A Low Complexity Fading Filter Design for Multipath Rayleigh Fading Simulation

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Abstract

A number of different methods have been proposed and used for the simulation of Rayleigh fading. In this paper a variant of the method of filtering of the white Gaussian noise is presented where the filter design is accomplished in the analog domain and transferred into the digital domain, and implemented as an ARMA or as an AR filter. Comparisons to other methods are then made by using quantitative measures to show the merit of the method.

1. Introduction

The fading caused by multipath propagation in wireless communication systems is commonly modelled by the Rayleigh distribution. It is well known that a Rayleigh fading process is characterized by its power spectral density and its auto-correlation function. The correlation function depends on the Doppler frequency which corresponds to the relative motion of the receiver and transmitter. In the communications literature, a number of different methods have been proposed and used for the simulation of Rayleigh fading. Jakes' model [1] has been of great interest which is based on *sum of sinusoids* approach. Simulators based on *white noise filtering methods* [2,3] and on the *Inverse Discrete Fourier Transform* (IDFT) *method* [4] have also become popular. It was shown in [3, and references therein] that the fading signals which are produced by classical Jakes' simulator are not wide-sense stationary (WSS). On the other hand simulators based on the IDFT method are of high-quality and efficient. Unfortunately, a disadvantage of the IDFT method is that all samples are generated with a single fast Fourier transform (FFT), hence the storage requirements make it useless for the generation of very large number of samples and for sample-by-sample simulations. In this paper, we consider the use of a fading filter that was first proposed in [2]. Unlike the other filter structures [1,3-5], a different optimization/design criterion is used to set the filter parameters in the analog domain as would yield the transfer function $G_\gamma(s)$ where the filter order is denoted by γ. *Bilinear transform* is then used to get the desired filter structure as an ARMA(γ,γ) filter. Comparisons to other methods are then made by using quantitative measures introduced in [6].

2. Rayleigh Fading Statistics

Rayleigh fading process is characterized by the Gaussian WSS uncorrelated scattering fading model [3], where the fading process is modeled as a complex Gaussian process. In this model, time variability of the channel is determined by its autocorrelation function. This statistic generally depends on the propagation geometry, the velocity of the mobile and the antenna characteristics. A common assumption is that the propagation path consists of two-dimensional isotropic scattering with a vertical monopole antenna at the receiver [1]. In this case theoretical spectral density of the in-phase or quadrature part of the received signal is represented as

$$S(f) = \begin{cases} \frac{\sigma^2}{2\pi f_d\sqrt{1-(f/f_d)^2}} & |f| \le f_d \\ 0 & \text{else} \end{cases} \tag{1}$$

where σ^2 is the rms value of the envelope of the waveform, or equivalently it is the fractional power of each lag. f_d is the maximum Doppler shift which is defined as the ratio of the vehicle speed, V, to the wavelength, λ, $f_d = V/\lambda$, and $\lambda = c/f_c$ where $c = 3 \times 10^8 m/sec$ is the speed of the light, and f_c(Hz) is the carrier frequency. The corresponding continuous time autocorrelation function of the received signal under these conditions is $R(\tau) = \sigma^2 J_0(2\pi f_d|\tau|)$, where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind. For the discrete-time simulation of this model, ideally generated in-phase and quadrature Gaussian processes should each have the autocorrelation sequence

$$R[n] = \sigma^2 J_0(2\pi f_m|n|) \tag{2}$$

where $f_m = f_d T$ is the maximum Doppler frequency normalized by the sampling rate $1/T$. Furthermore, in this model the in-phase and quadrature processes must be independent and each must have zero mean for Rayleigh fading.

3. Derivation of the Fading Filter

A straightforward method to simulate a faded signal is to amplitude modulate the carrier signal with a low-pass filtered Gaussian noise source [2]. In order to obtain time varying frequency selective fading channel we must have a bank of these fading filters where each filter generates the corresponding fading channel tap. A fading filter with impulse response $g[k]$ can be designed so that its output spectral density is an approximation to theoretical spectral density of the complex envelope of the faded signal $S(f)$. Consider the filter structures that were proposed in [2]:

$$G_1(s) = \frac{w_x}{s + w_x}, \text{ and } G_2(s) = \frac{w_x^2}{s^2 + \frac{w_x s}{Q} + w_x^2}. \tag{3}$$

