Finite Element Analysis of the Transient Ionospheric Energy Dissipation Associated with Lightning Induced Sprites

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Abstract

The research presented here is focused on identifying the distribution of energy dissipation (deposition) in the ionosphere during a Sprite event triggered by large-scale lightning. A Finite Element Model (FEM) of the atmosphere is used to investigate the high altitude electrodynamics. The model assumes the lightning event and Sprite can be effectively modeled within a right circular cylinder having a height and radius of 100 km. The electromagnetic behavior of the region is described using Maxwell's equations coupled with ionization equations. The charge displaced during the lightning event is assumed to be 100 C and centered at a height of 10 km.

1. Introduction

"Sprite Lightning", in addition to other transient luminous events (TLEs) including halos and elves, are currently being investigated using both experimental and numerical techniques [1, 2]. The focus of the research presented here is to identify trends in the electrical energy dissipation in the ionosphere in the vicinity of the Sprite. There is a tremendous variation in the power, energy, and electron density signatures in the ionosphere during the non-linear Sprite event. By accurately modeling the Sprite event, it is possible to gain valuable insight into where energy absorption peaks. This is especially important to note if the Sprite rapidity rate is high.

Following the first confirmed observations of Sprites, a host of researchers including [2, 3, 4, 5, 6], have investigated the electrodynamic behavior of Sprites resulting from the large scale lightning events. Today it is well known that the upper portion of the Sprite consists of large number of closely grouped streamers. This fact has been confirmed by both both simulations and observations [1, 4]. However, because of the complexity involved in modeling individual streamers over the entirety of the Sprite, it becomes necessary to use effective atmospheric parameters in order to estimate the transient field and energy signatures over any significant region where ionization takes place.

Prior research in the area of modeling the behavior of lightning induced Sprites has been largely based on simultaneous solutions of the diffusion equation (divergence of Ampere's law: $0 = \nabla \cdot (\sigma \mathbf{E} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}_s)$) and the electron ionization and attachment equations $\frac{dn_e}{dt} = (v_i - v_a) n_e$. Most current "bulk parameter" models referenced in the literature are based on Pasko's earlier research [3]. These models assume $\mathbf{E} = -\nabla \phi$ and use an expanded form of the divergence equation ($\frac{\partial \rho}{\partial t} = \nabla \sigma \cdot \mathbf{E} + \rho \sigma / \varepsilon_0$) to obtain a solution via the method of cyclic reduction [3]. In the research presented here, a Finite Element (FE) simulation of the Ampere's law is employed which accounts for the effects of ionization on the conductivity during the Sprite. The FE model uses a very dense grid (maximum element size $\sim$ 500 meters) in the vicinity where Sprites typically occur (altitudes $\geq$ 40 km) [7]. This modeling differs from earlier work in that the closed form of the divergence equation is solved using Galerkin's method without separating out the gradient operator.

2. Problem formulation of the Sprite event

Based on previous models of the Sprite event [7], an equivalent electrical representation of the atmosphere can be made by assuming a bounded right circular cylinder having an outer radius of 100 km and height of 100 km. The earth's surface and upper boundary ($z = 100$ km) are modeled electrically as a perfect conductors. Typical values of .001 to .01 mhos/meter are given for the conductivity [8] of the earth's surface, while 10^{-14} to 10^{-13} mhos/meter is typical for the adjacent atmosphere's conductivity. This is a difference of more than 10 orders of magnitude, making the earth's surface appear to be an electrically perfect conductor with respect to the atmosphere. The condition at the outer radial boundary is chosen to emulate the effects of $r \rightarrow \infty$ and second order elements are used for the modeling. The simulations of interest were found to be insensitive to increases in either the radial (100 km) or vertical (100 km) limit. The radial boundary condition (100 km) was tested as both a perfect conductor and as a perfect dielectric; for both conditions the simulations results were approximately the same and the former case was chosen. The selection of $z_{upper} \sim 100$ km for the height of the upper boundary was a necessary consequence of the atmospheric conductivity structure becoming extremely complicated by several factors. Many of these factors are likely to be of second order and dismissed in the study.

