

# Cooperative Relaying in Wireless Networks with Stochastic Traffic

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## Abstract

Cooperative Relaying techniques offer the potential for increasing the throughput of wireless networks by enabling multiple nodes to cooperate in forwarding information from its source to destination. However, when multiple flows are present, potential cooperative gains must be balanced among the competing traffic flows. In this paper we discuss models for cooperative relaying in networks with multiple stochastically varying end-to-end flows. Traffic from each flow is queued in the network until it can be transmitted. For such networks, we consider network control policies that dynamically determine how the current traffic is cooperatively forwarded. These policies are generalizations of *backpressure algorithms*, which have been widely studied for routing and scheduling in non-cooperative wireless networks. Such policies are *throughput optimal*, i.e., they stabilize all queues in the network whenever it is possible to do so.

## 1. Introduction

In cooperative relaying, multiple nodes jointly cooperate to forward packets in a wireless network, for example by forming a distributed antenna array. Such schemes have attracted much interest in recent years (e.g. [1–3]) and offer the promise of increased throughput for wireless networks. Cooperative communication has mainly been addressed from the physical layer viewpoint, i.e. by studying the achievable rates or diversity gains of given cooperative schemes, assuming that all sources are backlogued and often just focusing on a single end-to-end session. In this paper we instead focus on cooperative relaying in networks with multiple stochastically varying end-to-end sessions. In such a setting a key issue is that cooperation must be adapted to the time-varying traffic in the network. To address this we consider a variation of the *backpressure* policy first proposed by Tassiulas and Ephremidis [4]. This type of policy has been widely studied for dynamic resource allocation in different models of wireless networks without cooperative relaying (see [5] for a recent survey), where it is shown to be *throughput optimal*, meaning that it stabilizes all the queues within the network whenever it is possible to do so.

One key difference that cooperative relaying introduces into the model is that each packet no longer flows from the source to the destination over a single “store-and-forward” path. Indeed in cooperative networks, a “packet” may reside at several nodes in a network at a given time. In spite of this difference, we have shown that by appropriately redefining the network model, a generalization of the backpressure algorithm can be applied to networks that employ a wide class of cooperative schemes. Moreover, for these networks this algorithm is again throughput optimal. We begin by giving a model for cooperative wireless networks with a general topology. The model allows for multiple cooperative sets and includes the use of wireless broadcast to reach these sets. We give several examples to show how this model can capture different cooperative scenarios. We then move on to discussing our generalization of the backpressure algorithm to this network.

## 2. General Network Model

We consider a multi-hop network with an arbitrary topology. For simplicity, we consider only “two-hop” cooperative communication, i.e. a node may send a packet to a group of nodes to be cooperatively forwarded to a destination node; the cooperative group then forwards this packet to the destination. As discussed below, the cooperative group may include either the source or destination. This allows us to model several other cooperative schemes. Our network

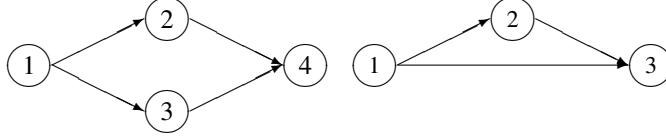


Figure 1: Left: A four node parallel relay network model. Right: A three node relay network.

model is a generalization of the model in [5] to include cooperative communication. A network  $\mathcal{G}$  consists of a set of nodes  $\mathcal{V}$ , and a set  $\mathcal{L}$  of feasible *direct links* that are ordered pairs  $(u, v)$  for  $u, v \in \mathcal{V}$ . These represent point-to-point links. There are also two other sets of “links” in the network. First, we define a set  $\mathcal{S}$  of feasible *cooperative links*. These are many-to-one links, denoted by ordered pairs  $(S, v)$ , where  $S \subset \mathcal{V}$  and  $v \in \mathcal{V}$ . The nodes in  $S$  all cooperate to forward a packet to  $v$ . Here, we restrict our attention mainly to “decode and forward” strategies in which each node in  $S$  will decode the packet before forwarding it to  $v$ . Second, we define a set  $\mathcal{T}$  of feasible *broadcast links*, denoted by ordered pairs  $(u, T)$  for  $u \in \mathcal{V}$  and  $T \subset \mathcal{V}$ . A packet sent over such a “link” is broadcast from  $u$  to all of the nodes in  $T$ . We assume that each cooperative link  $(S, u)$  is matched to at least one broadcast link  $(u, S)$  whose destination set is the same as the origin set of the cooperative link. Each broadcast link is also matched to at least one corresponding cooperative link. The only traffic that can be sent over a cooperative link is that which is received on one of the corresponding broadcast links.

The network operates in slotted time, where the length of each time-slot is normalized to 1. We assume there is no fading or changes in the topology over the time-scale of interest. Within time-slot  $t$ , let  $R(t) = (R_l(t))$  denote the vector of realized transmission rates for all  $l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}$ . For all  $t$ ,  $R(t) \in \mathcal{C}$ , where  $\mathcal{C}$  denotes the *instantaneous link capacity region*, which is a bounded subset of  $\mathbb{R}_+^{|\mathcal{L} \cup \mathcal{S} \cup \mathcal{T}|}$ . In other words,  $\mathcal{C}$  denotes the set of feasible link rates on all links (of each type) in any time-slot. Any constraints on the set of links that may be active are included in this set.

