GPS as a source of data generation about high-altitude distribution of electronic concentration

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1. Introduction

A new era of making total electron content (TEC) measurements using satellites of opportunity has begun with dual-frequency transmissions from the Global Positioning System, which is a valuable tool for investigating global and regional ionospheric structures. The use of GPS for ionospheric studies represents a revolution in coverage of the ionosphere because of the large number of satellites and by the large number of monitoring stations, which results in a global perspective.

Some advantages of the GPS are as- follows [1]:
1. Radio signals are provided, that is, no extra beacon satellites are required.
2. Global coverage is available using geodetic (e.g., International GPS Geodynamics Service (IGS» and regional (Continuously Operating Reference Stations (CORS» networks.
3. Receivers are commercially available.
4. Electron content does not depend on assumptions about the Earth's magnetic field and is the electron content up to the height of 20,000 km.
5. Dual-frequency data are available from several operational sources, for example, IGS, CORS, and sometimes over the internet.
6. Information is provided on the topside electron content of the ionosphere that is not available from ionosondes.
7. Data formats are standard, for example, RINEX (receiver independent exchange).
8. Frequencies are sufficiently high (Ll = 1575.42 MHz, L2 = 1227.6 MHz) so that ionospheric absorption and the effects of the Earth's magnetic field on the radio signals are small.

Now GPS is used for studies of such characteristic as total electron content generally. The limiting factor for the reconstruction of images of electron density in the ionosphere using computerized tomographic techniques based on GPS measurements is the inability to estimate vertical profiles of these distributions. This is due to the geometry of computerized ionospheric tomography systems, which limits information contained in data from large projection angles. A relationship exists between the amount of information that is gathered at a particular projection angle and the positions of the ground station receivers.

2. Algorithm of a radio-translucence method

The radio-translucence method is based on the transformation of normalized phase difference for radio waves. Mathematically, it corresponds to the transformation of integral equations of first kind.

Implementation of this method involves measurement of radio signal parameters along the path "satellite – ground receiver" carried out at one station. When the measurements are performed within the angle range Δ', the minimum size of which is determined by the inverse problem solution algorithm, the parameter Δ_I may be considered as a horizontal magnitude of averaging within the ionosphere in reconstructing the altitude profile of electron content. This profile abuts to the midpoint of the projection of the arc Δ_I onto the earth's surface. For angles E_o of 10^o - 90^o the projection of intersection of the line "satellite – receiver" may be located at the distance L up to 1,100 km from the station. The realization scheme of a radio-translucence method for one of observable from ground receiver and satellite is presented on fig. 1.

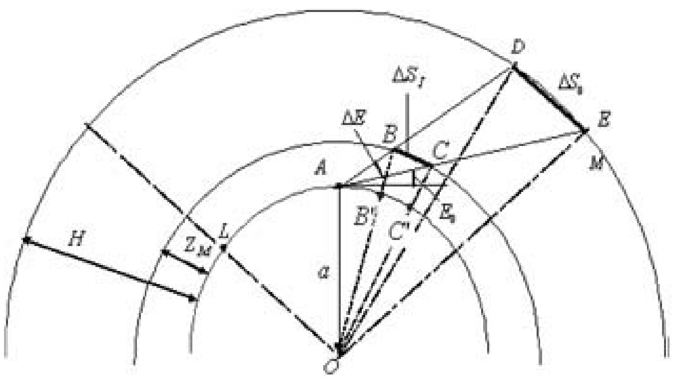

Figure 1. The scheme of a radio-translucence method.

For angles E_o of 10^o - 90^o the projection of intersection of the line "satellite – receiver" may be located at the distance L up to 1,100 km from the station. Numerical estimations of observation zones for the interval of time observations T = 600/30 s are given in Table 1.

Integral equations of first kind appropriate to such a technique of measurement of ionosphere parameters have no analytical solution and require methods of inversion in the class of so-called ill-posed problems to be developed. Such methods are well advanced thanks to the efforts of the mathematical school of A.N. Tikhonov [2]. They are essentially based upon digital algorithms for the inversion of radio signals' measurements and can be realized by means of modern computers.

Table 1. Evaluation of zones of navigation satellite observations

Parameter	Geometrical scales of radio-translucence of the ionosphere for time interval T=600/30 s				
E_0, degrees	10	30	50	70	90
Δ , degrees	5.8/0.29	6.3/0.31	6.8/0.34	7.1/0.35	7.2/0.36
Δ_l, km	282/16.8	96/5.4	53/2.9	40/2.1	38/1.9
L, km	1096	467	237	104	0

The ill-posed nature of the problem of reconstructing electron content distribution using the results of radio-translucence does not allow us to obtain an exact solution for the main integral equation that would be stable under small variations in input data. In this case it is necessary to look for some approximate solution, choosing an acceptable solution from all the possible ones. Mathematical difficulties encountered when trying to apply this approach quite often force us to abandon the idea of obtaining a general solution to the problem of determining environmental parameter distribution. Most frequently, the problem is to be reduced to some elementary cases, for which acceptable results could be obtained.

