Variance Analysis of a Hierarchical Decision-Directed Phase Detector for 16-QAM Constellation

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Abstract

In this paper we introduce and describe an optimized phase detector suited to 16-QAM receivers which prevents ambiguous locking and performs relatively rapid acquisition. Based on a new decision scheme, the proposed phase detector outperforms a classical decision directed phase detector over a Gaussian channel. An analytical analysis of the proposed phase detector variance is carried out and validated by means of computer simulation.

1 Introduction

Nowadays, even thought phase carrier recovery problem in data communication systems was properly resolved through the vast amount of synchronization research works and knowledge accumulated for several years, it still remains an issue of a great importance. In most practical applications, conventional adaptive phase carrier recovery techniques employing phase error detectors particularly Decision Directed Maximum Likelihood Feedback algorithms (DDMLFB) are widely used. While this approach yields satisfactory performance for classical transmission systems operating at high signal-to-noise (SNR) levels, it could fail in cognitive radio receivers or in low SNR transmission operating conditions.

In a carrier recovery system, the phase detector (PD) is of a crucial importance, the system performance is directly related to the phase detector performance. It is well known that the requirements of a PD in phase acquisition mode are first, a large linear response of its S-curve which increases the system locking capacity particularly in the presence of small frequency errors and second, a great gain of the linear response which permits fast phase acquisition. However, in phase tracking mode, the phase error variance should be as small as possible in order to provide an accurate phase estimate.

Most of the recent works have been directed toward improving the PD performance, for instance by using data decoding process of turbo or LDPC-coded transmissions [1]. This yields satisfying performance particularly at low SNR levels, but the approach requires an initialization step using relatively long data words which can be problematic when the carrier phase does not remain constant due to frequency offset or phase noise. In another work [2] a significant improvement has been brought by using a modified decision areas for QAM signals in the presence of phase noise and frequency offset. However, this system is not suitable for low signal-to-noise ratios since the modified decision boundaries depend on the additive noise variance.

The purpose of this paper is to study the variance of an optimized carrier PD we have proposed in [3] which has shown to achieve unambiguous locking and fast phase acquisition thanks to a new decision scheme based on a hierarchical decision concept.

The paper is organized as follows. In the next Section, we present the classical carrier recovery system. A brief review of the PD S-curve and variance definitions is given in section 3. In Section 4, the key idea behind the optimized PD is described where an investigation of its variance is depicted for 16-QAM signals. Section 5 concludes.

2 System Model

In the following study, we assume perfect symbol timing and frequency synchronization, no linear (e.g.,fading) or nonlinear distortion and no Inter-Symbol Interference (ISI).

In a complex baseband communication system, the received baud-rate sampled signal at the output of the matched filter can be expressed as

$$r_k = d_k e^{j\theta} + n_k \tag{1}$$

where d_k is the complex symbol transmitted at time kT, T is symbol duration and θ is the unknown phase-offset to be estimated. The channel adds white Gaussian noise whose components n_k are complex, independent and identically distributed, having variance σ^2, with independent real and imaginary parts.

In transmissions over AWGN channels, the conventional used phase recovery algorithms are those based on Maximum Likelihood (ML) estimation [4]. In this work we have considered one such algorithm, the commonly referred to as Decision-Directed ML (DDMLFB) algorithm.

A block diagram for the algorithm is depicted in Fig.1(a). The PD generates an error signal $e(k)$ which provides a measure of the difference between the carrier phase and its current estimate $\hat{\theta}(k)$, the error signal

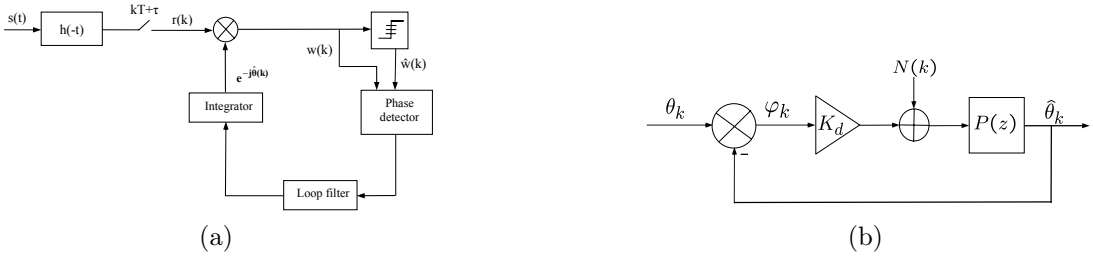


Figure 1: (a) DDMLFB carrier recovery system, (b) DDMLFB equivalent linear model

is then sent to a loop filter and a phase integrator operating together to update the phase estimate according to this equation

