Application of Least Squares Adjustment Technique for Estimation of TEC and Instrumental Biases for GAGAN Applications

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Abstract

The estimates of ionospheric TEC obtained from the GPS observables are affected by the instrumental delays (biases) introduced by the GPS satellite and receiver hardware. This paper describes the Least squares adjustment technique for estimation of TEC as well as the differential instrumental biases of the GPS satellites and receiver. Dual frequency GPS data of Hyderabad GAGAN station (78.47°E, 17.45°N) is used in this analysis. It is found that the absolute value of the estimated instrumental biases range from 4.22 to 24.24 TECU during the observation period. The estimated biases are of the same order that are reported elsewhere. The technique presented here is relatively simple and can be extended to other GAGAN stations to reduce the effects of instrumental biases.

1. Introduction

The Global Positioning System (GPS) is a satellite based navigation system developed by the U.S. Department of Defense (DoD). In recent years, there has been a widespread growth in the development of Satellite Based Augmentation System (SBAS), to meet the navigation accuracy requirements {16m (horizontal) and 6m (vertical), with 95% confidence interval}, for Category-I (CAT-I) precision approach landings of civil aviation [1]. The Indian Space Research Organization (ISRO) and Airports Authority of India (AAI) are jointly implementing a SBAS known as GPS Aided Geo Augmented Navigation (GAGAN) to provide seamless coverage over the Indian airspace [2]. The most predominant error affecting the accuracy of the GAGAN system is the ionospheric delay. The main source of error in the estimation of TEC is the effect of differential instrumental biases of the GPS satellite and receiver. These instrumental biases exist, as the signals at the two GPS frequencies experience different delays within the satellite and receiver hardware. Several methods for estimation of instrumental biases including Kalman filter, Self Calibration Of pseudoRange Error (SCORE), least squares fitting and neural networks are reported in literature [3-6].

2. Estimation Method

The ionospheric TEC can be estimated using the dual-frequency GPS observables (viz. code and carrier phase data), taking advantage of the dispersive nature of the ionosphere. The GPS code (P_k) and carrier phase (L_k) observables (in range units), on the two GPS frequencies (f_1=1575.42 MHz, f_2=1227.60 MHz) can be expressed as:

$$P_k = \rho + \frac{40.3TEC}{f_k^{\,2}} + \varepsilon_k \tag{1}$$

$$L_k = \rho - \frac{40.3TEC}{f_k^{\,2}} - \lambda_k N_k - \zeta_k \tag{2}$$

where, subscript k=1, 2 refer to the GPS frequency f_1 and f_2 respectively; ρ is the sum of geometric range, tropospheric error, and clock error (in metres); TEC is the total electron content along the line-of-sight direction from the satellite to the receiver (in electrons/m^2); ε_k is the sum of all errors due to instrumental delays, multipath, and random noise for code measurements at frequency f_k (in metres); ζ_k is the sum of all errors due to instrumental delays, multipath, and random noise for phase measurements at frequency f_k (in metres); N_k is the integer cycle ambiguity at frequency f_k (in cycles); and λ_k is the carrier wavelength at frequency f_k (in metres).

In this investigation, the carrier phase observables are used to compute the slant TEC for each tracked satellite at each observation epoch. A single station technique is used for resolving integer cycle ambiguities present in the phase observables [7]. The smoothed code measurements are used in the estimation of integer ambiguities. This helps in reducing the effect of code measurement noise. The multipath error is estimated using the TEQC software available in public domain. The carrier phase derived line-of-sight TEC still contains instrumental delays of the GPS satellite and receiver. This is modeled as the sum of actual line-of-sight TEC, the satellite differential instrumental bias, and the receiver differential instrumental bias, and is expressed as:

$$TEC_c = S(E) \times TEC_v + b^s + b_r \tag{3}$$

where TEC_c is the carrier phase derived slant TEC (in TECU); E is the elevation angle (in degrees) from the satellite to the receiver; $S(E)$ is the slant factor; TEC_v is the vertical TEC at the ionospheric pierce point (*IPP*); b^s and b_r are the satellite and receiver differential instrumental biases respectively, (in TECU). The IPP is the intersection of the line-of-sight ray path from the satellite to the receiver, with the mean ionospheric layer, assumed to be at a height of 350km above the earth surface. A single-layer model is used for defining the ionosphere, and the vertical TEC at the IPP is represented using a second-order polynomial function as [8]:

$$TEC_v = a_0 + a_1\phi_m + a_2\lambda_{cr} + a_3\phi_m{}^2 + a_4\lambda_{cr}{}^2 + a_5\lambda_{cr}\phi_m \tag{4}$$

where ϕ_m is the geomagnetic latitude of IPP; and λ_{cr} is the longitude of IPP in the co-rotating reference frame.

