Light Transmission through Two-Dimensional Photonic Quasicrystals

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Abstract

Light transmission through two-dimensional photonic-quasicrystal (PhQC) slabs fabricated from multiple exposure techniques is numerically studied using the finite-difference time-domain (FDTD) method simulation. The observed semi-forbidden band gaps are compared with band gaps of periodic photonic-crystal (PhC) structures having identical average refractive indices. It is demonstrated that the same argument of multiple reflections can be extended from PhCs to PhQCs to predict the location of the semi-forbidden band gaps of the PhQCs.

1. Introduction

The photonic crystal (PhC) refers to any structure periodically composed of materials of different dielectric constants, which features totally forbidden frequency bandgaps (named as photonic bandgaps (PBGs)), for no light can propagate through the crystal in this frequency range [1]. Traditional crystals are with translational symmetries. Althrough the study of three-dimensional (3D) space symmetry indicates that all normal crystals can be described by 230 space groups, in 2D structures only two kinds of lattice, either square or hexagonal, are available. Penrose [2] proposed a scheme to fully cover a plane using two different types of tiles in 1974; since then, the concept of quasi-crystals has been enthusiastically studied and has found applications in different areas including optics [3-6]. Aside from curiosity about those novelties, one reason to study on quasicrystals is to find better PBG materials. The underlying mechanism for PBGs has been known to be due to multiple reflections/refractions from the periodic structure. In order to have PBGs, the structure must provide total reflections in all directions for the forbidden frequency --- the discrete translational symmetries are associated with total reflections and the rotational symmetries are associated with angular degeneracies. Under this premise, the lattice is actually providing the first-order dominance over the PBGs; and indeed the lattice determines the fundamental features of the band diagram: only a hexagonal lattice can support a total PBG for both transverse-magnetic (TM) and transverse-electric (TE) modes. However, although there are only two kinds of perfect crystals in 2D space, there exist innumerable quasicryatals: they have no translational symmetries at all, but with more rotational symmetries instead. The combining effects are still unclear, but there are reports of promising results. In this paper, we study light transmission through 2D photonic-qusaicrystal (PhQC) slabs of different symmetries and compare its spectrum with a PhC slab case having an equivalent refractive index.

2. PhQC Structures and Analysis Method

Following [5], we consider PhQCs fabricated from multiple exposure techniques, which are more compatible with integrated optics. The 2D quasicrystal is assumed to be uniform in the *z* direction. The PhQC structure is generated from the diffraction pattern of optical beams with different incident angles, and the optical exposure causes the variation of the refractive index as: $n(\vec{R}) = n_0 - \Delta n[I(\vec{R})]$, where $I(\vec{R}) \approx \sum_{i=0}^{N-1} |E_i(\vec{R})|^2 = \sum_{i=0}^{N-1} |E_{0i}|^2 |\exp[j(\vec{k_i} \times \vec{R})]|^2$, with $\vec{R} = (x, y)$ and $\vec{k_i} = (2\pi/\Lambda)(\cos(2\pi i/N), \sin(2\pi i/N))$, $i = 0, \cdots, N-1$. The number of the optical beams, *N*, determines the desired rotational symmetry of the quasicrystal. The wavelength of the optical beams, Λ, is set to 430.55 nm, and the refractive index of the substrate, n_0, is set to be 3.4. For simplicity, all E_{0i}'s are set to be equal. The function Δn should be determined by empirical measurements. Due to the lack of this information, it is simply set in the linear form: $\Delta n = (n_0 - n_{air})I/I_{max}$ for an optical-burnt air holes when illuminated with the peak intensity, I_{max}. Resulting patterns of five, six, seven, and eight beams are demonstrated in Fig. 1.

We use the 2D finite-difference time-domain (FDTD) method [7] for the simulation study. The (x, y) coordinates of the 2D space are chosen as in Fig. 2(a) which shows a TM plane wave normally incident upon an air-hole PhC. The x axis is chosen as the main direction of propagation. Hence the TE mode only contains field components of H_x, H_y, and E_z, and the TM mode only contains field components of E_x, E_y, and H_z. This definition of TE and TM modes follows the convention in integrated optics; however, an opposite notation also exists, such as in [1]. For the PhQC slab in our transmission simulation, as shown in Fig. 2(b) for the five-beamed case, the x axis is in the vertical direction. We present the transmission spectrum for the PhQC slab under normal incident illuminations. The grid size is 10×10 nm^2, and the time step Δt is $\Delta x \times 10^{-9}$ s/m, about three times smaller than the constraint of Courant stability criterion.

