An Efficient Forward Scattering Model for a Three Layer Medium Representing Bodies of Fresh Water

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Introduction

Many geological survey, civil engineering and environmental applications require accurate estimates of depth and cross-sectional profile of fresh bodies of water such as lakes and rivers. Since covering large and often poorly accessible areas with in-situ sensors is slow, expensive and often unsafe and impractical, a remote sensing method is desired. The penetrating abilities of low frequency radiation make it a good candidate for this application provided a good resolution can be achieved and proper inversion schemes developed. A fast, reliable and efficient forward electromagnetic scattering model is therefore needed to relate the parameters of interest to the measured scattered field quantities. In this work a water surface is approximated as a periodic surface of given profile, while the bottom is treated as a rough surface with given statistics and dielectric properties. The algorithm is based on the extended boundary condition method (EBCM) and scattering matrix technique. The results are validated first with a scaled-down high frequency radar and then with a VHF radar.

Formulation

A water surface, especially on a windy day can be approximated as a one dimensional periodic surface as shown on the figure below with region 0 and 1 being air and water respectively. The surface height profile for the first interface can be expressed as

$$f(x) = f(x + nL), \quad \forall n \in \mathrm{N}$$

where L is the period of the surface

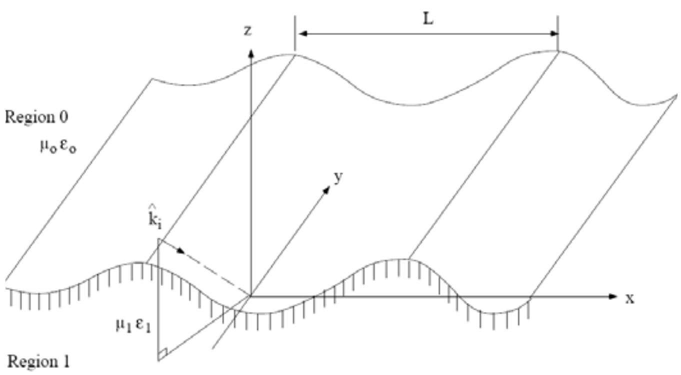

Fig. 1: air-water interface.

The incident Electric field can be expressed as:

$$\overline{E}_i = \hat{e}_i E_o e^{i\overline{k}_i \cdot \overline{r}}$$

The field components are then decomposed into TM and TE, so that once solution to TM problem is known, TE problem can be obtained using the concept of duality.
Writing a scalar Greens function for region 0 and 1 and using Extinction theorem the y components of the total electric fields can be written as:

$$E_{iy}(r) - \int_{o}^{L} dl \left\{ g_{0L}(r,r')\hat{n}_1 \cdot \nabla'_s E_{0y}(r') - E_{0y}(r')\hat{n}_1 \cdot \nabla'_s g_{0L}(r,r') \right\} = \begin{cases} E_{0y}(r'), & z > f_1(x) \\ 0, & z < f_1(x). \end{cases}$$

$$\int_{0}^{L} dl \left\{ g_{1L}(r,r')\hat{n}_1 \cdot \nabla'_s E_{1y}(r') - E_{1y}(r')\hat{n}_1 \cdot \nabla'_s g_{1L}(r,r') \right\} = \begin{cases} E_{1y}(r'), & z < f_1(x) \\ 0, & z > f_1(x) \end{cases}$$

$g_{jL}(r,r')$ is the periodic Greens function defined as:

$$g_{jL}(r,r') = \frac{i}{2L} \sum_{n=-\infty}^{\infty} \frac{1}{k_{jnz}} \exp\left(ik_{nx}(x-x')\right) \times \exp\left(ik_{jnz}|z-z'|\right)$$

$$k_{nx} = k_{xi} + \frac{2\pi n}{L}$$

$$k_{jnz} = \sqrt{k_j^2 - k_{nx}^2}.$$

There is an infinite number of total Floquet modes, but there is only a finite set of propagating modes. The mode is propagating if k_{jnz} is purely real and evanescent if k_{jnz} is purely imaginary. The equations for the surface fields are then transformed into a matrix equation which is used to obtain a reflection matrix for the first interface.
The bottom rough surface can also be treated as a periodic surface with an artificial period L_ rough which is an integer multiple of the period of the first interface. The same analysis is carried out for the bottom surface to generate a reflection matrix. Once reflection matrixes for both interfaces are generated they are cascaded to form a generalized reflection matrix. The last step is to deduce the scattering cross sections from the generalized scattering matrix.

