not explained until now. Significant progress in investigation of low frequency electromagnetic waves was achieved during last two decades. To completely calculate electromagnetic effects it is necessary to know hydrodynamic characteristics of the disturbance of the explosion. In the papers [7, 8] three solutions based on snowplow model of explosion has been obtained: "simple snowplow solution", Stuart's solution and Holway's solution. The first two solutions do not correspond to the real process. Holway's model agrees with experimental data but can be solved by numerical calculation only. So this model is difficult for using and does not allow obtaining analytical dependences of the process characteristics. That is why the analytical solution corresponding to experimental date is needed.

2. Explosive mass expansion

Energy release at the intense combustion in gas results in a rapid growth of the pressure leading to a hydrodynamic motion. Let us consider snowplow model of spherically symmetric point explosion [7]. We treat the regime where the initially dense explosive mass (sphere with radius R) expands into a homogeneous rarefied gas; the mean free path through the atmosphere for an explosive atom on the sphere surface is much longer than R, but the mean free path of atmospheric atoms into the dense sphere is very short; these atoms are swept into the sphere adding to its mass.

The condition for a uniform, or shape-preserving, expansion is that the expansion velocity v of the explosive mass be a function of r/R. We assume such form $v = \xi V$, where $\xi = r/R$ and $V = dR/dt$ is the velocity of the border of the explosive mass. This assumption seems reasonable since a collisionless expansion of a point source results in a linear velocity profile and also Taylor's similarity solution gives a velocity profile, which is not far from linear. Let us assume gas density so that $\rho(r,t) = K(t)\rho_r(\xi)$, where $k(0)$=1. $\rho_r(\xi)$ can be considered as time independent density and K(t) as time dependent coefficient. In such case the next expression is right:

$X = \int_0^1 \xi^4 \rho_r(\xi) d\xi \Big/ \int_0^1 \xi^2 \rho_r(\xi) d\xi$, where X is a constant just depends on gas distribution inside sphere. X is 3/5 for constant density and 1 for a thin shell $(\rho_r(\xi) = \delta(\xi - 1))$.

The simple snowplow model assumes that the sphere's radius expands so as to conserve the kinetic energy: $T \equiv \int_0^R \frac{1}{2} v^2 \rho(r,t) 4\pi r^2 dr = T_0$, where T_0 is initial kinetic energy. At the initial moment whole chemical energy E_0 of the explosive converts into kinetic energy, therefore $T_0 = E_0$. But this equation is right only when there are no any other channels of energy transmission except transmission to molecules of involved gas. Indeed there are many such channels, for example, internal energy and the frictional heating of the expanding sphere through the atmosphere, shock wave, electromagnetic radiation an so on. The main channels are frictional heating and gas moving outside the sphere including shock and acoustic wave. To take it into account we should add energy dissipation channel:

$$\int_0^R \tfrac{1}{2} v^2 \rho(r,t) 4\pi r^2 dr = \mu(t) T_0, \qquad \mu = T / E_0 . \tag{1}$$

μ is a factor that shows which part of explosive chemical energy has been transferred to kinetic energy of explosive mass together with involved gas, and which part has been dissipate by the other channels. So that μ is a monotone decreasing function; $\mu = 1$ at the moment t=0 and $\mu = 0$ at the end of the process.

Dissipative energy flow $Q = dT/dt$ from the kinetic energy T of disturbed mass to environment is proportional to value of kinetic energy: $Q = -\alpha T$. This assumption also seems reasonable. A lot of processes concerned with energy dissipation have such dependence; for example, the energy flow between hot and cold bodies is proportional temperature difference of them, i.e. internal energies. In any case the linear dependence can be considered as the second (linear) addend of Taylor's series. The first addend (constant) is equal zero. So we obtain such equation for T:

$$dT/dt = -\alpha T \tag{2}$$

The solution of this equation is $T = \exp(-\alpha t) E_0$, i.e. $\mu = \exp(-\alpha t)$.

The transformation Eq. (1) yields:

$$V^2 m = \mu(t) V_0^2 m_0, \qquad m = m_0 + \tfrac{3}{4}\pi R^3 \rho_1, \tag{3}$$

where m_0 is the explosive mass, V_0 the initial surface velocity, and ρ_1 the atmospheric density. Then we transform radius R and time t to dimensionless form: $R = \sqrt[3]{m_0/(4\pi\rho_1)}\ \bar{R}$, $t = \sqrt[3]{m_0/(4\pi\rho_1)}\ \bar{t}/V_0$. V_0 can be define from the kinetic energy formula: $T = \mu(0) E_0 = \frac{1}{2}\int_0^R v^2 \rho(r,t) 4\pi r^2 dr = X m_0 V_0^2 / 2$, $E_0 = q m_0$, q is specific heat of combustion. So, the expression of the initial velocity is

$$V_0 = \sqrt{\frac{2\mu(0)}{X} q}\ . \tag{4}$$

As it was mentioned $\mu(0) = 1$, but we kept μ in Eq. (4) to use it later. Eqs. (3) with dimensionless variables yields

$$\left(1 + \tfrac{1}{3}\bar{R}^3\right)\left(d\bar{R}/d\bar{t}\right)^2 = \mu\ . \tag{5}$$