3. Numerical Results for Transmittance Spectra

The transmittance of many-beamed PhQC slabs are numerically measured and compared with that of the PhC. All geometrical scales are chosen the same as shown in Fig. 2(b): the slab length is 11Λ and the slab width is 18Λ. For the PhC slabs having the lattice constant $a = \Lambda$ to be compared with, $n_{\text{substrate}} = 3.4$ and $n_{\text{hole}} = 1$ are fixed. One PhC slab and another PhQC slab are considered equivalent when their average refractive indices are identical. Under this constraint, the radius of air holes, r, in the equivalent PhC slab can be decided as: $r/a = 0.23534$ for the five-beamed PhQC slab (n_{average} =2.9178), $r/a = 0.21472$ for the six-beamed PhQC slab ($n_{\text{average}} = 2.9986$), $r/a = 0.19912$ for the seven-beamed PhQC slab ($n_{\text{average}} = 3.0548$), and r/a =0.18732 for the eight-beamed PhQC slab ($n_{\text{average}} = 3.0945$). All transmittance curves of TE and TM fields for the many-beamed PhQC slabs are measured and shown in the left panels in Fig. 3, and comparison of the transmittance curves of TM fields between the many-beamed PhQC slabs and their PhC counterparts are also presented in the right panels.

Two important features of the above results are the lack of complete-forbidden band gaps, and the presence of the semi-forbidden band gaps located at wavelengths around 1.3 μm (λ_0). The former result is a natural consequence of the lack of translational symmetries in the PhQCs. Moreover, the transmission curve of allowed bands of a PhQC is smoother then that of a PhC, which we suspect to be the consequence of the increased rotational symmetries. Also, there are more smooth rolling-off edges of transmission curves close to the semi-forbidden band gaps, whose underlying physical mechanism remains unknown and requires further investigations. The latter results of the semi-forbidden band gaps can also be explained using the concepts of multiple reflections. The locations of the semi-forbidden band gaps of various PhQCs are observed around ($\lambda_0/n_{\text{average}}$), which are all close to the structure constant of the PhQCs (the wavelength of the optical beams Λ, or the lattice constant a). This identical location of the semi-forbidden band gaps of one PhQC and the complete-forbidden band gaps of its equivalent PhC justifies our definition of n_{average} between one PhQC and its equivalent PhC. This also implies that we can extend the same argument of multiple reflections from PhCs to PhQCs to predict the location of the semi-forbidden band gaps of the PhQCs. Since this prediction is based solely upon the information of the first-order dominance of the lattice constant, and since there is no exact regular lattice for PhQCs, there must be some inevitable deviations. The second-order dominance of the aperiodicity manifests itself at the second semi-forbidden band gap located at larger wavelengths, which cannot be explained by the above rules-of-thumb and requires further investigations.

4. Conclusion

The concepts of PhQC have been explored in terms of light transmission behaviors, and comparisons with equivalent PhCs are made. We have demonstrated that multiple reflection mechanisms can explain the transmittance characteristics even for PhQCs, with simply an average refractive index and identifying the equivalent lattice constant with the wavelength of the optical beams from multiple exposures during the fabrication of the PhQC, so that the location of the semi-forbidden band gaps can be predicted.