Experimental Validation

Since the ultimate goal of this work is to remotely measure the depth of a body of water, the experimental validation of the forward model is especially important. The penetration depth of water decreases sharply with increased frequency, making the choice of frequency limited to VHF. However, the size of antenna and antenna footprint is so large that it is impractical to do an experiment in the controlled lab environment. Any change of the target (depth, roughness statistics) is difficult to achieve at this scale.

The penetration depth of a signal at several GHz is in the order of centimeters depending on temperature and salinity, making it impractical for realistic applications, but appropriate for validating the model in the lab.

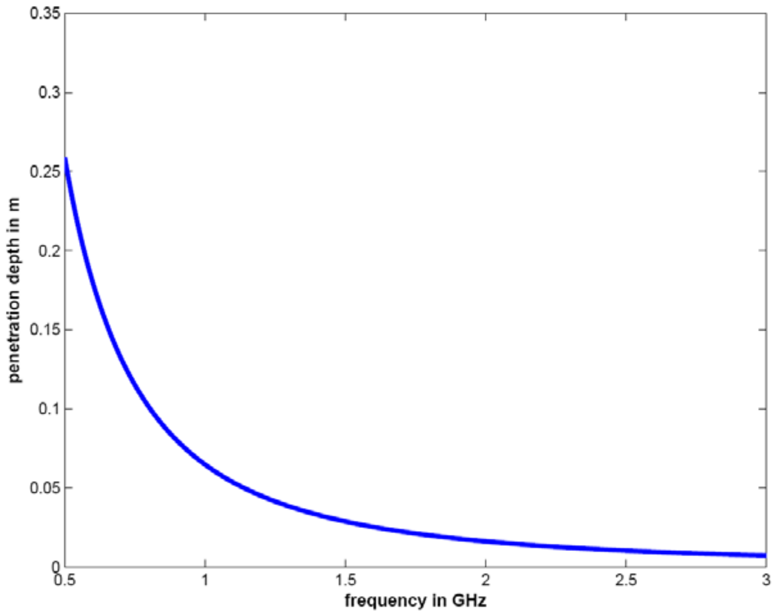

Fig. 2: the dependence of penetration depth on frequency at 30 degrees C for fresh water.

The laboratory setup used in this work consists of a shallow pool with rough homogeneous layer underneath a water layer. The depth of the water layer and the bottom layer were chosen so that the signal coming from the water soil interface could be detected to within a reasonable margin of the system noise floor, but the signal coming from the bottom of the pool would be completely attenuated. Absorbers are placed around the pool to minimize reflections. Both transmit and receive antennas are mounted on the telescoping tower to achieve better resolution by means of synthesizing an aperture. Proper phase shifts are applied to individual measurements to focus the beam at any point of the pool. A small wave-generating device is used to generate water waves of known amplitude and frequency on the surface of the water. The statistics of the rough bottom interface are also carefully estimated.

The validity of the presented model as well as the scaled down setup is further checked by measuring radar cross sections of a freshwater body system with known site parameters using dual polarization VHF radar.

References

[1] C. Kuo and M Moghaddam, “Electromagnetic scattering from a buried cylinder in layered media with rough interfaces”, Antennas and Propagation, IEEE Transactions, Volume 54, Issue 8, Aug. 2006

[2] A. Tabatabaeenejad and M. Moghaddam, “Backscattering of Electromagnetic Waves from Layered Rough Surfaces and Its Application in Estimating Deep Soil Moisture,” *IEEE IGARSS*, Anchorage, Alaska, September, 2004

[3] J. A. Kong, “Electromagnetic Wave Theory”, EMW Publishing, Cambridge, MA, 2000

[4] Ulaby, F. T., R. K. Moore, and A.K. Fung, “Microwave Remote Sensing: Active and Passive, Vol. I -- Microwave Remote Sensing Fundamentals and Radiometry, Addison-Wesley, Reading, Massachusetts, 1981