Time series modelling of spectrum occupancy for cognitive radio

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Abstract

Cognitive radio requires real time monitoring of the spectrum to determine the frequency of transmission. Spectrum analysers tend to employ a slow frequency sweep and hence such measurements can only be used for modelling of the spectrum, which can provide vital information for frequency planning and management. Occupancy measurements every hour over a seven-day period were performed in the UK and time series analysis has been applied to model different occupancy patterns in the Global System for Mobile communications (GSM) band. The results indicate that the auto-regressive integrated moving average model, gives a good fit for the measured data.

1. Introduction

There is an ever growing demand for access to frequency spectrum with the increasing popularity of new wireless applications and devices. Meanwhile, under current communication policies most prime spectrum has already been allocated for exclusive use. Spectrum efficiency becomes one of the pertinent aspects of radio system design and frequency management strategies. However, measurements indicate that approximately 80% of the lower band is essentially free of signals to within about 7dB of environmental noise [1]. Cognitive radio is a promising solution to address the problem of inefficient use of the radio spectrum.

Ongoing research project at Durham University is to measure the present spectral occupancy in the 100-2500 MHz range to assess the feasibility of cognitive radio technology in the UK. Spectrum monitoring supplies information used in determining compliance with rules and regulations, such as license conditions, and in achieving compliance with technical and operational standards. It provides general measurements which are used by the spectrum manager to understand and plan channel and band usage as well as to confirm the effectiveness of current planning and authorization activities. However, detailed measurements and analyses intended to quantify the performance of a particular band and a particular measuring period usually cannot be extended directly to others. An alternative is to develop statistical models for more general estimation and prediction. In [2] the mathematical definition of spectrum occupancy on one variable occupancy model was given. The model has been extended in [3] to a three variable model with Markov processes. A generalised linear model to regress the congestion value on power amplitude, the number of sunspots and time in the HF band was introduced in [4]. This paper proposes a seasonal autoregressive integrated moving average (ARIMA) model to analyse the spectrum occupancy.

2. Model identification

The occupancy of a single channel is defined as the Bernoulli probability that the signal level is above a certain threshold [1] and the occupancy of a radio band is defined as the binomial probability which groups all single channels in this band. For example, Fig.1 shows occupancy data of GSM downlink band range from 925 to 960 MHz which consist of a total of 3500 single channels which are defined by 10 kHz bandwidth of the intermediate frequency filter. At a given time, the grouped occupancy rate of this band with -100 dBm threshold is 17.9%. A sequence of occupancy data \( \{ X_t, t=..., -1, 0, 1 \ldots \} \) measured typically at successive time intervals can be analysed by time series analysis approach which attempts to understand the underlying theory of the time series, or to make forecasts. Panel A of Fig.2 shows the time series of the GSM band occupancy during 7 day measurements in 1 hour interval.
A stationary process $X_t$ is defined to be an ARMA(p, q) [5] if for every t, $\phi(B)X_t = \theta(B)Z_t$, where \( \{ Z_t \} \sim N(0, \sigma^2) \), normal distribution with zero mean and variance $\sigma^2$. B is the backward shift operator $B^j X_t = X_{t-j}$, and $\phi(B)$, $\theta(B)$ are the $p^{th}$ degree autoregressive (AR) and $q^{th}$ degree moving average (MA) polynomials respectively

$$
\phi(B) = 1 - \phi_1 B - \ldots - \phi_p B^p,
\theta(B) = 1 + \theta_1 B + \ldots + \theta_q B^q.
$$

Most of the spectrum occupancy data in our long term measurement period such as broadcasting band and fixed mobile band can be fitted with this model with lower p, q value since the measured data exhibit the stationary pattern due to their modulation schemes, which means that the occupancy data have constant mean value not varied with day and night, weekday and weekend. However, some other occupancy data have no stationary character and display trend patterns. For example, the occupancy data of the emergency services band (450-470MHz) shows that the activities of weekdays are slightly busier than weekends; meanwhile, the data of the TV broadcasting band (598-854 MHz) shows that the occupancy rate at midnight is 1-2% higher than the daytime data.

