Comparative Study of Singapore's Drop Size Distribution for Fixed μ

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Abstract

Modified gamma model with moment estimators is used to model the rain Drop Size Distribution (DSD) of Singapore (1°20'N, 103°41'E). Gamma model parameters N_0, μ and Λ are determined through measured data for ten different averaged rain rates. The estimates from the method of moments are obtained by equating a sufficient number of measured moments to the corresponding theoretical moments. The estimated parameters show that the gamma model parameter μ varies very slowly compared to the other two parameters. This slow varying parameter varies between 3.03 and 9.30 for rain rates from 1.50mm/hr to 147.70mm/hr for the tropical region of Singapore. In this paper, an attempt is made to simplify the modified gamma model with moment estimator by keeping this slow varying parameter μ as a constant. This study is performed using various constant μ values of 3, 4.58 and 5 to model rain drop size distribution along with the calculated μ values based on the measured DSD for all the considered rain rates. Fixed μ models deviate more from measured DSD for the lower three rain rates, except μ=4.58 which models well only at 4.99mm/hr. Root mean square errors (RMSE) are calculated to compare modeled DSD with the measured data. RMSE calculations along with rain drop size distributions show that modified gamma model can be used to model Singapore's DSD with fixed μ=4.58 and μ=5 for all the rain rates from 99.55mm/hr to 147.70mm/hr and μ=3 for all the rain rates from 23.29mm/hr to 76.15mm/hr. Results show that there is no fixed μ value that can be used for the entire range of rainfall rate.

1. Introduction

In tropical and equatorial regions, the attenuation of microwave signals caused by heavy rainfall is a major problem for the design of communication systems. Hence, detailed knowledge of rain drop size distribution is important for predicting the attenuation of microwave signals. In characterizing rain DSD, early studies commonly used exponential distributions with one or two parameters for its simplicity. This includes the Marshall-Palmer spectrum and the Laws-Parsons spectrum [1, 2]. Some observations, however, indicate that the natural rain DSD contains fewer of both very large and very small drops than exponential distribution [3]. Ulbrich (1983) suggested the use of modified gamma distribution for representing rain drop spectra as

$$n(D) = N_0 D^{\mu} \exp(-\Lambda D) \qquad (0 < D < D_{\max})$$

The gamma DSD with three parameters (Offset parameter: N_0, Shape parameter: μ and Slope parameter: Λ) is capable of describing a broader variation in rain DSD than an exponential distribution, which is a special case of gamma distribution with μ=0. It has been found that the three parameters are not mutually independent [3]. The shape of the DSD as a function of the drop diameter, D, is determined solely by the exponent μ.

In this paper, the parameters for the modified gamma model are estimated (1st and 2nd moments for μ and Λ and 1st and 0th moments for N_0) for a total of ten averaged rainfall rates 1.50, 4.99, 10, 23.29, 39.97, 66.88, 76.15, 99.55, 124.06 and 147.70mm/hr. Then all the gamma model parameters are plotted for the nine averaged rain rates. From the calculated values based on the measured DSD, the slow varying parameter μ is identified. In order to reduce complexity, the slow varying parameter is kept as a constant. Therefore, this paper studies the possibility of using a fixed μ value [4-5] of three different constant values of 3, 4.58 (mean of calculated μ values) and 5. Modified gamma modeled DSDs with calculated μ and fixed μ are compared with measured data for ten different rain rates. Root mean square errors (RMSE) are computed to compare modeled DSD and measured DSD. Comparisons along with DSD models show that the modeled DSD fits well with the measured data at rain rates from 23.29mm/hr to 76.15mm/hr with acceptable RMSE for a fixed μ=3. The RMSE produced is very small if a fixed value of μ=4.58 and μ=5 is used for modeling at the higher rain rates 99.55mm/hr and 147.70mm/hr.

2. Measurement of DSD and Root Mean Square Error

The Joss distrometer RD69 [6], which is an impact measurement device, measures the number of rain drops with diameters ranging from 0.3mm to >5.2mm and digitizes into 20 bands or channels with an integration time of T=60s.

For the computation of DSD and attenuation, the mean values of the diameters are used. The number of rain drops, n_i, in the i[th] channel with diameters, D_i, in the range of $D_i \pm \Delta D_i/2$ is collected over the sample area of S=5000mm^2. The measured drop size distribution $N(D_i)$ (m^{-3}mm^{-1}) can be expressed by

$$N(D_i) = \frac{n_i \times 10^6}{v(D_i) \times S \times T \times \Delta D_i} = \frac{n_i}{V(D_i) \times 0.3 \times \Delta D_i} \tag{1}$$

where $v(D_i)$ is the terminal velocity of rain drop in m/s obtained from Gunn and Kinzer's [7] terminal velocity of water drop. For more than one year from 1994, a total of about 270 rain events were recorded including about 1000 rain hours on 60000 samples. One sample is defined as recorded rain drops distributed in 20 diameter channels within one minute; one rain hour involves 60 minute samples; and one rain event refers to the rain from beginning to the end. For the purpose of modeling, rain rates ranging from 1.50mm/hr to 147.7mm/hr are considered. All the averaged rain rates are taken from the rain event occurred on 26 February 1995.