Acknowledgments

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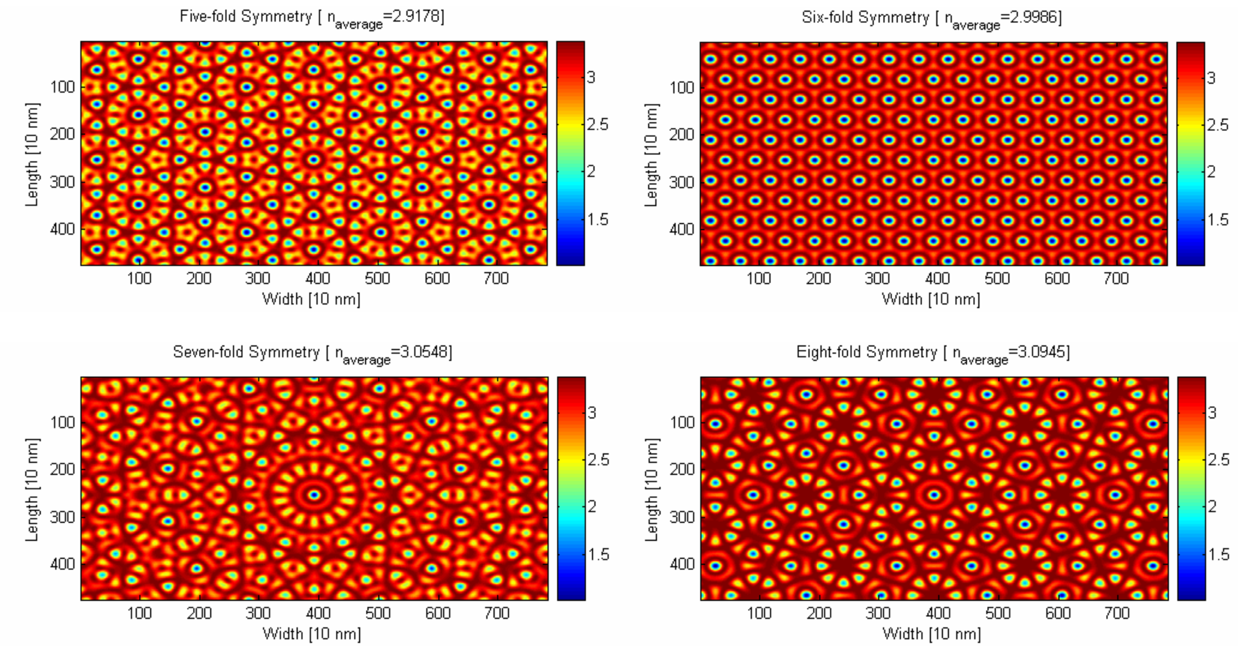

Fig. 1. Patterns of refractive indices for five-, six-, seven, and eight-beamed PhQCs.

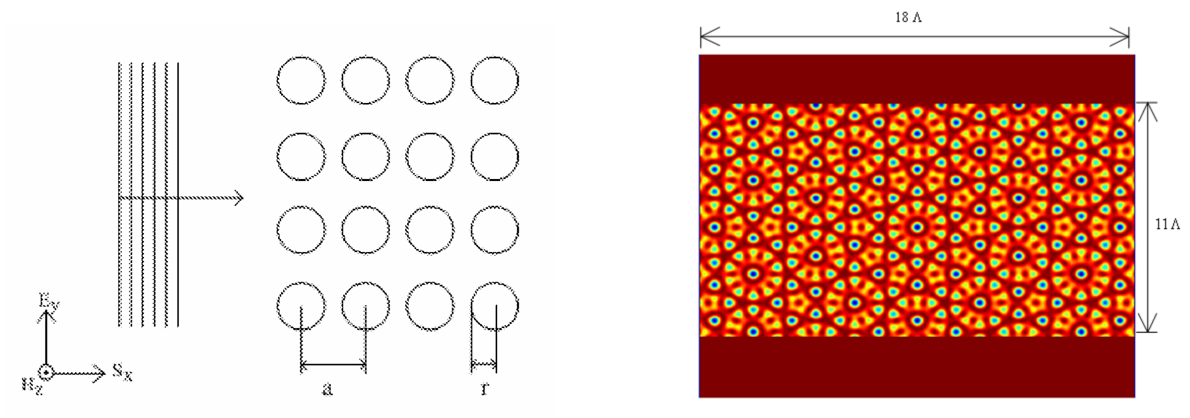

Fig. 2. (a) A TM plane wave normally incident upon a 2D PhC. (b) The geometrical structure of the five-beamed PhQC slab.

