Approximation of the average power variation for Geometric adding signal approach of PAPR reduction in context of OFDM signals

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Abstract

One of the major drawbacks of adding signal techniques of reducing peak to average power ratio (PAPR) is the variation of the average power of the transmit signal that undoubtedly degrades the performance of transmission. In this paper, assuming that the baseband OFDM signal is characterized as a complex Gaussian process, we derive an approximation by a simple closed form of the variation of the average power for geometric method [1] of PAPR reduction known as an adding signal technique. Comparisons of the proposed approximation with computer simulations show good agreement and convergence with an increase in the magnitude threshold.

1 Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has become popular in most of the wireless communication systems due to its high spectral efficiency, robustness against multi path fading channels and its efficiency hardware implementation that can be realized using Fast Fourier Transform (IFFT/FFT) techniques. One of the major drawbacks of this technique is the large Peak-to-Average Power Ratio (PAPR) of OFDM signals, which require large back-off in the transmitter amplifier and, as a consequence, inefficient use of high power amplifiers (HPA).

Several PAPR reducing methods can be found in the literature. A number of approaches have been proposed to deal with the PAPR problem. These techniques include amplitude clipping [2], clipping and filtering [3],[4], coding [5], tone reservation [6] and signal representation techniques such as partial transmit sequence (PTS) [7], selected mapping (SLM) [8], etc. Among these PAPR reduction schemes, the methods say adding signal seem to be promising. In [1], we proposed a technique for PAPR reduction of the OFDM signals known as adding signal principle. The major advantages of this technique include downward compatibility (this means that its implementation in a transmitter does not imply modifications in its associated receiver), low complexity, no performance degradation (no degradation of the Bit Error Rate) and no transmission of additional information. In general, one of the major drawbacks of methods say adding signal is the variation of the average power of the transmit signal. This variation of the average power undoubtedly degrades the quality of the transmission. It is therefore necessary and important to study and analyse the variation of the average power of the method.

In this paper, we first recall the principle of the geometric adding signal approach for PAPR reduction in section 2, assuming that the baseband OFDM signal converge to a complex Gaussian process for large subcarriers, we develop in section 3, a theoretical formula to evaluate the variation of the average power of the method proposed in [1], we present and analyse various simulation results in section 4 and finally we conclude in section 5.

2 Recall of the Principle of the PAPR reduction method

Let $\tilde{x}(t)$ be an OFDM baseband complex signal during a block period in continuous time-domain. It can be expressed below

$$\tilde{x}(t) = I(t) + jQ(t), \tag{1}$$

where $I(t)$ is the inphase signal and $Q(t)$ the quadrature phase signal. The corresponding passband signal $x(t)$ carried by f_c is expressed below

$$x(t) = I(t)\cos\{2\pi f_c t\} - Q(t)\sin\{2\pi f_c t\}. \tag{2}$$

According to (1), the complex envelope signal of the OFDM signal $x(t)$ is given by:

$$\varsigma(t) = \sqrt{I^2(t) + Q^2(t)}. \tag{3}$$

Let $a(t) = \Re e\left\{\tilde{a}(t)e^{2j\pi f_r t}\right\}$ to be the adding signal, where $f_r = f_c + \Delta f$ is the carrier frequency and $\tilde{a}(t)$ the adding baseband signal given by

$$\tilde{a}(t) = I_a(t) + jQ_a(t). \tag{4}$$

The principle of the PAPR reduction according [1] is as follows:
Let C to be the magnitude threshold.

(i) If $\varsigma(t) \geq C$, the adding signal $a(t)$ is generated such as

$$|\tilde{x}(t) + \tilde{a}(t)|^2 = C^2 \tag{5}$$

(ii) On the contrary, i.e for $\varsigma(t) < C$, the adding signal $a(t)$ is fixed at zero.

Let us suppose that at a time t: $\varsigma(t) \geq C$, we week the parameters $I_a(t)$ and $Q_a(t)$ which check (5). Developing (5), we obtain:

$$[I(t) + I_a(t)]^2 + [Q(t) + Q_a(t)]^2 = C^2 \tag{6}$$

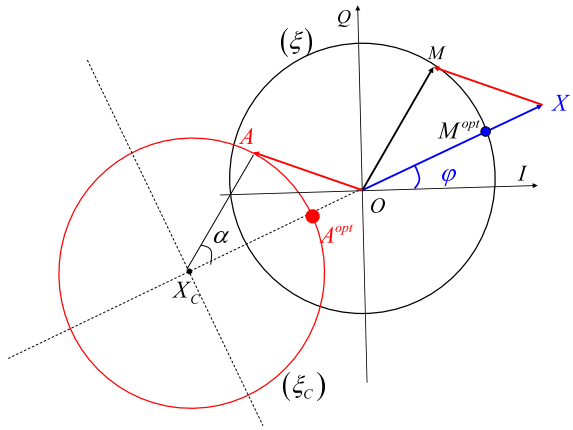

Figure 1: The plan (I/Q)
Where $\overrightarrow{OX}$ represents the baseband OFDM signal $\tilde{x}(t)$ and $\overrightarrow{OA}$ represents the baseband adding signal $\tilde{a}(t)$.