Then the fading filter continuous time transfer functions with higher orders (of order γ), $G_\gamma(s)$, can be given by

$$G_\gamma(s) = \begin{cases} G_2^{\gamma/2}(s), & \text{if } \gamma \text{ even}, \\ G_1(s) G_2^{(\gamma-1)/2}(s), & \text{if } \gamma \text{ odd}, \end{cases} \tag{4}$$

where the selection of Q is such that there is a pre-specified frequency response level at $w = w_x$ rad/sec. For example for the third-order filter if $Q = \sqrt{10}$, then the magnitude of $G(\cdot)$ will have a gain of 7dB at $w = w_x$ (10dB gain from the second order filter and -3dB from the first order part making the overall gain of 7dB). In order to find the parameters of the fading filter transfer function, $G_\gamma(s)$, we will first set the filter order γ and Q. Then let $S(f;\epsilon)$ be an approximation to the theoretical spectral density of (1)

$$S(f;\epsilon) = \begin{cases} \frac{\sigma^2}{2\pi f_d \sqrt{1-(f/f_d)^2}} & |f| \leq f_d - \epsilon \\ 0 & \text{else} \end{cases} \tag{5}$$

where $\epsilon \in \mathbf{R}^+$ is a small positive real number, which can be taken as the integer multiples of the smallest positive number the computing platform that can handle. Then the numerical optimization problem, for fixed γ, f_d and Q, $w_x = \arg\min \| S(f;\epsilon) - |G_\gamma(j2\pi f)|^2 \|$ is solved; where the result of this numerical optimization gives the minimizer of the norm of the distance between the modified theoretical spectral density and the fading filter spectrum. Theoretical and approximate spectral density (for the filter $G_3(s)$) are plotted in Fig.1. For the transfer functions provided in the s-domain, we can use the *bilinear transform* or *impulse invariance method* [7] to get a $G_\gamma(z)$ with an ARMA(γ,γ) model, or an AR(γ) model (all pole filter) respectively. Assume that bilinear transform is applied to the transfer function of (4), then ARMA(γ,γ) model is obtained with transfer function

$$G_\gamma(z) = \frac{\sum_{k=0}^{\gamma} g_k^M z^{-k}}{1 - \sum_{k=1}^{\gamma} g_k^A z^{-k}} \tag{6}$$

The generated Rayleigh fading process has an autocorrelation function, $R_{xx}[k]$, which can be found by directly using Wiener-Khintchine theorem [7]. That is,

$$R_{xx}[k] = \sigma^2 g[k] * g[-k]. \tag{7}$$

where, σ^2 is the variance of the complex zero-mean white Gaussian noise, and $g[k] = \mathcal{Z}^{-1}(G_\gamma(z))$ is the discrete time filter impulse response and as given as the inverse $\mathcal{Z}$- transform of the transfer function $G_\gamma(z)$.

4. Performance and Complexity Evaluation

In this section, we evaluate the suitability of our proposed filter design technique for producing high-quality Rayleigh fading sequence. Performance evaluations are made by using the quantitative measures $\mathcal{G}_{mean}$ and $\mathcal{G}_{max}$ which were defined in [6]. Comparisons of our proposed method provided in Section 3 are made to a WSS-improved Jakes' model of [5], AR fading filter approximation of [3], and to the IDFT technique which was shown in [4], and these methods are briefly outlined as follows:

1) Our Proposed Filter Design Method: Our filter design was accomplished in the analog domain and transferred into the digital domain and implemented via ARMA model or AR model by *bilinear transform* using the MATLAB function *bilinear* or *impulse invariance method* by MATLAB function *impinvar* respectively. After the filter coefficients were calculated, the Rayleigh fading sequence was generated by a direct structure using the MATLAB function *filter*.

2) IDFT Method: The simulator used was implemented as described in [4]. The MATLAB function *ifft* was used for IDFT computation.

3) AR Method: The method of [3] was implemented via MATLAB function *filtic*, to generate first p (model order) stationary Rayleigh fading samples and then MATLAB function *filter* was used to generate the other samples.

4) WSS-improved Jakes' Model: The method used was based on the sum of sinusoids technique of [5]. The normalized low-pass discrete fading process is generated by finite N_s, this WSS simulator is not autocorrelation ergodic. So, theoretical calculations of quality measures can't be done for this method.

Table 1. **Part (a)** Quality measures for the IDFT, our proposed filter design, AR, ARMA and sum of sinusoids methods of generating bandlimited Rayleigh random sequences for covariance sequence length 200, and **Part (b)** computational complexity (number of real multiplications required) comparison to generate 2^{20} samples.

Rayleigh Fading Random Sequence Generators		**Part (a): Quality Measures**				**Part (b): Computational Complexity**
		Theoretical (dB)		Empirical (dB)		Number of Real Multiplications Required to generate 2^{20} samples
		$\mathcal{G}_{mean}$	$\mathcal{G}_{max}$	$\mathcal{G}_{mean}$	$\mathcal{G}_{max}$	
IDFT Method		0.00076	0.00081	0.0035	0.0037	44×10^6
Proposed Filter Design	ARMA(2,2)	2.5066	2.5505	2.5068	2.5514	8×10^6
	AR(2)	2.6707	2.7247	2.6768	2.7313	2×10^6
	ARMA(3,3)	1.9777	1.9962	1.9775	1.9979	12×10^6
	AR(3)	2.0924	2.1173	2.1447	2.1727	6×10^6
AR Filtering	AR(20)	2.7	2.9	2.6	2.9	42×10^6
	AR(50)	0.29	0.43	0.26	0.40	105×10^6
	AR(100)	0.13	0.28	0.11	0.26	210×10^6
Sum of Sinusoids	8 Sinusoids	N/A	N/A	36.223	37.730	178×10^6
	16 Sinusoids	N/A	N/A	4.0264	6.4140	356×10^6
	64 Sinusoids	N/A	N/A	0.0211	0.0370	1424×10^6
	128 Sinusoids	N/A	N/A	0.0027	0.0049	2848×10^6