The equivalent charge removed during the the lightning event is assumed to be 100 C and centered at z = 10 km, having a spatial spherical Gaussian distribution and an exponential time decay constant of $\tau = 0.5$ msec. [8]. The partial differential equations in the form used for the simulations are based on the divergence of Ampere's law, and the electron ionization and attachment equations according to [7]. It should be noted that space does not allow the complete development of the mobility, electron density and ionization equations used in the research. For additional information please see [3]. **E** denotes the electric field (V/m), V is the The MKS system of units is used in all equations with $\mathbf{J_s}$ being the source current density associated with return stroke current , ϵ_0 the permittivity of free space, $\partial p_f/\partial t$ is the source of forced charge density associated with the return stroke current causing the sprite, v_i and v_a are the ionization rates [3], n_e is the electron density, and μ_e is the electron mobility. The quasi-static formulation of electric field is used ($\mathbf{E} = -\nabla\phi$) and the remaining equations in the differential form used are shown below:

$$\sigma = q \cdot \mu_e \cdot n_e \tag{1}$$

$$\frac{dn_e}{dt} = (v_i - v_a) \cdot n_e \tag{2}$$

$$\nabla \times \mathbf{H} = \sigma\mathbf{E} + \epsilon_\mathbf{0}\partial\mathbf{E}/\partial\mathbf{t} + \mathbf{J_s} \tag{3}$$

By using the definition for **E** and taking the divergence of the Maxwell current density (3), the continuity equation implemented in this study is derived and shown below:

$$\nabla \cdot (\sigma\nabla\mathrm{V} + \varepsilon_0 \frac{\nabla\partial\mathrm{V}}{\partial\mathrm{t}} + \frac{\partial\rho_f}{\partial t}) = 0 \tag{4}$$

Initially, the electric field and electric potential are set to zero everywhere and the ambient value of the conductivity is used. A detailed discussion of the prototype finite element code the simulations are based on is given by Baginski [8] and the references therein.

3. Results

Fig. 1 is a time snapshot of the total power absorbed at t = 3.4 μsec. The figure clearly shows that the most rapid energy dissipation occurs on axis at altitude from 85-95 km. This is in region where "Elves" and "Halos" are known to occur and agrees well with observations [5].

Fig. 2 is a later time snapshot of the total power absorbed at t = 5 msec. The figure clearly shows that the center where energy peaks moving downward and centered at $\sim$ 70 km. This is in the vicinity where the red Sprites are usually observed. The time frame agrees well with observations and shows there is a direct correlation between energy dissipation and visible photo ionization.

Representative schema of the temporal variation in the energy dissipation versus altitude is shown in Fig. 3 (altitudes = 70, 80, and 90 km). This is the region of the atmosphere where maximum enhancement in electron ionization and conductivity was observed, which is consistent with maximum luminosity of Sprite observations [5]. After a thorough examination of the simulations it was found that, for the time scales $\sim$ 1 - 10 msec., the maximum energy transfer occurred at an altitude of $\sim$ 70 km. This can be directly attributed to the significant magnitude and duration of the power density at 70 km for times $\leq$10 msec.

Fig. 4 shows the corresponding vertical electric field signatures. It is obvious that the relative maximum shown in all traces occurs at times that are directly proportional to the dielectric relaxation time at the corresponding altitude of the observation ($\tau = \epsilon/\sigma$). Following the onset of the peak value, the electric field decay appears to have two decay rates, each decaying in an approximately exponential manner at all altitudes. This is obvious for the higher altitude traces. However, this behavior was not clearly shown in the 60 and 70 km traces, since the time frame of interest was limited to 5 msec. to enhance the short time pulse characteristics.

4. Conclusions

A FEM simulation of the energy and electric field traits occurring in the atmosphere resulting from lightning induced electron enhancement was presented. The general traits were found to be in good agreement with observations and additional details will be presented at the conference.

References

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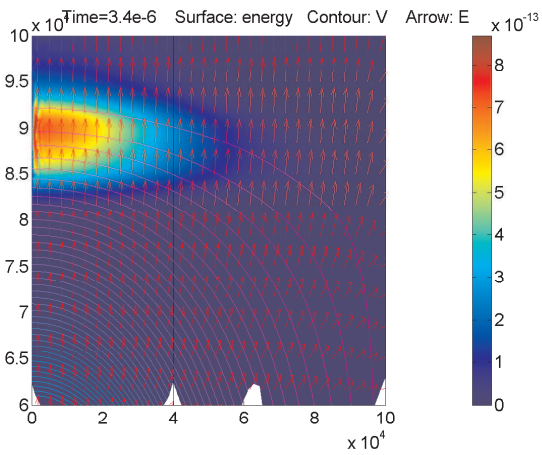

Figure 1: Surface plot of total energy dissipated at 3.4 micro-seconds above 60 km.

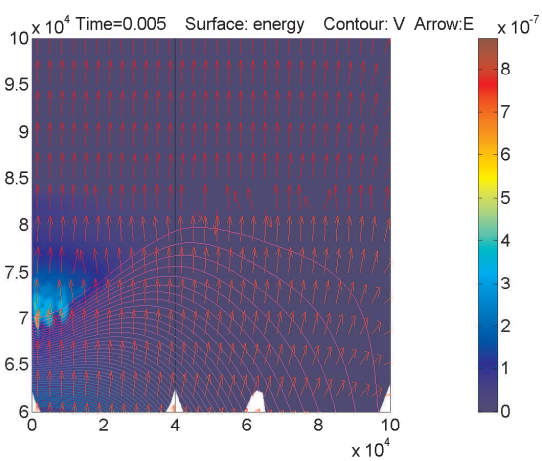

Figure 2: Surface plot of total energy dissipated at 5 milliseconds above 60 km.

