Spatial distribution of model transient electric field in the mesosphere above a bent stroke of lightning

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Abstract

We present the pulsed electric fields computed in the neutral atmosphere above a powerful positive lightning stroke with the bent channel containing vertical and horizontal sections, each 10 km long. The fine structure of field is demonstrated arising in the space due to combination of delayed pulses arriving from the stroke sections. The electric field distribution depends on time and on the stroke orientation with respect to an elevated observer. Characteristic size of ‘filaments’ in the transient electric field is about 1 km along the horizontal direction, and it reaches a few tens of kilometers in the height.

1. Introduction

The present paper is related to the red sprite phenomena, which are widely discussed in literature (see e.g. [1] and reference therein). The most effective and successful model of sprite formation was introduced in paper [2]. It exploits the quasi-electrostatic field, which remains in the atmosphere after a lightning stroke downloads one half of the cloud dipole charge. Recent development of cellular automaton network combined with a phenomenological percolation probabilistic approach resulted in effective time-space modeling of the sprite structure [3]. It provided structures similar to experimental video images of sprites. All these models are based on the static field of a thunderstorm while the pulsed radio emission is ignored.

The latest records of sprite images with exceptionally high time resolution [4] demonstrated that a sprite consists of many bright fast moving elements. We address the spatial distribution of electric field radiated from a parent stroke. Such an objective is justified by the fact that quasi-static field depends on distance \( r \) as \( 1/r^3 \), while radiation decreases as \( 1/r \). Therefore, radiation from a lightning stroke might be as important in the sprite formation as the static background. We show that radio emission from a simplest, bent discharge results in a sophisticated distribution of electric field in space and time. Such structures initiate a further development of filaments in the sprite body.

2. Model description

When computing electric pulses in the mesosphere, we use the model of a stroke with the bent channel [5, 6]. The intense positive discharge transfers 126 C to the ground from 10 km altitude, which agrees with typical charge moment of sprite-generating discharges [1]. We apply a standard model of a return lightning stroke [7, 8]. The current at the base of a positive stroke is the following sum: \( I(t) = \sum_{k=1}^{4} I_k \cdot \exp(-t/\omega_k) \), \( t \geq 0 \). Amplitudes \( I_k \) and relevant inverse time constants are equal to: \( I_1 = 569 \text{kA}, I_2 = -460 \text{kA}, I_3 = -100 \text{kA}, I_4 = -9 \text{kA}, \omega_k = 6 \times 10^5 \text{s}^{-1} \),
\[ \omega_2 = 3 \times 10^4 \, s^{-1}, \quad \omega_3 = 2 \times 10^3 \, s^{-1}, \quad \text{and} \quad \omega_4 = 147 \, s^{-1}. \]  The peak current of 480 kA is reached in 5.5 \, \mu s, after the stroke initiation. The electric charge transferred to the ground is \[ Q = \int_0^\infty I(t) \cdot dt = 126 \, C. \]

The current wave propagates along the stroke with the velocity \[ V(t) = V_o \cdot \exp(-t \cdot \omega_y), \] where \( V_o = 8 \times 10^7 \, m/s, \) and \( \omega_y = 4 \times 10^3 \, s^{-1}. \) The total length of the lightning channel is \[ L = \tau_y \cdot V_o = 20 \, km. \] The radiation moment of the stroke for is found from [8]:

\[
M_R(t) = V_o \sum_{k=1}^{4} I_k \exp(-\omega_k t) \left[ 1 + \frac{\omega_k}{\omega_y} \right] \exp(-\omega_y t) - \frac{\omega_k}{\omega_y} \right] \] \quad t > 0 \quad (1)

Initial vertical part of the channel is 10 km long, from the 10 km altitude the current proceeds horizontally along the X-axis. Therefore, a horizontal source moment appears after the ‘turning time’ \( t_k = \frac{1}{\omega_y} \ln \left( \frac{V_o}{V_o - \omega_y z_k} \right) \) necessary for reaching the altitude \( z_k = 10 \, km. \) Reflections from the perfectly conducting ground are also included. Radiation component of electric field (components \( E_x, E_y, \) and \( E_z) \) was computed for an arbitrary site in neutral atmosphere as described in [5, 6].

3. Results of computations

The vertical section of lightning channel and its reflection in the ground provide an initial electromagnetic pulse with a ‘doughnut’ field distribution in space. Radiation from the horizontal section appears after the current wave turns horizontally \( (t = 390 \, \mu s \text{ from the stroke initiation}). \) Finally, the wave reflected from the ground provides an additional pulse. Spikes in the pulsed waveform appear in accordance with the onset of radiation from definite parts of stroke and with appropriate propagation delay. The concept is rather straightforward if not a trivial one. However, little attention was given to the transient electric field distribution in space above the thunderstorm. We computed the temporal variations at a set of positions ranging from 50 to 100 km vertically and from –50 to 50 km horizontally. These data allowed for constructing the spatial distribution of three field components \( (E_x, E_y, E_z) \) at fixed moments of time. The 2D cross-sections of these 3D distributions show how individual pulses interact in space after arriving from different parts of stroke and its reflection in the perfectly conducting ground.

Temporal ‘motion’ is shown in Fig.1 of the spatial distribution of transient field \( E_z. \) The frames in this figure correspond to time \( t \in [320; 440] \, \mu s \) varying with step of 30 \, \mu s. General upward motion of pulsed fronts is clearly visible combined with considerable alterations in the filament structure. The Cartesian coordinate system is used with the origin at the stroke base and vertical \( Z - \) axis. The horizontal section of the stroke channel is directed along the positive \( X - \) axis. The “Along” plane is shown in figure being the XZ plane.