$$\hat{\theta}(k+1) = \hat{\theta}(k) + \alpha e(k) + \beta \sum_{j \leq k} e(j) \tag{2}$$

where α and β are positive coefficients depending on the well known loop parameters (the damping factor ξ and the noise equivalent bandwidth B_L).

3 The Phase Detector

It can be shown that the partial derivative of the simplified log-likelihood function with respect to $\tilde{\theta}$ is

$$\frac{d}{d\tilde{\theta}} \Lambda(\tilde{\theta}) = \sum_{k=0}^{k=L-1} Im(\hat{w}_k^* r_k e^{-j\tilde{\theta}}) \tag{3}$$

where $\tilde{\theta}$ is the trial value of the unknown phase offset, L is the number of symbols during the observation interval, and $\hat{w}_k$ represents the estimated symbol at time kT. $Im(\cdot)$ denotes imaginary part, and the asterisk denotes complex conjugate.
The error signal $e(k)$ in (2) is derived from the generic term in the sum (3) by setting $\tilde{\theta}$ equal to the current estimate $\hat{\theta}(k)$. $e(k)$ is called "phase detector".
Let $w_k = r_k e^{-j\hat{\theta}_k}$, $e(k)$ can be rewritten as

$$e(k) = Im(\hat{w}_k^* w_k) \tag{4}$$

The purpose of the FB loop is to iteratively solve for a maximum of the likelihood function by driving the PD output to zero, at which point the derivative (3) is zero (since the summation is performed in the loop filter). The use of a first-order filter followed by an integrator results on a second-order filter which permits to compensate for slight amounts of frequency offset [5].
The literature presents a wide variety of DD PDs derived from parameter estimation approaches or ad hoc reasoning methods [6, 7]. It is readily verified that the PD (4) is also the stochastic gradient of the mean square error (MSE) $E[|w_k - \hat{w}_k|^2]$ with respect to the phase error $\varphi = \theta - \hat{\theta}$ [7]. In the same way, adopting different criteria minimization, other simplified PD types have been proposed in [7]. Leclert and Vandamme showed in [7] that the already known PDs can be considered as a special case of a general form given by

$$e(k) = r\{f(w_I(k)) \cdot g(w_Q(k)) - f(w_Q(k)) \cdot g(w_I(k))\} \tag{5}$$

Where $f(\cdot)$, $g(\cdot)$, and $r(\cdot)$ are odd functions. Thus, based on the general form (5), other possible candidates can be suggested such as

$$e(k) = sgn(w_Q(k))(\hat{w}_I(k) - w_I(k)) - sign(w_I(k))(\hat{w}_Q(k) - w_Q(k)) \tag{6}$$

where we have considered $w(k) = w_I(k) + jw_Q(k)$ and $\hat{w}(k) = \hat{w}_I(k) + j\hat{w}_Q(k)$.
Such PD types can be seen as simplified versions of the ML based PD (4) which allow easier hardware implementations. In the present study, the PD (6) has been considered.

