New type of the ionospheric Alfven resonator

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In this work we consider a new model of the ionospheric Alfven resonator (IAR-II) differs from previous considered models. Briefly let’s describe works that had been devoted to this issue. Typically, the plane layered ionospheric model was considered in most of these works. The reflected Alfven wave from the magnetosphere caused own fluctuations in the stretch of power lines of the geomagnetic field falling on the ionosphere. In doing so, one way or another, self-resonant and resonant frequencies of Alfven oscillations were determined [1 - 3]. The spherical layered model was seen in another works. The power line of the geomagnetic field runs through two ionospheres and magnetosphere there. There is an object called us the ionospheric-magnetospheric Alfven resonator (IMAR). Self-resonant frequency range of the object was being studied in there case.

However, another possible case remained outside review. It can be realized at the low latitudes where the power line of geomagnetic field is located in the ionosphere entirely. We estimate that it corresponds to the value for the -35° to 35° geomagnetic latitude. The subject of this work is the research of this case.

We will be describing the propagation of hydromagnetic waves in the ionosphere and the magnetosphere (in the near-Earth space) using Maxwell’s equations for the time spectral components which dependence on taking time in the form exp(-iot), then:

\[ \text{rot } E = i\omega\mu_0 \mathbf{H}, \]
\[ \text{rot } \mathbf{H} = \sigma_1 \mathbf{E}_t + \sigma_\perp \mathbf{E}_\perp + \sigma_{\parallel} (\mathbf{E} \times \mathbf{h}), \]

where \( E \) and \( H \) are tensions of electromagnetic field waves, \( \mathbf{h} \) is a unit vector in the direction of the geomagnetic field, \( \mathbf{E}_t \) is a parallel electric field component of the wave vector \( \mathbf{h} \), \( \mathbf{E}_\perp \) is a perpendicular electric field component of the wave vector \( \mathbf{h} \), \( \sigma_1 \) is a plasma conductivity in the direction of the geomagnetic field, \( \sigma_\perp \) is a plasma conductivity in the perpendicular direction of the geomagnetic field, \( \sigma_\parallel \) is a Hall plasma conductivity.

We would be considered the low frequency range from 10-2 Hz to 10 Hz. In this range we believe that \( |\sigma_{\parallel}| \to \infty, E_\parallel=0 (\sigma_{\parallel}E_{\parallel}\neq0), |\sigma_\perp| \gg |\sigma_\parallel| \). Thus, \( E_\parallel=-E_{\parallel}\mathbf{h}_\parallel/E_{\parallel}\mathbf{h}_\parallel \) in spheric coordinate system (r, \( \theta \), \( \varphi \)).

Let’s assume that the environment properties do not depend on the \( \varphi \) angle as well as the properties of the fields. Propagation takes place in the magnetic meridian plane, \( \partial/\partial\varphi=0, h_\varphi=0 \). We will neglect the \( \sigma_\parallel \) component of the conductivity tensor to make the solution of this problem more visible because this is the most interesting for the IAR-II concept. Then, just as it was done in work [4], an equations system for Alfven waves can be written in the form:

\[
\begin{align*}
\frac{\mathbf{E}}{r} &= \left( \frac{h}{r} \right) \mathbf{E}\mathbf{h}_r \frac{1}{r} \mathbf{E} h_r + i \mathbf{B} , \\
\frac{\mathbf{B}}{r} &= \frac{k^2}{i h_r^2} \mathbf{E} + \frac{1}{r \sin h_r} (\mathbf{B} \sin \theta). 
\end{align*}
\]

But \( \mathbf{E}' = r\mathbf{E} \), \( \mathbf{B}' = r\mathbf{B} \), etc, \( k^2 = i \), where \( k \) is a wave number. Then the modified spherical impedance of the Alfven wave is:
\[ U(r, \theta) \cos \left( i \frac{\hat{E}(r, \theta)}{L B(r, \theta)} \right), \]

where \( L \) is a constant with a length dimension. Further we will be assumed that \( \sigma_a \) depends only on the \( r \) coordinate and does not depend on the \( \theta \) coordinate. Then using \( \frac{dU}{dh} = (h) U \) and believing \( h = \cos \ h, + \sin \ h, \) (\( h, h' \) are unit vectors in the direction of \( r \) and \( \theta \) axes) we get:

\[ \frac{dU}{dh} = 1 + U^2 k^2. \]  
\[ (1) \]

Boundary conditions are met at the Earth's surface, \( U(a) = 0 \). Let's build the solution of the equation (1). In a homogeneous environment exact solution of a nonlinear Riccati type equation (1) has the form:

\[ U(h) = \frac{\tan(kh)}{k}. \]  
\[ (2) \]

For the model of the geomagnetic field \( r \) and \( \theta \) coordinates are related by:

\[ r = a \frac{\sin \theta}{\sin \alpha}, \quad h = a \frac{\alpha}{\sin \alpha}. \]  
\[ (3) \]

\( a \) is Earth’s radius, \( \alpha \) is an angle by which the geomagnetic field power line goes from the Earth’s surface in relation to the radial direction, \( h \) is coordinate along the geomagnetic field power line in the ionosphere, \( k = \frac{c_\alpha}{a} \) is a wave number in ionosphere, \( C_\alpha \) is the Alfven speed in ionosphere. For the case during a night time with the maximum solar activity we believe that \( C_\alpha = 430 \text{ km/sec}, \quad 66^0 \leq a \leq 114^0 \). (The power line get through the ionosphere [4]). Function (2) satisfies the boundary conditions at \( h=0 \). Let’s requiring compliance with the boundary conditions at \( h = a \frac{\alpha}{\sin \alpha} \) (entry point of power lines in the Earth's surface). The self-resonant frequency range of the IAR-II will be determined from the condition of \( U(h) = 0 \):

\[ \operatorname{tg} \ k a \frac{2}{\sin \alpha} = 0, \quad f_n = n \frac{c_\alpha \sin \alpha}{2a} \left( \frac{2}{\alpha} \right). \]  
\[ (4) \]

Quasi-equidistant resonance frequency spectrum with monotonically increasing quality in the ionosphere in accounting for the frequency dispersion of the Alfven speed and for Joule losses. We have \( f_0 = 0.904 \text{ Гц}, \ Q_n = 19 \) at \( \alpha = 700, \ n = 20 \) for the variant during a night time with the maximum solar activity.

The IAR-II existence is possible in the low latitudes area as can be seen from the work done by the present analysis.

**References**