The equation of radio-translucence built with the assumption of local spherically stratified medium links the measured difference of pseudo ranges and the function of altitude distribution of electron content $N(z)$ [3, 4]

$$\int_{z_1}^{z_2} N(z)\frac{(a+z)dz}{[(a+z)^2 - a^2\sin^2\vartheta]^{1/2}} = 2{,}475\cdot 10^{-8}\frac{f_1^2}{k}\left(\Delta R(f_1,f_2) - \delta\right) \tag{1}$$

where z_1 and z_2 are the assumed lower and upper boundary of the ionosphere, $\vartheta = 90 - E$ is the zenith angle of the satellite observed from the ground station, a is the radius of the Earth, z is the current altitude from the Earth's surface, δ is the error in radio measurements, $f_{1,2}$ are GPS frequencies, $k = (f_1/f_2)^2$.

The left part of this equation represents total electron content of ionosphere (TEC) along the path of radio signal propagation:

$$TEC = \int_L N(l)dl = 2{,}475\cdot 10^{-8}\frac{f_1^2}{k}\left(\Delta R(f_1,f_2) - \delta\right). \tag{2}$$

The search for a possible solution to equation (1) is more expediently carried out using the method of conjugate gradients [2, 4]. It is a mathematically strict method for the solution of inverse problems, with imposed restrictions enabling us to obtain admissible solutions on convex sets.

When the finite-dimensional approximation is applied, the functional

$$\Phi(\varphi) = \|A\varphi - J_\delta\|^2 \tag{3}$$

converts into the quadratic function $\phi(z)$ which can be represented by the following generalized expression:

$$\varphi(z) = (z, Qz) + (b, z) + c, \tag{4}$$

where Q, b, c are the coefficients of quadratic polynomial.

When using the difference approximation, the problem of minimization is reduced to the minimizing series, the elements φ_i of which should minimize quadratic function $\varphi(z)$ and should comply with the restrictions imposed by a priori information. The procedure of minimization consists of following operations. The elements φ_i of the minimizing sequence are determined using the following rule: each successive element φ_{i+1} is calculated from the previous one φ_i as follows:

$$\varphi_{i+1} = \varphi_i - \beta_i p_i, \tag{6}$$

where $p_i = -grad\varphi|_{z=z_i} + \frac{\|grad\varphi(z_i)\|^2}{\|grad\varphi(z_{i-1})\|^2}p_{i-1}$ is the gradient of the function,

$p_0 = -grad\varphi(z_0)$, $\beta_i = \frac{1}{2}\frac{(grad\varphi(z_i), p_i)}{(Qp_i, p_i)}$ is the gradient step, $\varphi(z_0)$ is the initial approximation.

Any model of the Earth ionosphere, for example IRI model, may be used as the initial approximation since in the method of conjugate gradients the initial approximation is the zero point for iterative process. The use of model accelerates considerably speed of a convergence of iterative process.

3. Simulation results and restoration on GPS data

In [4] the method of numerical modeling investigated influence of a priori information and inaccuracy of measurements GPS data on accuracy of the decision of a return task of radio-translucence of the Earth ionosphere and the field of application of satellite navigating measurements was determinate. The simulation results depending on a class of functions, in which the decision of a return ill-posed problem is searched, are presented on fig. 2.

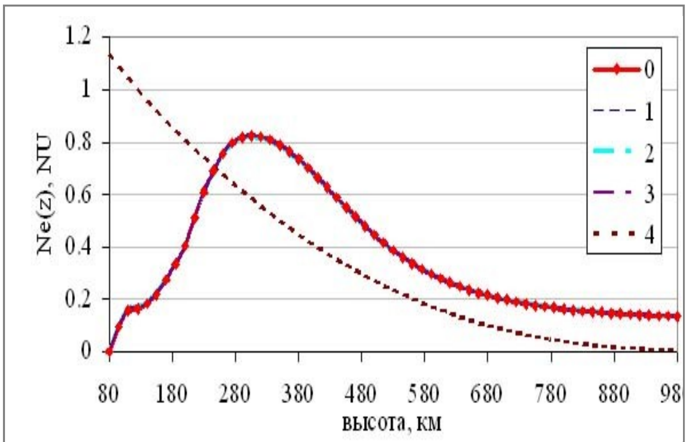

Figure. 2. Influence of functions class on definition of high-altitude distribution of electronic concentration (0 - model; 1 - monotonous; 2 - monotonously-decreasing, convex upwards; 3 - convex upwards; 4 - monotonously-decreasing, convex downwards).