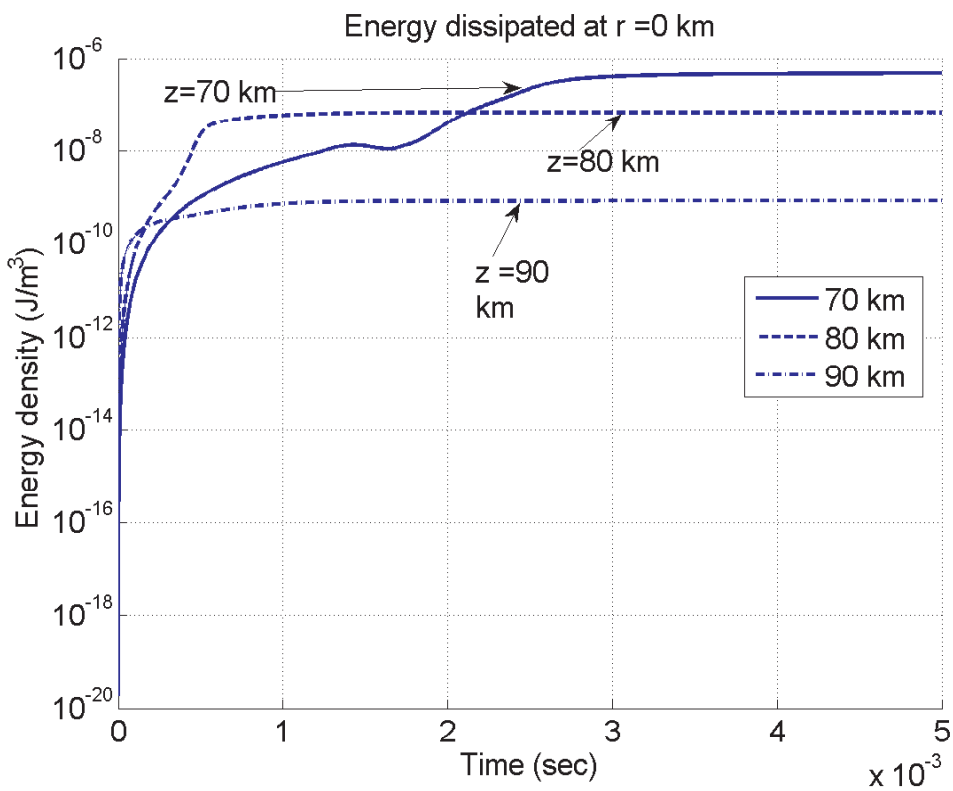

Figure 3: Total energy dissipation versus time at a radial distance of 0 km for altitudes of 70 - 90 km.

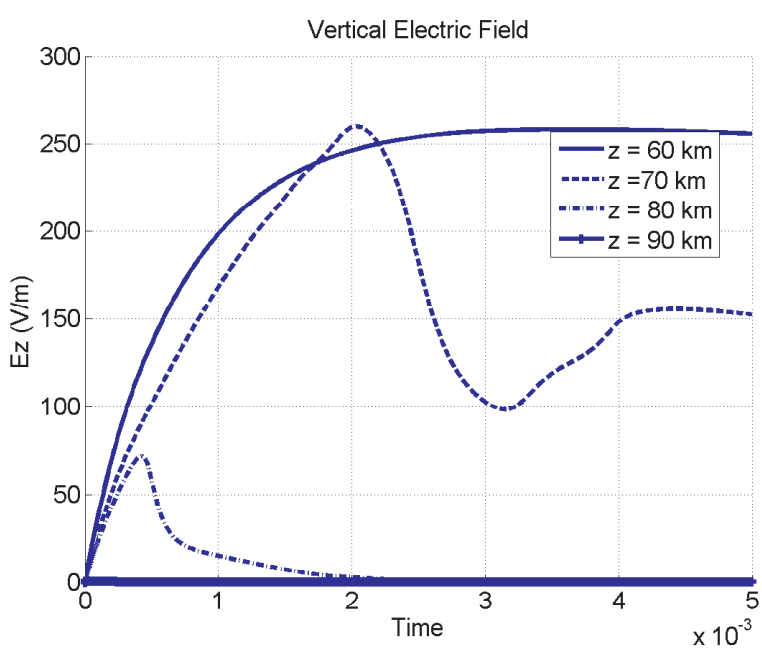

Figure 4: Vertical electric field signatures at r =0 km for altitudes of 70 - 90 km.