The quality measure comparison results, which are presented in Part (a) of Table 1, compare the quality of the real part of the simulator outputs. Similar results were achieved for the imaginary sequences and these are omitted for brevity. Perfect Rayleigh fading sequence generation method corresponds to 0 dB for both measures. In all cases, the reference autocorrelation function is (2) with a normalized maximum Doppler of $f_m = 0.05$. An autocorrelation sequence length of 200 was considered for evaluation of all theoretical results. For the empirical results, time average correlations were calculated based on 2^{20} generated samples. The computed quality measures were then averaged over 50 independent simulation trials. Plots of the empirical autocorrelation functions of the IDFT method, AR model and our proposed Rayleigh fading generator via AR and ARMA models are shown in Fig. 2.

The results show that the IDFT method generally provides closer the highest quality Rayleigh samples. The AR model of [3] provides a more precise match to the desired autocorrelation function as the order of the model used increases. But our proposed filter design method provides same accuracy with much lower order models. Our ARMA(3,3) generator has a significant advantage over AR(20) generator of [3]. Similar accuracy can be achieved by the WSS sinusoidal generator when a large number of sinusoidal oscillators are used.

The main advantage of our low complexity Rayleigh fading generator is that the samples of the fading sequence can be generated as they are required while achieving the lowest complexity of all the Rayleigh fading generators mentioned. The computational efficiency of the IDFT method brings a cost in storage requirements as all samples are generated using a single IFFT. Our proposed fading generator and the all other generators don't have such a limitation. As provided in Table 1 Part (b), to generate 2^{20} samples, IDFT method requires 44040193 real multiplications, our proposed filter design technique via ARMA(3,3) model requires 12582912 real multiplications, AR(20) model of [3] requires 41943040 multiplications and the improved Jakes' model of [5] requires 356515840 multiplications.

5. Conclusions

A low-complexity high performance Rayleigh fading simulator has been proposed. Our proposed ARMA(3,3) has been compared with improved Jakes' model of [5], AR fading filter approximation of [3], and to the IDFT technique of [4], in terms of performance measures and computational complexity. Our ARMA(3,3) Rayleigh fading generator, outperforms AR(20) generator of [3], by about 1dB in both performance measures provided, while requiring approximately a quarter of the multiplications required by the AR(20) generator. Similarly, our ARMA(3,3) fading generator outperforms modified Jakes' generator with 8 and 16 sinusoids by 32dB and 2dB respectively, while requiring less than one-tenth of the multiplications required by the Jakes' generators with 8 and 16 sinusoids. While the IDFT method of [4] achieves the best performance in terms of the quality measures, it brings a significant cost in storage requirements as all samples are generated using a single IFFT. Thus the IDFT method is undesirable from simulation point of view when the Rayleigh fading samples are generated as they are required. The main advantage of our ARMA(3,3) Rayleigh fading generator is that the samples of the Rayleigh fading sequence can be generated as they are required while achieving the lowest complexity of all the Rayleigh fading generators mentioned.

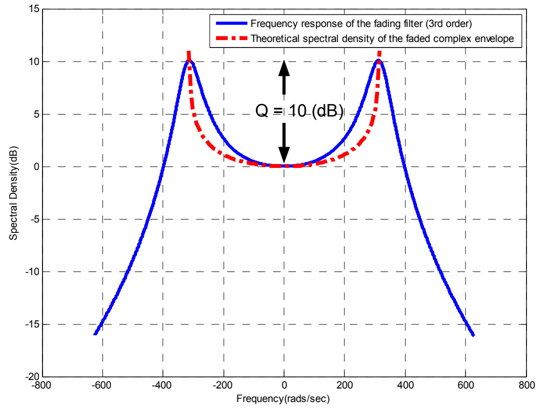

Fig. 1 The theoretical and approximate spectral density (for the filter $G_3(s)$)

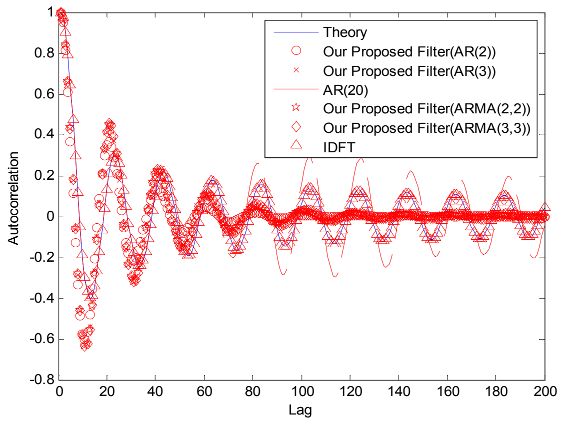

Fig.2 The empirical autocorrelations for AR method, IDFT method and our proposed filter

6. References

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