3.1 The Phase Detector S-curve

The PD S-curve $e(\varphi)$ (also called PD characteristic) is the key tool to investigate phase acquisition features of the phase recovery loop, it is defined as the average of the PD output, conditioned on a fixed phase error [7]. Such an information provides useful indications about the PD capabilities regardless of the synchronizer implementation, it is given by

$$e(\varphi) = E_{d_k,n_k}[e(k)|\varphi] \tag{7}$$

where $E_{d_k,n_k}[\cdot]$ denotes the statistical average over data and noise

3.2 The Phase Detector Variance

Despite the fact that the PD S-curve is of paramount importance to assess the phase acquisition performance of the synchronizer loop, it does not suffice to investigate the impact of the PD on the tracking behavior of the loop. That is why we deal here with computing the PD variance.
The PD variance $\sigma_d^2(\varphi)$ measures the power of the PD output fluctuations originating from the additive noise and symbol estimation errors with respect to the S-curve given the phase error φ, it is defined as follows:

$$\sigma_d^2(\varphi) \quad = E_{d_k,n_k}[e(k)^2|\varphi] - e(\varphi)^2 \tag{8}$$

In principle, the most common approach for investigating tracking performance is to measure the steady-state variance of the phase error φ_k and compare it to a lower limit bound known as Cramer-Rao bound. It is nevertheless possible to establish an approximate relationship between the steady-state variance of the phase error σ_φ^2 and the PD variance in the origin ($\varphi = 0$) since the phase estimate is a filtered version of the PD output. This goes through the linearization of the block diagram in Fig.1(a) around the equilibrium point $\varphi = 0$. Let us consider the equivalent linear model described in Fig.1(b).
In this figure, K_d is the PD gain, $N(k)$ accounts for thermal noise, and $P(z)$ is the open loop transfer function. The tracking phase error variance is expressed by

$$\sigma_\varphi^2 = \int_{-1/2T}^{1/2T} S_N(f)|H_N(e^{j2\pi fT})|^2 df \tag{9}$$

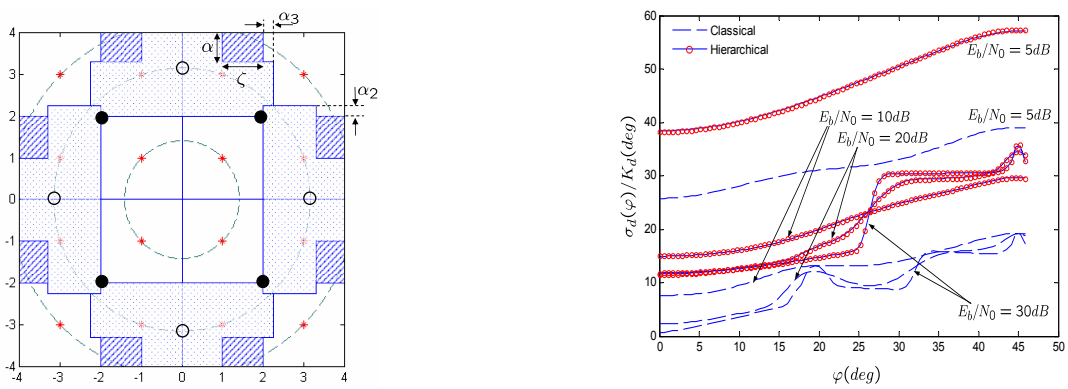
where $S_N(f)$ denotes the power spectral density of $N(k)$ and $H_N(e^{j2\pi fT})$ is the loop transfer function corresponding to the response to $N(k)$.
Taking in account that $N(k) \approx e(k)$ (in steady-state conditions), we can assume $S_N(f)$ nearly flat over the range where $H_N(e^{j2\pi fT})$ takes significant values.
Using the loop bandwidth definition: $B_L = \frac{1}{2|H_N(0)|^2}\int_{-1/2T}^{1/2T}|H_N(e^{j2\pi fT})|^2 df$, equation (9) becomes

$$\sigma_\varphi^2 \approx \frac{2B_L T}{K_d^2} S_N(0)$$

since $S_N(0) = \sigma_d^2(0)$ we get $\qquad \sigma_\varphi^2 \approx \frac{2B_L T}{K_d^2}\sigma_d^2(0)$

4 The Hierarchical Phase Detector

Referring to [3], the noiseless S-curve of the considered PD is piecewise linear. The size of the linear response can be defined as the lowest phase error (the first discontinuity) which is equal to 16.88 degrees (for 16-QAM constellations) and at which the symbol having the largest amplitude changes decision region. Indeed, if we examine the error signal (6) we note that when the signal point crosses a decision boundary, at least, the real or the imaginary part of one of the terms $(\hat{w} - w)$ or $sgn(w)$ changes value.
In 16-QAM constellation, the signal points describe three circles in case of synchronization loss (dashed



