FALSE LOCKING IN OPTICAL PHASE LOCK LOOP: REVISITED

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Abstract

A homodyne optical phase locked loop is considered. Even if the loop is stable, the presence of loop propagation delay can cause acquisition difficulties in the form of false locks. In this paper, we not only consider the false locking phenomenon in an OPLL, but also illustrate a method of calculating the locking range. Moreover, we consider the effect of finite line-width of the laser and suggest a method of reducing the deleterious effect of false locking. It appears to the authors that no one has reported in this regard.

Indexing terms: False locking, Optical phase locked loop, Locking range, Laser phase noise.

1.0 Introduction

The annoying circuit component in optical phase lock loop is the loop propagation delay. The presence of this delay along with the finite line-width of the laser adds instability to the loop and manifests as a spurious locking. Frequency search halts and the loop appears to lock at a frequency that bears no obvious relation to the input frequency. We assume that the loop is operating in the unlocked mode. Under this condition, the output of the phase detector comprises of a dc voltage and alternating voltage component. But due to the presence of the loop filter, the output of the filter consists of a dc component and only the fundamental component of the ac voltage.

2.0 Theory

Let us assume that the incoming signal is $A\sin[w_i t + \alpha(t)]$ and the free running laser VCO output is $2\cos[w_2 t + \beta(t)]$, where $\alpha(t)$ and $\beta(t)$ are due to laser phase noise. It can be shown that the governing equation of the OPLL is

$$\frac{d\phi_t(t)}{dt} = \Omega_0 - AKF(s)(1 + s\tau_p)\sin(\phi(t-\tau)) + \frac{d}{dt}(\alpha(t) - \beta(t)), \quad \text{.....} \quad (1)$$

where $\Omega_0 = w_1 - w_2$ = open loop frequency error

AK = open loop gain

τ_p = phase modulation constant

We assume a solution of equation (1) of the form given by $\phi_t = \phi_s + \phi_n$, ... (2)

where ϕ_s is the steady state solution and ϕ_n is a random variable due to the incoming noise having zero mean. From equations (1) & (2)

$$\frac{d\phi_s}{dt} + \frac{d\phi_n}{dt} = \Omega_0 - AK\,G(s)[\sin\phi_s\cos\phi_n + \cos\phi_s\sin\phi_n] + n_\phi(t) \quad \text{.....} \quad (3)$$

where $n_\phi(t)$ = laser phase noise component.

Thus using quasi-linearization technique in the locked state, one finds

$$\frac{d\phi_s}{dt} = \Omega_0 - AK.G(s)e^{-\sigma_\phi^2/2}\sin\phi_s \quad \text{.....} \quad (4)$$

$$\& \quad \frac{d\phi_n}{dt} = -AK(\cos\phi_s).\mathrm{G(s)}e^{-\sigma_\phi^2/2}\phi_n(t) + n_\phi(t) \quad \text{.....} \quad (5)$$

Now when the loop is beating, the output of the VCO can assumed as

$2\cos[(w_2 + \Delta\Omega)t - M\cos(\Omega t) + \beta(t)]$, where the closed loop frequency error is given by $\Omega = (w_1 - w_2) - \Delta\Omega = \Omega_0 - \Delta\Omega$.

Here $\Delta\Omega =$ frequency shift of the VCO

$M\cos(\Omega t) \rightarrow$ appears because of the fluctuating component

Ψ = phase angle introduced due to loop propagation delay and the phase modulator.

The phase detector output is $v_\phi(t) = A\sin[\Omega t + M\cos(\Omega t) + \Psi_n(t)]$..... (6)

If we assume the equivalent filter to be of the form $G(s) = (1 + s\tau_p)\left(\frac{1 + s\tau_1}{1 + s\tau_2}\right)e^{-s\tau}$,

where Fo = asymptotic gain of the filter = $\left(\frac{1 + s\tau_1}{1 + s\tau_2}\right)$

τ = loop delay time constant

τ_p = phase modulation time constant

$s = j\Omega$.

then the output of the filter is $v_f(t) = G(s).v_\phi(t)$

$= \sqrt{1 + \Omega^2\tau_p{}^2}.F_0\, e^{-j[\Omega\tau - \tan^{-1}(\Omega\tau_p)]}.v_\phi(t)$ ..... (7)

It can be shown that $\Psi = (\Omega\tau - \tan^{-1}\Omega\tau_p)$ and thus equation (7) reduces to

$v_f(t) = \sqrt{1 + \Omega^2\tau_p{}^2}.F_0\, e^{-j\Psi}v_\phi(t)$..... (8)

Equation (6) further simplifies to

$v_\phi(t) = A[\sin(\Omega t + \Psi_n)\cos\{M\cos(\Omega t)\} + \cos(\Omega t + \Psi_n)\sin\{M\cos(\Omega t)\}]$

$\cong A\, J_1(M)e^{-\sigma_\phi{}^2/2} + A\, J_0(M)e^{-\sigma_\phi{}^2/2}\sin(\Omega t)$ ..... (9)

In deriving the above equation all the higher order terms are removed due to low-pass filtering.