In [1], the equation (6) is solved geometrically in the plan (I/Q)(see Figure.1) and the solution is given by (7).

$$I_a = \left(\frac{C}{\sqrt{I^2+Q^2}} - 1\right) I \quad , \quad Q_a = \left(\frac{C}{\sqrt{I^2+Q^2}} - 1\right) Q \ . \tag{7}$$

Therefore, the adding signal $a(t)$ is expressed as $a(t) = \cos\{2\pi f_r t\} I_a(t) - \sin\{2\pi f_r t\} Q_a(t)$, where I_a and Q_a are given in (7).

3 Approximation by a simple closed form of the average power variation

In this session, we derive a theoretical formula to evaluate the variation of the average power ΔE in order to analyse and understand the magnitude threshold influence on the average power.

Let us consider the baseband OFDM signal $\tilde{x}(t)$ defined in (1) and $\tilde{a}(t)$ the adding baseband signal given by (4). The baseband signals $\tilde{x}(t)$ and $\tilde{a}(t)$ are carried respectively by frequencies f_c and f_r, where $f_r = f_c + \Delta f$.

The resulting passband signal is obtained by

$$y(t) = \Re e\left\{\tilde{x}(t)\, e^{j2\pi f_c t} + \tilde{a}(t)\, e^{j2\pi f_r t}\right\}. \tag{8}$$

In order to analyse the variation of the average power of the PAPR reduction scheme, we derive a theoretical formula to evaluate this variation by assuming that the baseband OFDM signal converge to a complex Gaussian process for large subcarriers. Let us suppose the baseband signal $\tilde{y}(t)$ given below

$$\tilde{y}(t) = \tilde{x}(t) + \tilde{a}(t)e^{j\theta(t)}, \tag{9}$$

where $\theta(t) = 2\pi\Delta f t$. The square of the envelope of the complex baseband signal $\tilde{y}(t)$ normalized by its average power is defined as

$$Q(t) = \frac{|\tilde{y}(t)|^2}{\sigma_{\tilde{y}}^2}. \tag{10}$$

Consider that the signal of (10) is sampled at the Nyquist rate. Let Q_l denote the real samples of $Q\left(\frac{lT_s}{N}\right)$ and let r_l denote the real samples of $\left|\tilde{x}\left({}^{lT_s}/_{N}\right)\right|\Big/_{\sigma_{\tilde{x}}}$, where $\sigma_{\tilde{x}}^2$ is the baseband OFDM average power.

From the method principle, the real samples Q_l can be written:

$$Q_l\left(r_l, \theta_l\right) = r_l^2 . 1_{r_l \leq \rho} + \left|r_l + (\rho - r_l)\, e^{j\theta_l}\right|^2 . 1_{r_l > \rho}, \tag{11}$$

where $\rho = C/\sigma_{\tilde{x}}$ is the magnitude threshold normalized by the square root of the average power of the baseband OFDM signal $\tilde{x}\left(t\right)$ and θ_l is the real samples of $\theta\left({}^{lT_s}/_{N}\right)$.

In [9], it is shown that r_l is an i.i.d Rayleigh random variable and θ_l can be likened to an uniform random variable defined on $[0, \theta_s]$ which probability density functions are respectively given by

$$\begin{array}{ll} f_{r_l}\left(r\right) = 2re^{-r^2}, & r \geq 0 \\ f_{\theta_l}\left(\theta\right) = \frac{1}{\theta_s}, & \theta \in [0, \theta_s] \end{array}, \tag{12}$$

where $\theta_s = 2\pi\Delta f T_s$. Q_l is function of two random variables r_l and θ_l and satisfies the conditions to be a random variable. In the following, let us consider ΔE, the variation of average power which expression is given by $\Delta E = \sigma_{\tilde{y}}^2/\sigma_{\tilde{x}}^2$, where

$$\sigma_{\tilde{x}}^2 = \int_0^\infty r^2 f_{r_l}\left(r\right) dr \quad , \quad \sigma_{\tilde{y}}^2 = \int_0^{\theta_s}\int_0^\infty Q\left(r, \theta\right) f_{r_l\theta_l}\left(r, \theta\right) dr d\theta \ . \tag{13}$$

$Q\left(r, \theta\right)$ is expressed in (11). We verified easily that $\sigma_{\tilde{x}}^2 = 1$. Now let us look for a simple closed form of $\sigma_{\tilde{y}}^2$. Using (13), the mean of the random variable Q_l is written by

$$\sigma_{\tilde{y}}^2 = \int_0^{\theta_s} \left\{ \int_0^\rho r^2 f_{r_l}\left(r\right) dr + \int_\rho^\infty \left|r + (\rho - r)\, e^{j\theta}\right|^2 f_{r_l}\left(r\right) dr \right\} f_{\theta_l}\left(\theta\right) d\theta. \tag{14}$$