It is shown [4], that application of a radio-translucence method expediently at accuracy of carrying out of complex phase and pseudo range measurements is not worse than an ionosphere 0,2 m. Results from computer-aided simulation show that the root mean square error of determination of the ionosphere electron content altitude distribution function does not exceed value $\delta = 0{,}02NU$ $(NU = 10^{6} el/cm^{-3})$. The discrepancy between the modeled values and reconstructed values using inverse problem solution, is evaluated by value $\delta' = N_{e\max mod} - N_{e\max rec} = 0{,}014NU$. These parameters are obtained at inaccuracy of measurements, at which the ill-posed task solution of radio-translucence has a physical meaning.

Let's note one important circumstance connected with restriction of altitude resolution. One the basic restrictions computing procedures are. On the one hand, with increase in number of elements of splitting of the chosen area accuracy of approximation of integral the integrated sum grows. On the other hand, the increase in number of elements leads to increase in quantity of used initial data that leads to increase in an error owing to their discrepancy. Results of practical realization of algorithm of the decision of a return task of radio-translucence have shown that optimum quantity of points is equally 40.

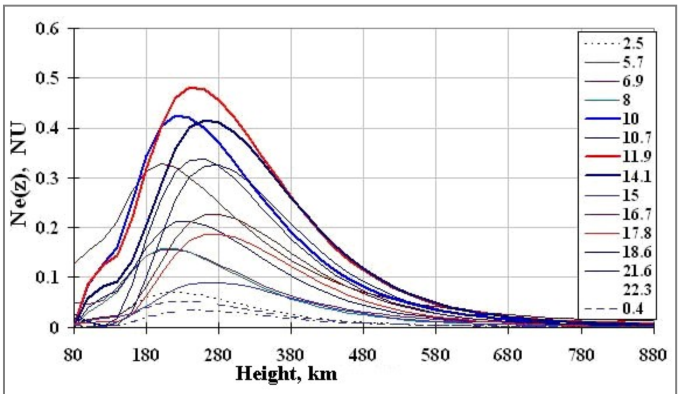

Figure 3. The restored electron concentration profiles

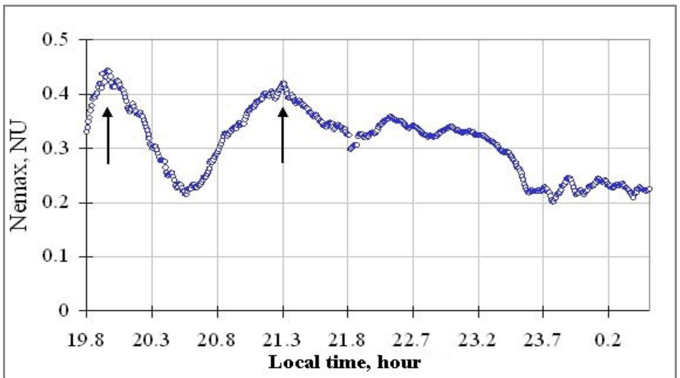

Figure 4. The ionosphere variations in F2 layer maximum

Continuous observations of navigating satellites of systems Glonass and GPS allows to receive values of time variations, both a maximum of an ionosphere, and its high-altitude structure of electronic concentration of an ionosphere simultaneously on several azimuthal directions. The high-altitude structures of the ionosphere received as a result of inversion of satellite measurement data are presented on fig. 3. These structures are received for the nearest distances of a projection of subionospheric points from receiver depending on time on observation of satellites. The opportunity to restore a full high-altitude structure of distribution of electronic concentration of an ionosphere allows analyzing time variations practically any part of an ionosphere.

The restoration opportunity high-altitude structure of electronic concentration on observation on small time intervals allows to fix a big spatial the heterogeneity, observable in an ionosphere. Use of short time intervals of observation, sufficient for determination of a high-altitude structure of an ionosphere, allows localizing rather narrow areas of an ionosphere. Change of electronic concentration of an ionosphere (presence of the irregularities, failures in an ionosphere) leads to change of a difference pseudo range and to change of phase increments. As a result of work of restoration algorithm the ionosphere area, containing irregularity, can be successfully localized. For example some results are presented on fig. 4.

4. Conclusions

The radio-translucence method of the Earth ionosphere based on use of GPS data and the method of conjugate gradients is a promising technique for two- and three-dimensional imaging of electron density profiles. This method allows determine one-dimensional cuts of an ionosphere along a line similar under the form of a trajectory of movement of subionospheric point. Structures received at it concern to a vertical cut of an ionosphere which geographical coordinates are determined by position of subionospheric point projection. Presence of the high-altitude structures received for different geographical points, allows synthesize time-spatial sections of an ionosphere, using only one ground receiver.

5. References

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