Interfacing 3D Cartesian TLM Computational Regions with Unstructured Tetrahedral Layers

P Sewell, C Christopoulos, TM Benson, DWP Thomas, A Vukovic, JG Wykes

George Green Institute for Electromagnetics Research, The University of Nottingham, University Park, Nottingham NG72RD, UK, phillip.sewell@nottingham.ac.uk, christos.christopulos@nottingham.ac.uk, trevor.benson@nottingham.ac.uk, david.thomas@nottingham.ac.uk, ana.vukovic@nottingham.ac.uk, jim.wykes@nottingham.ac.uk

Abstract

Transmission Line Modeling, TLM, is a flexible and established numerical simulation technique that has been adopted for a wide variety of electromagnetic applications. Traditionally, structured, usually Cartesian, meshing is used to describe the computational geometry even though this inevitably introduces staircasing errors of smooth boundaries and provides restricted scope for power conserving and stable multi-scale meshing. Recently, the feasibility of unstructured tetrahedral TLM has been demonstrated which alleviates many of the difficulties associated with Cartesian meshes. Although the computational effort and memory requirement for each unstructured cell is greater than those of its Cartesian counterpart, a net computational advantage is generally recovered, as fewer unstructured cells need to be used to represent complex geometries. The use of hybrid structured-unstructured meshes provides further computational advantages: Computationally efficient Cartesian cells are employed to describe large uniform regions of space with thin layers of tetrahedra providing smooth boundary descriptions and stable and conservative interfaces between the structured regions. This paper will describe the properties of such hybrid schemes.

1. Introduction

Numerical techniques are the robust workhorse for many simulation tasks offering geometric flexibility and algorithmic simplicity, usually suitable for large-scale parallelization. However, the recurrent difficulty with such approaches is their voracious appetite for computational resources, both in terms of memory and run times. In order to alleviate this problem it is preferable to employ a description of the geometry in terms of regular computational cells connected in a structured manner. This reduces the number of computational parameters that the overall simulation must store as well as minimizes the need to explicitly store connectivity information. However, such descriptions entail *snapping* the geometry to a discrete grid so that material and computational boundaries which are not naturally aligned with the grid become *staircased*. The consequences of this approximation are analogous to introducing surface roughness into the problem, causing unphysical scattering of the fields as well as distortion of wiring routes resulting in shifts in their resonant lengths and field coupling strengths. To avoid these phenomena, it is often necessary to mesh to the scale of the geometry's fine features, rather than to the scale of the wavelength of the fields of interest and typically this negates the computational advantages of using a structured mesh of regular computational cells. The alternative approach is to adopt unstructured meshes with canonical cells such as tetrahedra or non-cubic hexahedra which provide piecewise linear rather than piecewise constant boundary approximations. In this case, the electromagnetic response of each cell is different which entails explicitly storing parameters to characterize it. Furthermore, the inter-connectivity of the cells must also be explicitly stored. Additionally, the issue of generating suitable mesh descriptions of complex geometries satisfying the constraints imposed upon the cell geometry by the computational algorithm is a major concern not to be underestimated. However, as unstructured meshes provide smooth boundary approximations and a natural mechanism for grading the size of the cells in different parts of the problem, as dictated by the local feature size, a net computational advantage is usually obtained for a given level of simulation accuracy.

Multi-grid methods seek to compromise between these two approaches, [1,2]. The geometry is described in terms of a number of uniform regions each decomposed into structured cells, but with each region using different size cells. Adopting this approach hierarchically, leads to Octree and similar meshes, which use wavelength-scale cells in large empty parts of the problem and local feature-scale cells near boundaries and sub-wavelength objects such as thin wires, [3]. Although multi-grid methods are well established and provide good computational efficiency, this is often bought at the expense of guaranteed computational stability and energy conservation, as interfacing the different computational regions typically entails numerical interpolation. Moreover, the fine featured regions are still described in a piecewise constant manner requiring finer grids than absolutely necessary, albeit that now local feature sizes only impact on the local, rather than the global, mesh scaling.

In this paper, a hybrid structured-unstructured mesh approach is presented for 3D Transmission Line Modeling, TLM. In this case, the multi-grid philosophy is used to decompose the geometry into different regions each consisting of structured meshes of regular cells, but now the interfaces between the different regions are thin layers of unstructured tetrahedra. Additionally, smooth boundary descriptions are also achieved by using layers of unstructured tetrahedra. Using tetrahedral interfaces provides both assured stability and energy conservation and as the unstructured cells are typically only used in proximity to surfaces their proportion of the total cell count diminishes, as the mesh is refined.