Using equations (7), (8) and (9) we have

$v_f(t) = \sqrt{1 + \Omega^2\tau_p{}^2}.F_0\, e^{-j\Psi}AK[\, J_1(M)e^{-\sigma_\phi{}^2/2} + J_0(M)e^{-\sigma_\phi{}^2/2}\sin(\Omega t)]$ ..... (10)

Since the output of the filter modulates the instantaneous frequency of the VCO, we have

$\Delta\Omega = AK\, J_1(M)e^{-\sigma_\phi{}^2/2}$ ..... (11)

& $M\,\Omega\sin(\Omega t) + \frac{d\Psi_n}{dt} = v_f(t)\big|_{ac}$

$= AK\, J_0(M)e^{-\sigma_\phi{}^2/2}.F_0\sqrt{1 + \Omega^2\tau_p{}^2}\sin(\Omega t - \Psi)$..... (12)

This ac component will cause VCO phase modulation. If we ignore the noise modulation of VCO, then

$M = \frac{AK\sqrt{1 + \Omega^2\tau_p{}^2}\,F_0}{\Omega}J_0(M)e^{-\sigma_\phi{}^2/2}\cos\Psi$ ..... (13)

For small M, $J_1(M) \cong \frac{M}{2}$ and thus

$$V_f\big|_{dc} = \left(\frac{A}{2}\right)\frac{\sqrt{1 + (\Omega\tau_p)^2}}{(\Omega/AK)}F_0\, e^{-\sigma_\phi{}^2}\cos\left[\left(\frac{\Omega}{AK}\right)(AK\tau) - \left\{\tan^{-1}\left(\frac{\Omega}{AK}\right)AK\tau_p\right\}\right]\ldots \quad (14)$$

Similarly,

$$\Delta\Omega = \left(\frac{AK}{2}\right)\frac{\sqrt{1+(\Omega\tau_p)^2}}{\left(\Omega/AK\right)} F_0\ e^{-\sigma_\phi^2} \cos\left[\left(\frac{\Omega}{AK}\right)(AK\tau) - \left\{\tan^{-1}\left(\frac{\Omega}{AK}\right)AK\tau_p\right\}\right] \text{.....} \quad (15)$$

Since the beat error frequency is given by $\Omega = \Omega_0 - \Delta\Omega$, equation (15) reduces to

$$2\Omega^2 = 2\Omega\Omega_0 - (AK)^2\sqrt{1+\Omega^2\tau_p{}^2}\,F_0 e^{-\sigma_\phi^2} \cos\left[\Omega\tau - \tan^{-1}\Omega\tau_p\right] \text{.....} \quad (16)$$

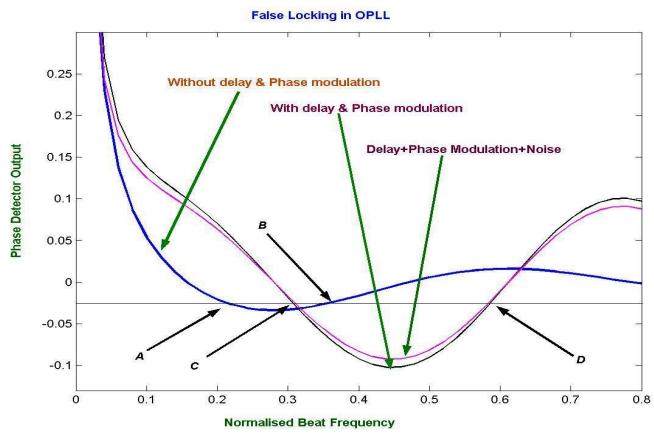

Fig. 1 False locking in OPLL

Fig.1 shows the variation of phase detector output with beat frequency error.