Let the channel between each pair of nodes  $i, j$  be an additive Gaussian noise channel with gain  $\sqrt{h_{ij}}$ , unit variance noise, and bandwidth  $W = 1$  Hz. Each transmitter has a power constraint of  $P$  during each time-slot. If link  $(i, j)$  is the only link activated, then we model the feasible transmission rate by  $R_{ij} = \log(1 + h_{ij}P)$ , i.e., the Shannon capacity of this point-to-point channel.

**Example 1** Let  $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{S}, \mathcal{T})$  be a model for the four-node parallel relay network in Figure 1 from [6]. There,  $\mathcal{V} = \{1, 2, 3, 4\}$ , and  $\mathcal{E}$  consists of the four direct links, shown by the arrows in the figure. Additionally, nodes 2 and 3 can cooperate to relay a message to 4, i.e.  $\mathcal{S} = \{\{(2, 3), 4\}\}$  and  $\mathcal{T} = \{(1, \{2, 3\})\}$ . Let  $R = (R_{1S}, R_{12}, R_{13}, R_{S4}, R_{24}, R_{34})$  be the vector of transmission rates for the 6 links in this model, where  $S = \{2, 3\}$ , e.g.  $R_{1S}$  is the rate of the broadcast link  $(1, \{2, 3\})$  and  $R_{S4}$  is the rate on the cooperative link  $(\{2, 3\}, 4)$ . Only one of the following two sets of transmitters may be active in any time-slot:  $\mathcal{A}_1 = \{1\}$  or  $\mathcal{A}_2 = \{2, 3\}$ . This enforces a half-duplexing constraint. With this assumption, the link capacity region,  $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$ , where  $\mathcal{C}_i$  is the set of feasible rates corresponding to activation set  $\mathcal{A}_i$ .

In [6],  $\mathcal{C}_1$  is modeled as the capacity region of a Gaussian broadcast channel, where the traffic sent over the broadcast link  $(1, \{2, 3\})$  is viewed as common information. The region  $\mathcal{C}_2$  is modeled as a “cooperative MAC channel” in which there are three “users” corresponding to the two direct links from 2,3 to 4, and the cooperative link  $(\{2, 3\}, 4)$ . Over the cooperative link, the nodes cooperate via beamforming.

**Example 2** Next, consider the three node relay network shown in Figure ???. Again, all feasible direct links are indicated via an arrow. Furthermore, assume that  $h_{12} > h_{13}$ . We discuss two ways in which packets from node 1 can be cooperatively relayed to node 3. First, consider a cooperative link  $(\{1, 2\}, 3)$ , in which nodes 1 and 2 cooperatively forward a packet to node 3, e.g. using distributed beamforming. To utilize this link, node 1 must first send a packet to node 2 and save a copy of the packet for itself. Then in the next time slot, both nodes can transmit the packet to node 3. We model the first transmission as occurring over a broadcast link  $(1, \{1, 2\})$ , i.e. a link in which the source is also one of the destination nodes. Of course, node 1 need not actually transmit a packet and thus the maximum transmission

rate on the link is simply  $\bar{R}_{\{1,2\}} = \log(1 + h_{12}P)$ . In this case,  $\mathcal{C}$  contains vectors of the form  $(R_{1S}, R_{12}, R_{13}, R_{S3}, R_{23})$ . Given duplexing and multiplexing constraints, we can again specify this rate region.

A second cooperative scenario is for node 1 to first transmit a packet to node 2, but for node 3 to also store the received signal from this transmission (even though it can not decode it). When node 2 then forwards the packet to node 3, node 3 can use the information from both transmissions to decode the packet. We model this by including a broadcast link  $(1, \{2,3\})$  and a cooperative link  $(\{2,3\}, 3)$ . The maximum rate for the broadcast link  $(1, \{2,3\})$  is again the rate at which node 1 can transmit to node 2 (since node 3 is not decoding), i.e.,  $\bar{R}_{\{1,2\}} = \log(1 + h_{12}P)$ . The corresponding rate on the cooperative link  $(\{2,3\}, 3)$  is

$$\bar{R}_{\{2,3\}3} = \log(1 + h_{23}P) + \log(1 + h_{13}P). \quad (1)$$

Here, the first term reflects the mutual information received from node 2's transmission and the second term is the mutual information received from node 1's original transmission to node 2. We can again define  $\mathcal{C}$  for given duplexing and multiplexing constraints. For example, the cooperative link can still achieve rate  $\bar{R}_{\{2,3\}3}$  in (1), while node 1 can simultaneously send at rate

$$\bar{R}_{1,3} = \log\left(1 + \frac{h_{13}P}{1+h_{23}P}\right). \quad (2)$$

This can be accomplished by having node 3 successively decode both transmissions starting with the direct transmission.