Substituting Eq. (4) in (3), for n observations, Eq. (3) can be represented in matrix form (AX = B) as:

$$\begin{bmatrix} sf_1 & sf_1\phi_{m1} & sf_1\lambda_{cr1} & . & . & . & . & sf_1\lambda_{cr1}\ \phi_{m1} & 1 \\ sf_2 & sf_2\phi_{m2} & sf_2\lambda_{cr2} & . & . & . & . & sf_2\lambda_{cr2}\ \phi_{m2} & 1 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ sf_n & sf_n\phi_{mn} & sf_n\lambda_{crn} & . & . & . & . & sf_n\lambda_{crn}\ \phi_{mn} & 1 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ . \\ . \\ a_5 \\ b^s + b_r \end{bmatrix} = \begin{bmatrix} TEC_{c1} \\ TEC_{c2} \\ . \\ . \\ . \\ TEC_{cn} \end{bmatrix} \tag{5}$$

where A is design matrix or coefficient matrix, B is observation vector and X is unknown parameter vector. Applying least squares technique to the Eq. (5), the combined satellite plus receiver differential instrumental delay ($b^s + b_r$), for each tracked satellite, along with the polynomial coefficients can be estimated. Having the polynomial coefficients determined, the vertical TEC at the IPP can be determined.

3. Results and Discussion

The dual frequency GPS data in Receiver INdependent EXchange (RINEX) format corresponding to the Hyderabad GAGAN station (78.47°E, 17.45°N) is used in this analysis. The data is provided by the Space Applications Centre, ISRO, Ahmedabad, India. The ambiguity-free slant TEC computed from carrier phase data is validated with that obtained from the Bernese software (version 4.2). Table 1 compares the computed slant TEC, with that obtained from Bernese software for a particular epoch for satellite PRN 9.

Table 1 Comparison of Computed and Bernese Slant TEC (PRN 9)

Inputs (RINEX observation file)		**Outputs**		
		Parameter	**Computed Value**	**Bernese software**
P_1	21639441.20300 (m)	Ambiguity free L_1 (cycles)	113715979.79572	113715980.07419
P_2	21639444.37340 (m)	Ambiguity free L_2 (cycles)	88609841.704758	88609841.54719
ϕ_1	-18561100.92600 (cycles)	**Slant TEC** (TECU)	29.48	30.35
ϕ_2	-14463178.45340 (cycles)			

Further, the satellite position, receiver position in Earth-Centered Earth-Fixed (ECEF) coordinates, slant factor, IPP coordinates (in geographic and geomagnetic reference systems) are computed from the GPS data for processing. Using these parameters and the computed slant TEC for each epoch, the design matrix (A) and the observation vector (B) are formulated. Applying the least squares adjustment technique, the polynomial coefficients a_0, a_1, ….a_5 defining the vertical TEC, and the combined satellite plus receiver bias ($b^s + b_r$) are estimated. Table 2 gives the estimated combined instrumental biases due to various satellites for March 4, 2005. The absolute value of the combined instrumental bias due to various satellites is found to range from 4.22 to 24.24 TECU (1 ns of differential delay = 2.852 TECU) during the observation period.

Table 2 Estimated combined instrumental biases due to various satellites (March 4, 2005).

S.No.	PRN #	Combined satellite plus receiver instrumental delay (TECU)
1.	4	23.80
2.	9	7.61
3.	10	4.22
4.	14	19.57
5.	21	-24.20
6.	22	-10.57

Figure 1 and 2 compare the estimated vertical TEC (TEC_v) obtained from the least squares adjustment technique and the corresponding biased vertical TEC $\{TEC_c / S(E)\}$.