2. Unstructured Tetrahedral TLM

TLM is time domain numerical modeling approach that maps the behavior of electromagnetic fields onto an equivalent 3D network of commensurate transmission lines, [4]. The network comprises a set of transmission *link-lines* that are connected together at *nodes* of the network. By propagating ideal voltage impulses throughout the network, alternating between scattering at nodes of the network and propagation to adjacent nodes, the evolution of the analogous electromagnetic fields is mimicked. As it is based upon a passive network paradigm, the algorithm is both stable and conservative. The transit time along all the transmission lines is the same and determines the time step of the algorithm, being constrained by the requirement that all line characteristic impedances remain positive and is consistent with the Courant condition. Previously, the TLM approach has been extended to the case of unstructured meshes in both 2D and 3D, [5]. In 3D the canonical cell is the tetrahedron and this appears in the equivalent network as a scattering node connected to four adjacent nodes which correspond to the tetrahedra with which it shares common faces. The scattering characteristics of such cells have been determined and are straightforward to evaluate in practice. The network's scattering nodes geometrically correspond to the circumcenters of the tetrahedra and the link-line lengths to the distances between adjacent tetrahedral circumcenters and requiring that these lengths remain positive, demands that the tetrahedral mesh is strictly Delaunay. As the smallest link length dictates the time step of the overall algorithm, it is desirable that adjacent circumcenters are as evenly spaced as possible. However, this is not generally the case even for good quality meshes and to overcome this problem, clusters of adjacent nodes interconnected by short links are explicitly pre-processed to create larger scattering objects, [6]. In particular, clusters of tetrahedra forming regular cuboids and triangular prisms commonly occur and it has been demonstrated that the merging process employed yields the conventional scattering characteristics of the HSCN TLM cubic cell, [4], for example.

3. Hybrid Meshes

As the tetrahedral clusters just described naturally encompass the case of Cartesian cells and, more importantly, their seamless integration into an unstructured tetrahedral mesh, it is straightforward to deliberately introduce regions of structured cubes into the approach. The mesher recovers a basic tetrahedral description of the problem ensuring that all required boundary facets have been recovered. Subsequently, a coarse regular cubic grid is overlaid on the geometry which is then recursively refined as in Octree meshing according to user-defined scaling criteria in different regions of the problem space. The resulting cubic multi-grid is then trimmed to lie within the geometry, imposing proximity criteria to ensure that the cubic cells neither lie too close to boundaries nor their vertices too close to existing mesh points. Having extracted the cubic regions, the remaining space between them and between the cubes and the geometrical boundaries is refined using incremental Delaunay insertion. The usual Delaunay refinement approach selects bad cells for splitting on the basis of their circumradius to minimum edge length ratio, [7]. However, it has been found that this strategy often results in link line lengths which are neither short enough to justify cluster formation with adjacent cells, yet are small enough to result in small TLM time steps. Mesh smoothing techniques based upon Centroidal Voronoi, [8], and Optimal Delaunay, [9], meshes have been tried and although these do improve the regularity of the mesh, they do not generally alleviate the short link line problem. A generally successful approach has been to change the criteria for selecting bad cells in the Delaunay refinement: Bad cells are simply defined as those which result in a small TLM time step. Interestingly this strategy also appears to recover good meshes in terms of the circumradius to minimum edge length ratio, which is unsurprising given that these measures are inherently linked.

4. Results

The example selected to illustrate the approach is the simulation of a small TEM cell used for the characterization of the impact of RF fields on biological samples. The cell comprises a rhombic outer metal surface, which tapers rapidly in width and within which a thin tapered metal septum acts as the central conductor. The ends of the septum are connected to coaxial conductors that provide electromagnetic feeds to the cell. The cross-section of the cell has been designed to support a $50\,\Omega$ TEM field at all points.

This geometry is problematic for uniformly sized structured grids as the sloped metal walls become staircased, the consequences of which are observed as non-physical scattering and as non-uniformity of the fields at the cell center where the samples are placed. Moreover, the cell cross-section tapers by a factor of 12, from 2cm to 24cm, over a distance of 17cm and therefore as a fine grid is necessary to correctly capture the geometry of the feed connection, this results in the use of a large number of cells for the whole problem. Although graded rectangular cells could be beneficially employed here, the worse case scenario of regular cubes is considered for comparison.

Using the hybrid approach described above, each half of the TEM cell is split into 3 regions for cubic meshing; coarse in the center, fine at the feed with an intermediate region between. Feeds are obtained by explicitly extruding the end of the cell to form square cross-section $50\,\Omega$ TEM waveguides. To excite the simulation, the triangulated input cross-section is extracted from the mesh and a very short length, Δ_z, of waveguide is extruded from it comprising prismatic triangular cross-section elements. A frequency domain eigenmode solver variant of the unstructured TLM algorithm recovers the fundamental waveguide mode by requiring that the fields on the start and end facets of the waveguide are related by $e^{-j\beta\Delta_z}$ where β is the propagation constant, [6]. This provides both the field distribution and the numerical wave impedance for exciting the full 3D simulation. The cell is excited by two cycles of a 300MHz sinusoidally varying unit power fundamental mode and the power entering a matched load on the output of the cell observed against time. All simulations are undertaken on a 4 CPU workstation and have been parallelized using OPENMP.