2.1 Modified gamma model

Modified gamma model is expressed [1, 6] as follows:

$$N(D_i) = N_0 D_i^{\mu} e^{-\Lambda D_i^{q}} \tag{2}$$

where D_i is the rain drop diameter (mm), N_0, μ and Λ are parameters to be determined through measured data. The value of q is assumed to be 1 for calculations. The estimates from the method of moments are obtained from equating a sufficient number of measured moments to the corresponding theoretical moments. Equations given in [8] are used to estimate modified gamma model parameters from measured moments.

2.2 Root Mean Square Error

Root mean square error is calculated for all the models by using the following formula for comparison.

$$RMSE = \sqrt{\sum_{i=1}^{n} \frac{\left[\log(N(D_i)) - \log(ModeledN(D_i))\right]^2}{n}} \tag{3}$$

where $N(D_i)$ is calculated from measured data and n is the number of channels considered.

3. Results and Discussion

Fig 1 illustrates the 3 modified gamma model parameters N_0, μ and Λ for nine different rain rates. All of the gamma model parameters vary with rain rate. Calculated gamma model parameters for the rain rate 1.5mm/hr are not plotted in this graph, since N_0 value at this rain rate is large compared to the other calculated N_0 values. But this rain rate's calculated values will be considered in this analysis. N_0 variations from the calculated values for all nine rain rates are large as expected. μ varies from 2.78 to 9.29 whereas Λ values vary from 3.23 to 13.22. Based on these variations, μ is identified as a slow varying parameter. It is not possible to keep the intercept parameter N_0 and slope parameter Λ constant. But it is possible to fix μ as constant [4, 5] to simplify the modeling process. Rincon [4] used μ =4 and 5 for the estimation of path-average drop size distribution in his paper. He [5] used μ=2 to study the variability in rain DSD. In this paper, the mean μ value is calculated from ten rain rates considered and used as one of the constant μ to model DSD using the modified gamma model. Two other constants μ=3 and 5, one at the lower end of the range and another at the higher end of the range are also selected. Calculated μ values vary between 2.75 and 9 to suit Singapore's gamma DSD. Unlike the DSD in America, where μ=2 is selected [5], this value is out of range from Singapore's DSD.

3.1 Modified gamma modeled DSD

Fig. 2 shows the modeled DSDs using equation (1) with measured data using equation (2) for two different rain rates 4.99 and 66.88mm/hr. In Fig. 2, there are 5 plots, the measured DSD, the modeled DSD using the calculated μ value, and the modeled DSD when μ is fixed at values of 3, 4.58 and 5.

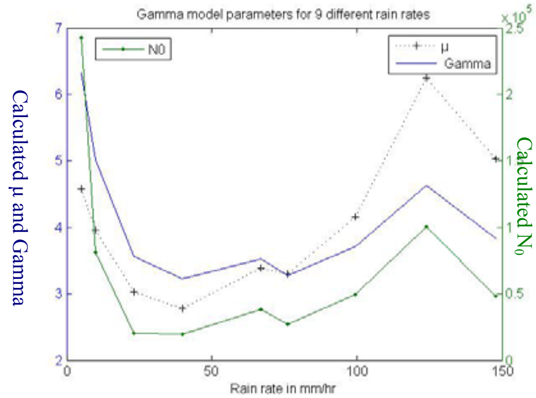
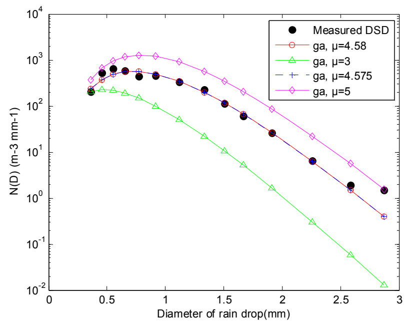
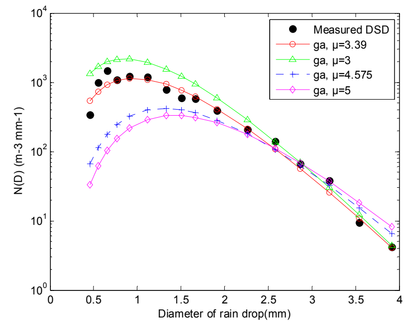


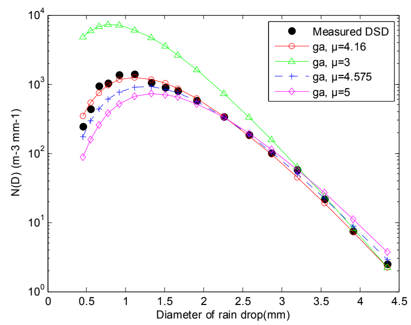
Fig. 1: Estimated gamma model parameters for 9 different rain rates