2.0 Lock Range Calculation without Phase Modulation

Here $\tau_p = 0$ and thus equation (16) reduces to

$$2\Omega^2 = 2\Omega\Omega_0 - (AK)^2 F_0 e^{-\sigma_\phi^2}\left[1 - \frac{(\Omega\tau)^2}{2}\right]$$

or, $\Omega = \Omega_0 \pm \sqrt{\Omega_0{}^2 - 2(AK)^2 F_0 e^{-\sigma_\phi^2}[1 - 0.25(AK\tau)^2 F_0\ e^{-\sigma_\phi^2}]}$

To find the pull-in range, we equate $\frac{\partial\Omega}{\partial\Omega_0} = \infty$ and thus we get the pull-in range as

$$\frac{\Omega_0}{AK\sqrt{2F_0}} = e^{-\sigma_\phi^2/2}\left[1 - 0.25(AK\tau)^2 F_0\ e^{-\sigma_\phi^2}\right]^{1/2} \text{.....} \quad (17)$$

3.0 Lock Range Calculation with Phase Modulation

We approximate $\cos[\Omega\tau - \tan^{-1}\Omega\tau_p]$ as $\frac{1-(\Omega\tau)^2 m}{\sqrt{1+(\Omega\tau_p)^2}}$, where $m = \left(\frac{1}{2} - \frac{\tau_p}{\tau}\right)$. Thus equation (15) reduces to $\Omega = \Omega_0 - \frac{(AK)^2}{2\Omega} e^{-\sigma_\phi^2} F_0\left[1 - (\Omega\tau)^2 m\right]$. Proceeding in a similar manner, one obtains the pull-in range as

$$\frac{\Omega_0}{AK\sqrt{2F_0}} = e^{-\sigma_\phi^2/2}\left[1 - 0.5(AK\tau)^2 mF_0\ e^{-\sigma_\phi^2}\right]^{1/2} \text{.....} \quad (18)$$

Remembering that τ_p and τ are of the same order one obtains the pull-in range for $m \cong -\frac{1}{2}$ as

$$\frac{\Omega_0}{AK\sqrt{2F_0}} = e^{-\sigma_\phi^2/2}\left[1 + 0.25(AK\tau)^2 F_0\, e^{-\sigma_\phi^2}\right]^{1/2} \quad \text{..... (19)}$$

It is seen that the phase modulator improves the locking range of the OPLL in presence of loop-propagation delay.

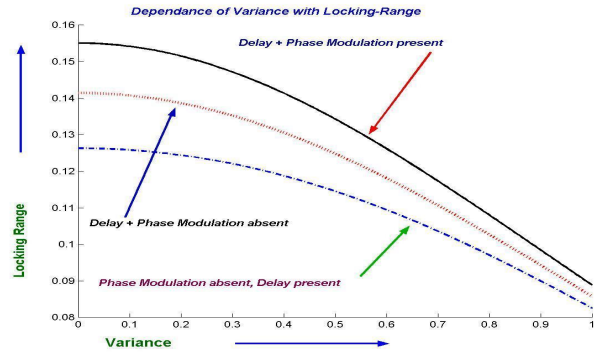

Fig. 2 Variation of locking range with noise variance

4.0 Noise Variance Calculation

In order to calculate the noise variance, we take the help of equation (5) form which one obtain $\phi_n = \dfrac{s(\alpha - \beta)}{s + (AK)\mathrm{G(s)}\, e^{-\sigma_\phi^2/2}\cos\phi_s}$. Thus the noise variance is given by $\sigma_\phi^2 = \dfrac{1}{2\pi}\displaystyle\int_0^\infty |H(s)|^2\left[4\pi(\Delta\gamma_\alpha + \Delta\gamma_\beta)\right] dw$

$$= \frac{1}{2\pi}\int_0^\infty \frac{w^2 dw}{w^2 + \left[AK\cos\phi_s\right]^2 G(s) e^{-\sigma_\phi^2}}\left[\frac{4\pi(\Delta\gamma_\alpha + \Delta\gamma_\beta)}{w^2}\right]$$

where $\Delta\gamma = \Delta\gamma_\alpha + \Delta\gamma_\beta$, is the summed laser line-width. The above integration can be solved to study the dependence of laser line-width to locking range ratio with noise variance.

5.0 Conclusion

Referring to Fig.1 it is at once clear that if the loop jumps into the nulls A, B, C, D etc., the pull-in voltage disappears; consequently it appears that the loop is locked, but actually not out of the nulls. The one with positive slope is stable whereas the null with negative slope is unstable. It is thus concluded from Fig.1, that the immunity to false locking of the modified loop is better than the conventional OPLL.

6.0 Acknowledgement

The authors are extremely grateful to Professor J. Banerjee, Chairman Academy of Technology for providing all assistance for carrying out this work.

7.0 References

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