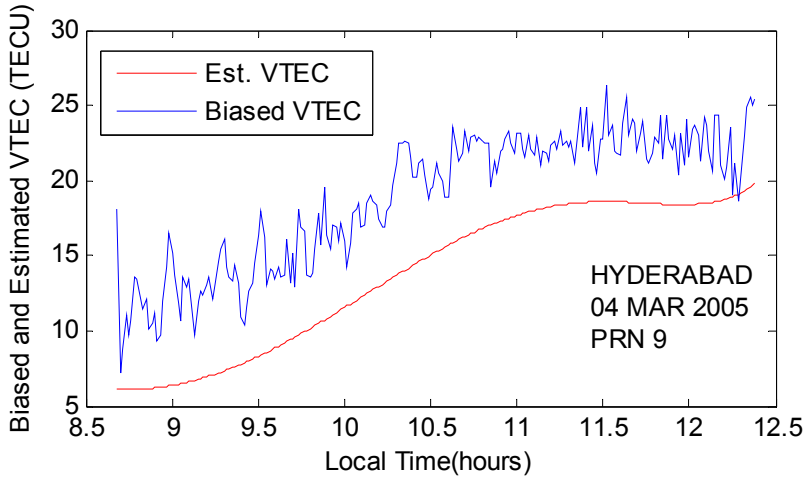

Figure 1 Biased and Estimated Vertical TEC (Fore-noon)

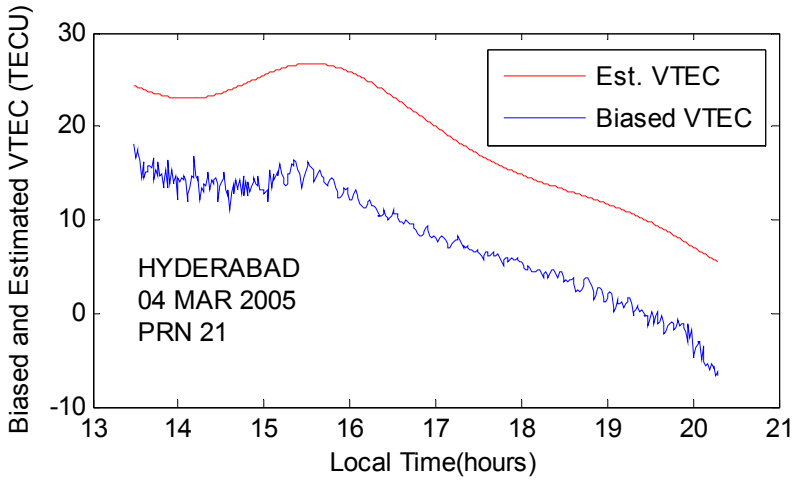

Figure 2 Biased and Estimated Vertical TEC (After-noon)

These plots show the diurnal behaviour of TEC considering two satellites (PRN 9 and 21) tracked during the observation period. From figure 1, it is observed that the estimated vertical TEC increases from 6.11 to 19.7 TECU, with increase in local time {LT = Universal Coordinated Time (UTC) + 0530 hours}. As the sun rises, ionization caused by the solar radiation builds up the electron density (TEC). The TEC reaches a peak value of 26.7 TECU around 1530 hours (LT) (See figure 2), and then starts declining. As there is no further ionization during nighttime, the ions and free electrons recombine, resulting in reduced TEC. The vertical TEC is estimated to be 6.71 TECU at 2018hours (LT).

4. Conclusions

The development of Indian SBAS (GAGAN) is under progress to meet the precision approach and landing requirements of civil aviation. The ionospheric delay is the major source of error in GAGAN. The GPS carrier phase measurements precisely track TEC, but are affected by integer ambiguities. A single site technique for estimation of integer ambiguities is implemented. The computed slant TEC is validated with the Bernese software. For estimating the combined satellite plus receiver instrumental bias, along with the vertical TEC, the least squares adjustment technique is adopted. TEC is modeled using a second order polynomial function. The estimated biases are under validation. The technique presented here can be extended to other stations in the GAGAN system to reduce the effects of instrumental biases and achieve the Category-I precision approach requirements.

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