(a) (b)

Figure 2: (a) The new decision regions for 16-QAM constellation, (b) The simulated hierarchical PD standard deviation

circles in Fig.2(a). The first S-curve discontinuity occurs inside the constellation quadrants for the points which are on the outer circle. Indeed, whenever one of these points crosses a decision boundary, the real or the imaginary part (depending on the quadrant and on the rotation sense) of $(\hat{w} - w)$ changes value from 1

to -1 (or inversely) and the S-curve exhibits a jump of 2 (or -2) degrees. To eliminate this sudden change, we propose to modify symbol decision rules. The idea is to make a 4-QAM decision (small filled circles in Fig.2(a) instead of the classical 16-QAM decision when the received signal point falls within appropriate regions (lined windows in Fig.2(a).

Now, we focus on the second discontinuity (at 18.43 deg) which occurs when one of the signal points of the middle circle crosses a decision boundary. Following the same strategy as for the first discontinuity, a QPSK decision (small empty circles in Fig.2(a) is made instead of the 16-QAM only when the received signal point falls within the dotted regions of the constellation, illustrated in Fig.2(a).

Since the additive noise effect makes received signal points close to decision boundaries, we define parameters ζ and α (Fig.2(a) taking values within the range [0,2] which are used to determine the appropriate size of the lined windows in the presence of additive noise. In the same way, we introduce parameters α_2 and α_3 which serve to slightly "adjust" the common decision boundaries between the 16-QAM and the QPSK decision regions. The optimum values of these parameters are chosen according to a tradeoff between reducing noise effect and phase error sensitivity.

4.1 The S-curve

In [3], we have investigated the S-curve of the hierarchical PD in detail and we have shown that with the approach described above, we have so far succeeded in increasing significantly the size of the S-curve linear response and the PD gain.

4.2 The variance

Substituting $e(k)$ given by (6) in (8), we find $\sigma_d^2(\varphi) = \Sigma_{i=1}^4 e_i(\varphi) - 2\Sigma_{i=5}^7 e_i(\varphi) - e(\varphi)^2$
$e_i(\varphi)(i = 1, ...7)$ are first order moments given d_k including w_I, w_Q, $sign(w_I)$ and $sign(w_Q)$. Their analytical expressions are derived but are somewhat long and due to a lack of space, we have omitted to include them in this section, so only the result is plotted here. Fig.2(b) plots the normalized standard deviation $\sigma_d(\varphi)/K_d$ for both the hierarchical and classical PDs versus φ in the interval $[0, \frac{\pi}{4}]$ for different values of the signal-to-noise ratio per bit ($\frac{E_b}{N_0}$). We observe first that the curves exhibit a low variance of the PD output within the linear region of the S-curve and a high variance outside. This is obvious since the PD output is not representative of the phase error outside the linear region. Secondly, we note that inside the linear region, the variance increases with noise increasing. Such behavior is a consequence of the increasing symbol decision errors at high noise levels. We see also that the variance of the hierarchical PD output is higher that that of the classical one, that is due to the additional noise introduced by the hierarchical decisions. This is the main drawback of the proposed hierarchical decision approach, it is however possible to cancel this additional noise during the tracking mode by the use of some phase lock detector devices which indicate when the synchronizer gets lock and thus switch the decision to the true 16-QAM decision. Fig.2(b) shows also the comparison between simulation (solid line) and theoretical analysis results (curve of small circles) for the hierarchical PD variance. Clearly, they are very close, i.e., the theoretical results are accurate.

5 Conclusion

We have presented an optimized carrier recovery phase detector for 16-QAM systems by using a new decision scheme. Theoretical analysis of the variance of this phase detector has been performed, and confirmed by computer simulations. The proposed phase detector suffers from a phase error noise originating from hierarchical decisions which can be reduced by the use of a phase lock detector information in order to take up the classical 16-QAM decision once the loop gets lock. Future works would include the refinement of this solution.

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