To non-stationary processes $X_t$ which include trend component, Box and Jenkins [6] applied the difference operation

$$
X_t - X_{t-1} = (1 - B)X_t
$$

to eliminate the trend. A process $\{X_t\}$ is an ARIMA(p, d, q) if the differencing process $(1 - B)^d X_t$ is an ARMA(p, q) process. Moreover, some radio bands show seasonal pattern. For instance, the GSM (880-915 MHz matched 925-960 MHz) band shown in panel A of Fig.2 shows the clear 24 hour seasonality. As expected, in which the everyday’s peak occupancy happened in 10:00-18:00 local time and minimum occupancy happened in 2:00-6:00 local time. The differencing technique can be adapted to eliminate the seasonality of period d by introducing the lag-d difference operation

$$
X_t - X_{t-d} = (1 - B^d)X_t.
$$

We can then consider ARIMA models for the sub-series sampled at a period s unit time apart. This corresponds to replacing B by $B^s$ in the ARMA model. To incorporate dependence between these sub-series, a process $\{X_t\}$ is a seasonal ARIMA(p, d, q)×(P, D, Q)s process with period s if the differenced process

$$
Y_t = (1 - B)^d (1 - B^s)^D
$$

is an ARMA process,

$$
\phi(B)\Phi(B^s)Y_t = \theta(B)\Theta(B^s)Z_t
$$

![Fig. 1 The recorded spectrum of GSM downlink band](image-url)
where \( \{ Z_t \} \sim N(0, \sigma^2) \) and \( \phi(B), \Phi(B), \theta(B), \Theta(B) \) are the \( p^\text{th}, P^\text{th}, q^\text{th}, Q^\text{th} \) degree polynomials.

3. Analysis of GSM band

The occupancy data set of GSM downlink band range from 925 to 960 MHz was collected at Durham campus each hour from 14:00 27/06/2007 to 13:00 03/07/2007 with 10 kHz bandwidth. A log-periodic antenna which has a typical 8.2-9.5 dBi gain in 90-degree horizontal beamwidth range from 800-2500 MHz has been employed as the monitoring antenna. The received signals which have been filtered and amplified by 20 dB were fed into a spectrum analyzer. In order to increase the spectrum resolution, 10 kHz bandwidth has been set with 6 MHz span so there is a total of 3500 samples in each sweep.

Panel A of Fig.1 shows the occupancy percentage versus time. The grouped occupancy data show apparent seasonal and trend components with threshold -100 dBm after local polynomial regression fitting smoothing, and panel C of Fig.2 shows the trend component where the least occupancy happened on Sunday 31/07/2007 (day 5-6). The trend component is caused by the peak traffic of each day in 10:00-18:00 local time; the flat off-peak occupancy is caused by signalling traffic. The seasonal component in panel B of Fig.2 has exactly 24 hour periodical.

![Fig. 2 The GSM occupancy data with seasonal and trend components](image)

Generally, the higher the values of parameters in the chosen seasonal model \( \text{ARIMA}(p, d, q) \times (P, D, Q)_s \), the smaller the residuals will be. However, the over-fitted model will produce gross errors. Akaike information criterion (AIC) has been chosen in the model identification to prevent over-fitting. Finally, the optimal model seasonal \( \text{ARIMA}(0,1,1) \times (0,1,1)_{24} \)

\[
(1 - B)(1 - B^{-24})X_t = (1 - 0.56B)(1 - 0.98B^{-24})Z_t, \, Z_t \sim N(0,0.97)
\]

has been chosen in fitting GSM occupancy data \( X_t \). Fig. 3 shows the fitted series, where good agreement with the measured data is achieved.
4. Conclusions

While monitoring of the radio spectrum occupancy plays a crucial role in frequency management and planning, the complementary statistical model approach describes and summarizes comprehensive occupancy situations. Besides generalized linear models and Markov models, in this paper, concise time series models are introduced to analyze the complex radio spectrum occupancy, which is convenient especially in analyzing the occupancy variation with time. The stationary occupancy data can be fitted with lower order ARMA(p, q) models; the seasonal occupancy data can be fitted with ARIMA(p, d, q) \times (P, D, Q) models. This application can be useful for cognitive radio design and frequency spectrum management. Currently the frequency agile multiple receive chirp sounder is used to estimate occupancy versus time, frequency and angle of arrival [7].

5. Acknowledgements

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6. References