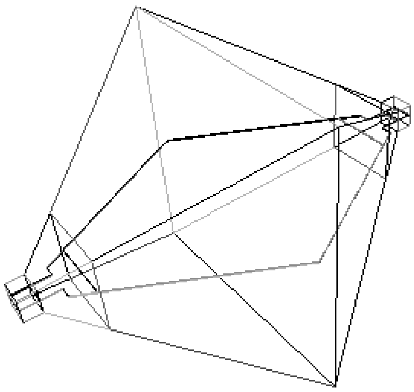
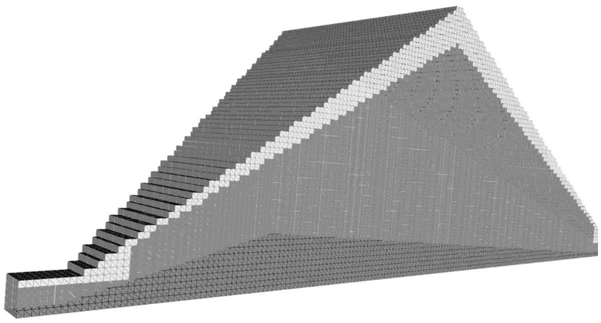

Figure 1: Left, the TEM cell geometry. Overall length 37cm, with 1.5cm of waveguide feed at each end. Square cross-section metal outer shield tapering from 2.1cm to 24cm. Zero thickness metal septum of width 85% of the cell width and length 30.5cm, stepping to feed sections of width 1.785cm. Right 2mm cubic approximation to one quarter of the structure.

In this brief space we present just an indication of the quantitative results, concentrating on run time savings due to hybridization, with full characterization being made in the presentation. The accuracy of the unstructured and unstructured-with-clusters TLM has been established in [5, 6]. Figure 1 shows the detail of the geometry and figure 2, typical unstructured and hybrid meshes. Even the 2mm regular grid employing 214,877 cubes shown does not maintain the electrical isolation between the septum and the outer wall and fails to provide physically meaningful results. Two hybrid meshes are used, that of figure 2 and one with the cube sizes halved. These employ 566 (9113) ideal cubes, 5589 (22040) tetrahedra and 1688 (9588) interface clusters, of which 480 (2813) are cubes with one or two link lines of non-standard length. As each ideal cube can be geometrically split into 5 tetrahedra, this provides a count to determine unstructured meshes with approximately the same sampling density and with which run times are compared. The hybrid meshes required 478s and 13708s compared to 633s and 26955s for the unstructured cases. In terms of accuracy, the results are in good agreement and consistent with experimental values as will be shown in the presentation. Typically, the unstructured and hybrid simulations presently operate with (0.1 to 0.01 times) smaller time step than the equivalent density regular grids and therefore their accuracy is presently limited by dispersion errors rather than the geometrical errors of the latter. The main contributor to this limitation is the mesh characteristics as discussed above although it is felt that there is significant scope to improve this, which will not only benefit the accuracy, but also yield shorter simulation times.

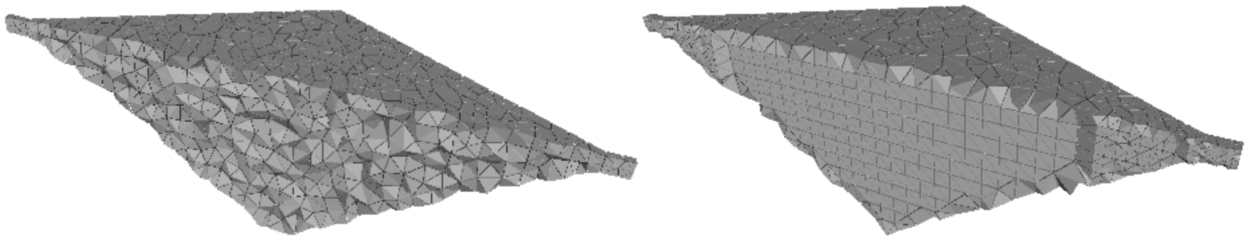

Figure 2: Cut aways of comparable density (left) unstructured and (right) hybrid meshes using 10, 5 and 2.5 mm cubic regions interfaced with tetrahedra.

5. Conclusion

Hybrid structured-unstructured mesh TLM has been investigated. This provides a valuable degree of flexibility for efficiently performing simulations of structures with a diverse range of scales. In particular, as the scattering from the regular cells can be implemented more efficiently than that of the unstructured cells. Useful run time savings, here of 50%, are recovered.

6. Acknowledgments

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7. References

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