4. Discussion and conclusion

Pulses in Fig.1 were computed for the observer altitude \( Z = 80 \, km \) and horizontal position varying from –50 to 50 km from the stroke base. The origin of coordinate system is set at the base of the cloud-to-ground section. Each frame in figure depicts a spatial distribution of the field for a fixed moment of time, the latter increases from the bottom to the top. The radiation pattern is clearly seen in all frames. The lowest frame depicts the field when distribution when the positive electric pulse from the vertical part of channel (red) is accompanied by the negative transient field arriving from the horizontal part of the stroke channel.

The second frame depicts the vertical shift of the field with time combined with its spatial re-distribution caused by interference in space of pulses from horizontal and vertical parts. Particular arrival times depend on the distance and orientation of horizontal branch in respect to the observer. Therefore, different ‘hair comb’ distributions arise depending on the time. The positive fields correspond to reddish palette, and negative – to blue palette. The first will attract the free electrons, while the second will repel them. As one may see, electric fields exhibit various complicated structures. As a result, the free atmosphere particles are subjected to irregular transient electric field.
The reflected from the ground pulsed field appears after 0.4 ms from the stroke initiation. It is seen as red area in the plots, since the sign is opposite to that of direct pulse from the horizontal segment of the source current. Computations indicate that fine structure persists for a long time. The particular pattern varies rather fast, but filaments tens to be always present in transient (pulsed) electric field. Substantial changes occur in a hundred of microseconds, while the stroke current has scales smaller than 10 $\mu$s. The physical reason of such a slowing down is clear: the radiated pulses depend on the velocity of the current wave moving along the stroke channel. Its velocity reduces when moving along the vertical part, and it has already moderate value when entering the horizontal section. Thus, the completing pulsed radiation from a bent stroke becomes extended in time.

Model computations show that electric field in atmosphere remains structured and variable within the YZ and an arbitrary plane. Similar conclusion is valid for the pulsed horizontal electric field. Clearly, there is a complicated spatial distribution of field, different for particular field components and depending on time in individual way. We show here the dynamics of a single distribution, demonstration of other components or other planes will take too much place.

We confine ourselves to modeling of the radiation fields of a bent stroke. Nevertheless, simplistic model results in rather complicated field distributions related to geometry of lightning discharge. Data obtained present dynamics of the ‘driving force’ that would interact with the free charged particles of the atmosphere above the thunderstorm. Such forces might be used to integrate the motion equations and obtain a feasible bunching of particles and formation of ‘beams’ above the thunderstorm. Our simple treatment indicates that a powerful stroke with a complicated geometry of current channel causes noticeable transient electric forces in the mesosphere. These could in turn not only accelerate free particles, but also to ‘warm up’ different ‘cells’ in the air thus supporting further formation, motion, and branching of streamers that are observed experimentally. It looks like that highly structured electric field produced by a 'parent' stroke may serve as a ‘seed’ for future filaments in sprites.

We obtained the structured pulsed electric field of a simple bent stroke. Its fine structure in the space arises from the sequential radiation from the vertical and horizontal sections of lightning channel. The simplistic model was applied of a bent positive return stroke that consists of a single vertical and a horizontal section, each 10 km long.

![Fig.1. Temporal development of $E_z$ field component with 30 $\mu$s time steps](Image)

The current waveform and velocity of the current wave along the channel were borrowed from the usual models return strokes (s.f. [7, 8]). Physics of a bent lightning stroke was not addressed, as this is separate and a
complicated problem itself. Computational data indicate that presence of two sections in the stroke channel is obligatory for the effect demonstrated. Particular distribution of the field and its temporal variation are governed by the stroke morphology and by details of the current wave progress along the channel.

The standard explanation of sprite development exploits the sole quasi-electrostatic field [1. 2]. Radiated fields are usually ignored. In contrast, we ignored the static field and turned to the radiation component alone that was computed in the neutral atmosphere. In reality, the radiation fields supplement the quasi-static component when forming a sprite. Radiation field decreases as $1/r$, and it starts from smaller value than the static field, but the static field decreases as $1/r^3$. Both the fields become equal at distance where $kr = 1$ (the classical condition for the ‘far zone’). The equality is held for $f = 1000$ Hz when the distance $r = 47.8$ km, and we know that the frequency band lies well above 1 kHz occupied by the transient fields from typical lightning strokes. Hence these fields must be added when modeling the sprite development. The influence of conductivity is usually ignored at such ‘high’ frequencies [1, 2]. If one accounts for the finite conductivity of air, the transient field amplitude will decrease faster with distance; the details will ‘smoothed’ in the spatial structure – separate filaments might ‘blur’ or ‘merge’ owing to the wave dispersion. However, the structure will not vanish completely. We hope that the data presented here attract attention and will be included in future modeling of sprite development.

To summarize the material of the present study, we list the main results.

1. Spatial distribution was computed of transient electric fields in the air above a thunderstorm with the bent stroke of lightning. The simplest model was applied: the extended in length and amplitude positive stroke having the single vertical and a horizontal sections in the current channel.

2. Owing to the finite velocity of current wave, a series of pulses arrives to the elevated observer. Each pulse originates from different parts of the bent stroke or from its reflection in the ground.

3. Superposition of pulses in space forms a structured electric field above the parent discharge.

4. Characteristic size of ‘filaments’ in transient electric field is about 1 km in horizontal direction and reaches a few tens of kilometers along the vertical.

5. References