After some mathematical developments, we obtain:

$$\sigma_{\tilde{y}}^2 = \left[1 + \left(1 + \rho^2\right) e^{-\rho^2}\right] + 2\frac{\sin\theta_s}{\theta_s}\left[\rho \int_\rho^\infty e^{-r^2} dr - \left(\frac{\rho^2}{2} + 1\right) e^{-\rho^2}\right] \ . \tag{15}$$

For small θ_s i.e $\theta_s \simeq 0$, $\frac{\sin\theta_s}{\theta_s} \simeq 1$ and the variation of the average power expression becomes

$$\Delta E = \frac{\sigma_{\tilde{y}}^2}{\sigma_{\tilde{x}}^2} = \left[1 + \left(1 + \rho^2\right) e^{-\rho^2}\right] + 2\left[\rho \int_\rho^\infty e^{-r^2} dr - \left(\frac{\rho^2}{2} + 1\right) e^{-\rho^2}\right] \ . \tag{16}$$

4 Numerical results

Figure.2 shows the gain in terms of PAPR reduction and the average power variation in dB according to ρ. The average power variation decreases as ρ increases to stabilise at 0 dB for ρ larger than 2. At the same time, ΔPAPR increases to a maximum at 1.92 dB for $\rho = 1.70$ and gradually decreases to stabilise at 0 dB. Indeed, where ρ is high (e.g $\rho > 3$), the peaks of the OFDM signal fall below the magnitude threshold and thus, there are no further PAPR reduction. The parameter ρ describes the behaviour of the method and according to simulation results, a maximum of PAPR reduction is reached at 1.92 dB for $\rho = 1.70$ which corresponds to $\Delta E = -0.12$ dB. Of course there is a slight variation of average power that can be acceptable if we focus on the gain in terms of reducing PAPR. For the values of $\rho < 0.5$, there is some loss of the method performance, it would be for specific applications to choose suitable ρ.

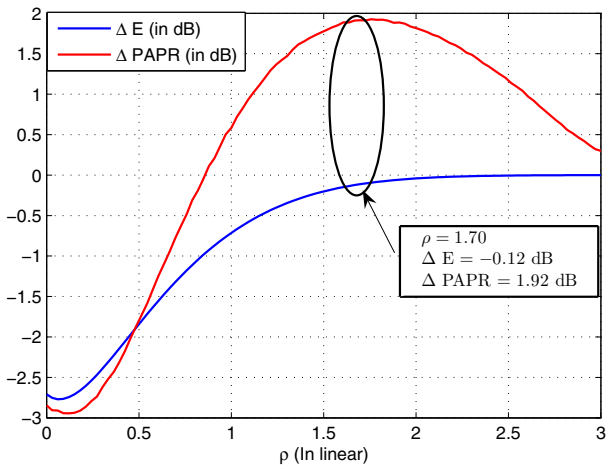

Figure 2: Gain in terms of PAPR reduction and average power variation according to ρ.

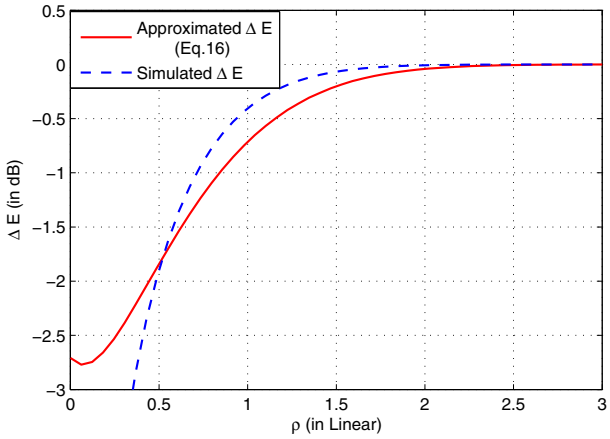

Figure 3: Comparison of proposed approximation of ΔE with simulation results according to ρ.

Figure.3 compares the proposed approximation of ΔE (eq.16) with simulation results for different values of ρ. Computer simulations were performed for an OFDM system with 100 subcarriers ($N = 100$) with 16 QAM data symbols. Although the proposed approximation diverges from the simulation results for low ρ ($\rho < 0.5$), it shows good agreement with the simulation results for ($\rho > 2$). Indeed, for $\rho < 0.5$ the approximation error in the equation (16) becomes important whereas for $\rho > 2$ this approximation error becomes weak and even negligible for large values of ρ.

5 Conclusion

In [1], we have proposed a technique of reducing PAPR known as an adding signal technique for PAPR reduction. The major advantages of this technique include downward compatibility, efficient implementation, and good transmission performance. One of the major drawbacks of adding signal techniques is the variation of the average power of the transmit. In this paper, we have proposed a simple closed form of the variation of the average power for technique proposed in [1]. The proposed approximation shows good agreement with computer simulations for large values of ρ.

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