We have focused on relatively simple networks to illustrate some possibilities for cooperation. The following results apply to a general network, in which several of these scenarios, as well as others, could exist at different locations in the network. Next we turn to describing the traffic in the network.

Following [5], all traffic is classified as particular "commodity," which specifies its desired destination. Let  $\mathcal{K} \subset \mathcal{V}$  denote the set of commodities in the network, where commodity  $k$  has destination node  $k$ . Exogenous traffic corresponding to each commodity  $k \in \mathcal{K}$  is assumed to arrive into the network at node  $i \in \mathcal{V} \setminus k$ , according to an ergodic process  $B_i^k(t)$ , where  $B_i^k(t)$  is the number of exogenous bit arrivals to node  $i$  in time-slot  $t$ . Each node buffers all arriving packets for each commodity until they are transmitted.

Let  $U_i^k(t)$  be the number of untransmitted bits (unfinished work) of commodity  $k$  at node  $i$ , which is to be sent over a direct or broadcast link (we refer to this as the direct traffic). Additionally, for each cooperative link  $(S, u) \in \mathcal{S}$ , let  $U_S^k(t)$  be the unfinished work of commodity  $k$  traffic, which is to be forwarded cooperatively by the nodes in  $S$ . Each of these quantities corresponds to the backlog of a separate queue at the corresponding nodes, i.e., each node keeps separate queues for each commodity of the direct traffic as well as each commodity of traffic for each cooperative set  $S$  to which it belongs.

Let  $U(t) = ((U_i^k(t))_{i \in \mathcal{V}}, (U_S^k(t))_{S \in \mathcal{U}, k \in \mathcal{K}})$  denote the joint queue state at time  $t$ , i.e. the unfinished work of each commodity in every direct or cooperative queue in the network. We consider the case where given  $U(t)$  at time  $t$ , a network controller specifies a joint rate allocation/routing assignment denoted by  $(R_l^k(t))_{l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}, k \in \mathcal{K}}$ , where  $R_l^k(t)$  denotes the rate allocated to commodity  $k$  over link  $l$  at time  $t$ . For feasibility,  $(R_l^k(t))_{l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}, k \in \mathcal{K}}$  must satisfy

$$\sum_{k \in \mathcal{K}} R_l^k(t) \leq R_l(t) \quad \forall l, \text{ and } R(t) \equiv (R_l(t))_{l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}} \in \mathcal{C}, \quad (3)$$

where  $R_l(t)$  is the total rate allocated to link  $l$  at time  $t$ .

### 3. Cooperative Backpressure

The following theorem gives our generalization of the backpressure algorithm to a cooperative network.

**Theorem 1** A throughput optimal rate allocation/routing policy  $\mathcal{R}^*(u)$  for a network with two-hop cooperative forwarding is given by first finding a rate allocation  $R^*$  which is a solution to the following optimization:

$$\max_{R \in \mathcal{C}} \sum_{(i,j) \in \mathcal{L}} b_{ij}^* R_{ij} + \sum_{(i,T) \in \mathcal{T}} b_{iT}^* R_{iT} + \sum_{(S,i) \in \mathcal{S}} b_{Si}^* R_{Si} \quad (4)$$

where

$$b_{ij}^* \equiv \max_{k \in \mathcal{K}} u_i^k - u_j^k, \quad (5)$$

$$b_{iT}^* \equiv \max_{k \in \mathcal{K}} u_i^k - |T| u_T^k, \quad (6)$$

$$b_{Si}^* \equiv \max_{k \in \mathcal{K}} |S| u_S^k - u_i^k. \quad (7)$$

The corresponding routing policy is implemented by sending only bits from traffic class  $k^*$  which attains the maximum in (5) ((6) and (7), respectively) at rate  $R_{ij}^*$  ( $R_{iT}^*$  and  $R_{Si}^*$ , respectively) for all  $(i, j) \in \mathcal{L}$  ( $(i, T) \in \mathcal{T}$  and  $(S, i) \in \mathcal{S}$ , respectively). That is, over link  $l \in \mathcal{L} \cup \mathcal{S} \cup \mathcal{T}$ ,  $R_l^k = R_l^*$  for  $k = k^*$  and  $R_l^k = 0$  otherwise.

We note that the policy in (4) is the not the same as the conventional backpressure policy. In particular, the terms  $u_i^k - |T| u_T^k$  and  $|S| u_S^k - u_i^k$  reflect the *queue coupling* effect induced by the cooperative transmission structure.

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## 4. Conclusions

We considered wireless networks with cooperative relaying and stochastically varying traffic. For a large class of such networks, we gave a variation of the backpressure policy, which we proved to be throughput optimal. This policy is modified to incorporate the potential gains of cooperative communication. We focused on a centralized implementation. In practice, a distributed solution is more desirable. Moreover, in a large network, there may be many potential cooperative sets. Allowing all of these would likely result in prohibitively high complexity. A useful direction for future work would be to develop a means for determining the most “fruitful” of these sets which are to be used. Finally, we considered only decode and forward models for cooperation. Incorporating other cooperative models into this framework is also of interest.

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