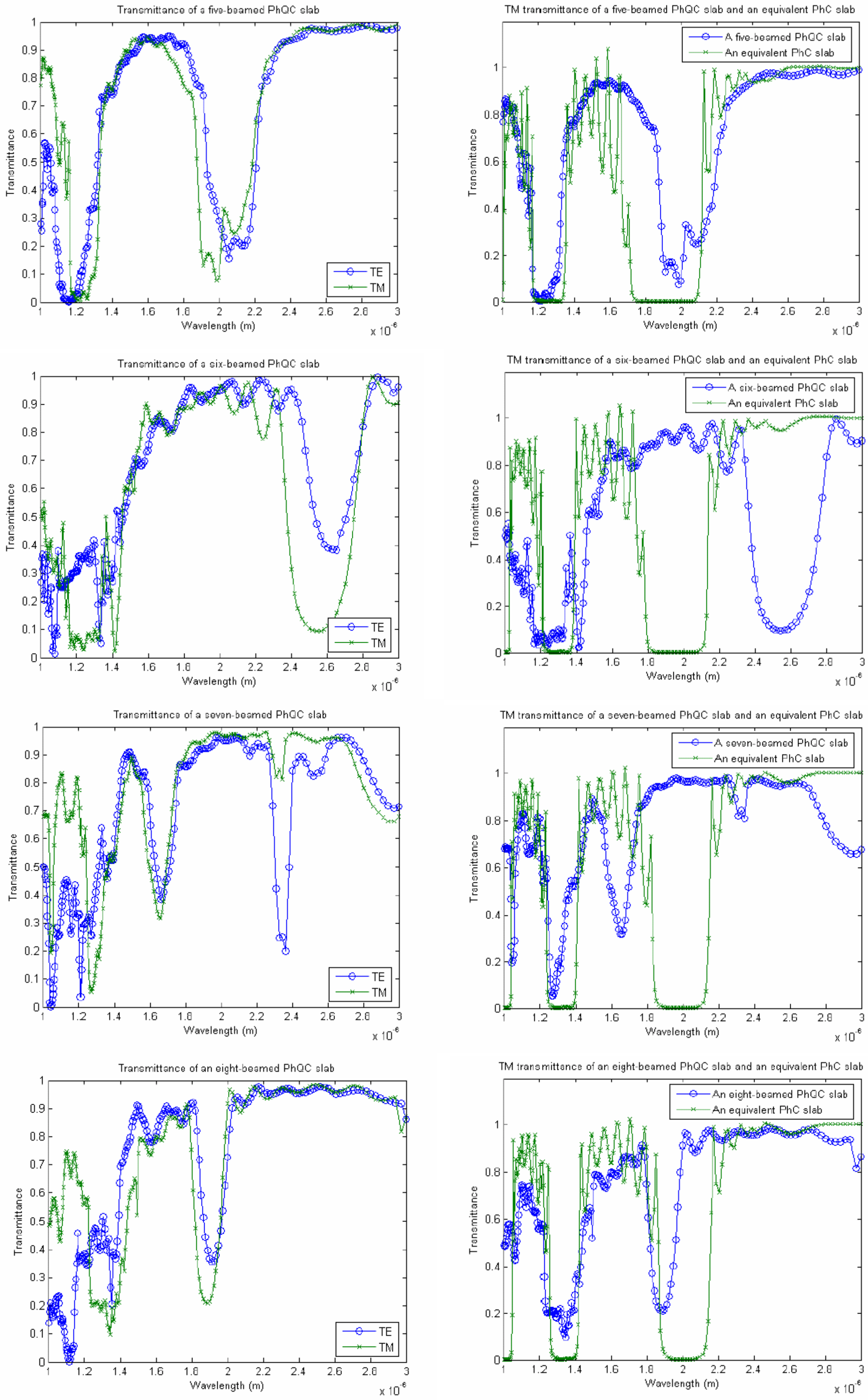

Fig. 3. Left panels: TE and TM transmittance spectra of five-, six-, seven-, and eight-beamed PhQC slabs. Right panels: TM transmittance spectra of PhQC slabs and corresponding equivalent PhC slabs.