In this equation we consider μ as a variable parameter. The exact analytical solution of Eq. (5) is

$$\begin{aligned} V_0\sqrt{\mu}\ t = \frac{b}{5}\Bigg[& \sqrt[4]{27} F\left(\frac{2\sqrt[4]{3}\sqrt{1+R/b}}{\sqrt{3}+1+R/b}, k\right)\mathrm{sgn}\left(\sqrt{3}-1-R/b\right) + \\ & + 2\sqrt[4]{27}\eta\left(1+R/b-\sqrt{3}\right) F(1,k) + 2\frac{R}{b}\sqrt{1+(R/b)^3} - \sqrt[4]{27} F\left(\frac{2\sqrt[4]{3}}{\sqrt{3}+1}, k\right)\Bigg] \end{aligned}, \tag{6}$$

where $b=\sqrt[3]{3m_0/4\pi\rho_4}$, $k=\sin(75^\circ)\equiv\sqrt{2+\sqrt{3}}/2$, $F(x,k)=\int_0^x 1/\sqrt{(1-x'^2)(1-k^2x'^2)}dx'$ is the incomplete elliptic integral of the first kind, sgn is the sign function of a real variable ($\mathrm{sgn}(x)=1$ if $x>0$ and $\mathrm{sgn}(x)=-1$ if $x\le 0$), η is the step function ($\eta(x)=1$ if $x>0$ and $\eta(x)=0$ if $x\le 0$).

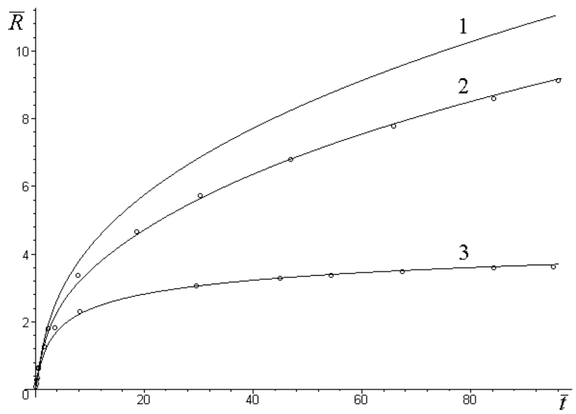

Fig. 1. Comparison of result of analytic model (Eq. (6)) with numerical solution of Stuart's (Eq. (7)) and Holway's (Eq. (9)) models.

1 - μ=1 (simple snowplow model)

2 - $\mu=(\gamma-1)/\gamma+\exp(-\bar{t}/3)/\gamma$ (circles - Stuart's solution)

3 - $\mu=\exp(-2\bar{t}/3)$ (circles - Holway's solution, $l=6$)

If $\mu(t)=\mu$ is a constant, the left side of Eq. (8) is $\sqrt{\mu}V_0t$ and it shows linear expansion with velocity V_0 in the beginning of the process; V_0 is taken from Eq. (4) with substitution $\mu(0)=\mu$. μ equal constant means that at the initial moment just the part of the explosive chemical energy E_0 has been transformed into the kinetic energy and then dissipation of the kinetic energy is zero. However it does not correspond to the real process. In the paper [7] author obtains equation that allows take into account the frictional heating of the expanding sphere through the atmosphere:

$$\bar{R}\left(1+\tfrac{1}{3}\bar{R}^3\right)d^2\bar{R}/d\bar{t}^2 + \\ +\left[\tfrac{3}{2}(\gamma-1)+\tfrac{1}{2}(\gamma+1)\bar{R}^3\right]\left(d\bar{R}/d\bar{t}\right)^2=1 , \tag{7}$$

where $\bar{t}$ satisfies expression $t=\sqrt[6]{X^3m_0{}^5/\left[432(\gamma-1)^3\pi^2\rho_1{}^2E_0{}^3\right]}\,\bar{t}$ and $\bar{R}$ was defined above, γ is the ratio of specific heats.

Let us examine some properties of Eq. (7). In the limit of small $\bar{R}$, the expansion is linear in time: $R=\sqrt{\frac{2}{X}q}\,t$, i.e. V_0 corresponds Eq. (4) with substitution $\mu(0)=1$. In the limit of large $\bar{R}$,

$$R=\sqrt[5]{\frac{75}{8\pi}\frac{\gamma-1}{\gamma}\frac{E_0}{X\rho_1}}\,t^{2/5} , \tag{8}$$

and $T/E_0=(\gamma-1)/\gamma$. If we take $\mu=(\gamma-1)/\gamma$, Eq. (5) yields Eq. (8) - the same result as Eq. (7) gives. So, it seems reasonable to take μ as a simple function so than μ is equal 1 at t=0 and $(\gamma-1)/\gamma$ for a big value of time, for example, $\mu=(\gamma-1)/\gamma+\exp(-\bar{t}/3)/\gamma$. This formula as $T/E_0=(\gamma-1)/\gamma$ does not correspond to basic assumption about energy dissipation $\mu(t=\infty)=0$, but we use it to compare our analytic model with model given by Eq. (8). The result of Eq. (5) is presented in Fig. 1, where, for comparison, the numeric solution of Eq. (7) is also plotted.