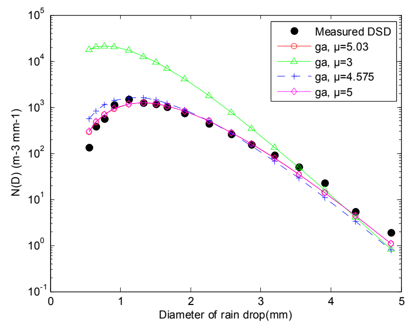
(a) At 4.99mm/hr



(b) At 66.88mm/hr



(c) At 99.15mm/hr



(d) At 147.70mm/hr

Fig. 2: Modeled N(D) with measured data for different fixed gamma values

Modeled N(D) using fixed μ deviates more at 1.50mm/hr, since calculated μ is large compared to the fixed μ values. In Fig 2(a), measured data fits well with the gamma model DSD with μ=4.58. However, for μ=3 and μ=5, the modeled DSD fits better at smaller drop size of less then 0.7mm and deviates far for larger drop diameters. For higher rain rates, all the μ values result in a good fit of the modeled DSDs for diameters above 2mm for the rain rates from 23.29mm/hr to 147.70mm/hr. This is clearly seen from Fig 2(b) at 66.88mm/hr. By looking at the trend from Fig. 2(a) to Fig. 2(d), it can be concluded that, as the rain rate increases, the more possible it is to use a constant μ value of either 4.58 or 5 to model the DSD. As can be seen from the figure, modeled DSD deviates from measured data if the fixed μ moves away from the calculated μ for all the rain rates. Calculated μ values are 4.58 and 3.391 for rain rates 4.99mm/hr and 66.88mm/hr respectively. The aim of this paper is to find whether is it possible to keep μ as a constant to model DSD. In order to find the amount of deviation between modeled and measured DSD, root mean square errors are calculated using equation (3) and analyzed next.

3.2 Root Mean Square Error

Table I shows the RMSE between measured DSD and fixed μ modeled DSD for all the rain rates. The RMSE is very large for the lower rain rates of 1.50mm/hr and 4.99mm/hr. Although at 4.99mm/hr, μ=4.58 models the DSD very well. The RMSE for 4.99 for μ=5 is 0.92 since the differences at this rain rate occurs mainly at the higher rain drop sizes where the DSD is small (less then one) resulting in a small RMSE. As shown in the table, μ=4.58 and 5 cannot be used to model the DSD at 10mm/hr, similarly for μ=3 for 124.06mm/hr. Also shown in Fig. 2 and Table I, for rain rates between 23.29mm/hr to 76.15mm/hr, a constant μ=3 can be used with high accuracy (small RMSE). For rain rates from 99.55mm/hr to 147.70mm/hr, a fixed μ=4.58 or μ=5 can be used with high accuracy. However, it is should be noted that, as shown in Table I, when the fixed μ value is close to the calculated μ, the RMSE will be small. When the fixed μ value is far from the calculated μ, the RMSE is large.

Table 1. RMSE between measured and modeled DSD for fixed μvalues

Rain rate Mm/hr	μ=3	μ=4.58	μ=5	Calculated μ	
1.50	6.15	4.21	3.76	9.29	
4.99	2.38	0.39	0.92	4.57	← μ=4.58
10.00	3.20	NA	NA	3.96	
23.29	0.45	1.75	2.13	3.03	
39.97	0.44	1.89	2.28	2.78	μ=3
66.88	0.55	1.07	1.40	3.39	
76.15	0.43	1.09	1.42	3.30	
99.55	1.45	0.32	0.64	4.16	
124.06	NA	1.35	1.03	6.24	μ=4.58 or μ=5
147.70	2.28	0.56	0.31	5.02	

4. Conclusions

In order to model the drop size distribution of Singapore, modified gamma model with fixed μ is proposed. This method reduces complexity of modeling. Ten averaged rain rates are considered for modeling. Gamma model parameters are calculated by estimating moments and then used to model DSD. Then two of the parameters N_0 and Λ are taken without any change for all the rain rates and different constant μ values are used for modeling. Root mean square errors are calculated for all the models. Modified gamma models using constant μ=4.58 and 5 fit well with the measured data for the higher average rain rates from 99.55mm/hr to 147.70mm/hr and μ=3 for all the rain rates from 23.29mm/hr to 76.15mm/hr. However results also indicate that the fixed μ is to the calculated value, the smaller the RMSE value. The error for lower rain rate is larger for larger drop diameters than smaller drop diameters. Similarly, the error for higher rain rate is smaller for larger drop diameters than smaller diameters. Results at lower rain rates need more analysis and no single constant μ value is applicable for all rain rates in Singapore.

5. References

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