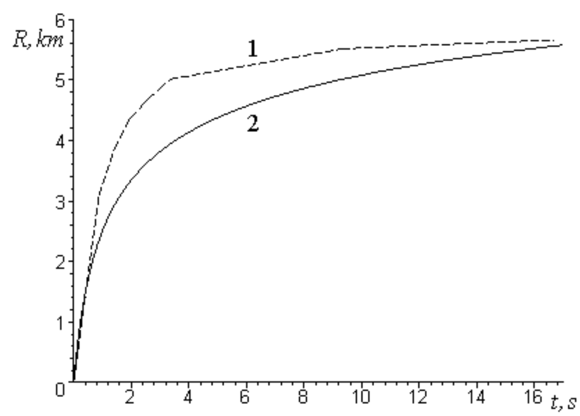

Fig. 2. Explosion mass expansion in SPOLOH-1 experiment.

1 - Experimental data

2 - Result of analytical model (ρ_1=1.1*10^{-13} g/cm^3, m_0=10kg, q=1.5*10^7 J/kg, X=1, α=1)

Eq. (7) assumes that the system is unconfined, i.e., that the surface integral of the pressure over the spherical disturbance can be neglected. In the paper [8] authors avoid such assumption by requiring that the momentum and energy conservation equations, including source terms, are satisfied at each point within the disturbance. It results in the equation:

$$[(1+\overline{\mathrm{R}}^3/3)(\mathrm{d}^2\overline{\mathrm{R}}/\mathrm{d}\bar{\mathrm{t}}^2)+\overline{\mathrm{R}}^2(\mathrm{d}\overline{\mathrm{R}}/\mathrm{d}\bar{\mathrm{t}})^2]\{(1+\overline{\mathrm{R}}^3)\overline{\mathrm{R}}\mathrm{d}^2\overline{\mathrm{R}}/\mathrm{d}\bar{\mathrm{t}}^2+[(3/2)(\gamma-1)+(\gamma/2)\overline{\mathrm{R}}^3](\mathrm{d}\overline{\mathrm{R}}/\mathrm{d}\bar{\mathrm{t}})^2-1\}=$$
$$=l/6\overline{\mathrm{R}}(\mathrm{d}\overline{\mathrm{R}}/\mathrm{d}\bar{\mathrm{t}})2[1-(3/2)(\gamma-1)(1+\overline{\mathrm{R}}^3/3)(\mathrm{d}\overline{\mathrm{R}}/\mathrm{dt})^2 \quad (9)$$

The examination of Eq. (9) indicates that in the limit of large $\overline{R}$ the ration T/E_0 equal zero. It agrees with basic assumption about energy dissipation. Also examination shows the main properties of Eq. (9) corresponds with analytical model Eq. (5) if $\alpha = 2/3$, i.e. $\mu = \exp\left(-\tfrac{2}{3}\bar{t}\right)$. The results of analytical model Eq. (6) and numeric solution of Eq. (9) are presented in Fig. 1. Comparison shows that including energy dissipation channel in the simple snowplow model can describer different explosion models with good accuracy.

We have used our model to explain explosion mass expansion observed in the SPOLOH-1 experiment [5]. The result is shown in Fig. 2., where, for comparison, the experimental data is also plotted.

3. Electromagnetic disturbance caused by explosion

Spherically symmetric motion of the conductive gas leads to the generation of axisymmetric electric current $\vec{j} = \sigma_P\left(\vec{v}\times\vec{B}\right)+\sigma_H\vec{B}\times\left(\vec{v}\times\vec{B}\right)/B$, where σ_P, σ_H are Pedersen and Hall conductivities, $\vec{B}$ is geomagnetic field. v can be calculated by Eq. (6). Green function of the axisymmetric current has been obtained in [9]. The convolution $\vec{j}$ with Green function results in electromagnetic disturbance. The disturbance depends on altitude and mass of the explosive. An example is plotted in the Fig. 3, where z-axis directed along the external magnetic field. Fig. 3 shows that the electromagnetic disturbance propagates mainly along magnetic field. Such result agrees with the data obtained by experiments TOR, Trigger and Ariel [1-4].

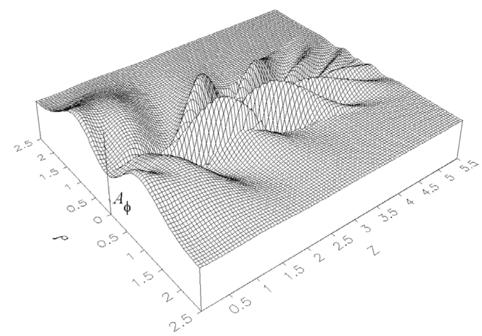

Fig. 3. An example of the electromagnetic disturbance.

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5. References

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