Impact of Concentrated Azimuth Power Spectrum for Performance Evaluation of User Terminal Array Antenna

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Abstract

In the previous study, the author has proposed a simplified propagation modeling and a performance evaluation scheme of MIMO antennas, which utilizes the ideal base-station antenna configuration so that the angular power spectrum model for the user terminal is applicable for Kronecker MIMO model. By utilizing this modeling concept, this paper considers the impact of the concentrated azimuth power spectrum in the propagation modeling for the diversity or MIMO antenna performance evaluation.

1. Introduction

Nowadays, multiple-input multiple-output (MIMO) transceivers become a reality for the wireless LAN for the increase of data rate without additional spectrum occupancy. Moreover, MIMO technologies are integrated into the next generation mobile access networks such as IMT-Advanced.

As is already pointed out in the context of the adaptive array antennas, classical parameters of the antenna such as directivity and efficiency are not sufficient to evaluate the performance of the antenna system as a whole. Propagation channel should be taken into account for the appropriate evaluation. Recently, very excellent double-directional channel models have been proposed for the MIMO system modeling and evaluation [1, 2]. They are, however, too complicated for the comparative evaluation of the array antennas of the user terminal as these models have been originally developed for the purpose of the system level simulation. Therefore, the author has proposed a simplified approach to evaluate the MIMO antenna performance [3]. With the assumption of the ideal vertically-polarized base-station (BS) array antenna, the conventional angular power spectrum model for the user terminal (UT) is applicable for Kronecker MIMO model. Taga model [4], which is commonly used as the angular power spectrum model for UT, was considered in [3]. In Taga model, the azimuth power spectrum (APS) is uniform. However, it is mentioned in [4] that the uniformity is due to the random user rotation in azimuth and not due to the uniform arrival of the power in azimuth. For example, APS in the macrocell is modeled as the wrapped Gaussian in [2], which is concentrated within some certain azimuth range.

This paper examines the influence of concentrated APS together with the user rotation to the performance of the array antennas. To focus just on the impact of APS, very simple modeling and evaluation scheme are deployed, i.e. the APS and the antenna directivity are further simplified from [3], and the performance is evaluated by the maximum ratio combining diversity instead of MIMO mutual information.

2. Summary of MIMO Antenna Evaluation Approach

This section summarizes the simplified approach of the MIMO antenna evaluation presented in [3].

It is common to evaluate the MIMO mutual information (MI) [5, 6, 7] as the potential measure of the MIMO channel quality including the array antennas at both link ends, although the final quality-of-service may not coincide with MI [8]. MI of MIMO channel I is defined as the function of the MIMO channel matrix $\mathbf{H}$ as

$$I = \log\det\left(\mathbf{I} + \rho\mathbf{H}\mathbf{H}^H\right), \tag{1}$$

where ρ is the reference value of signal-to-noise ratio. Therefore, it is necessary to model channel matrix including the propagation environment and antennas.

In the non-line-of-sight (NLoS) environment, each element of $\mathbf{H}$ is according to the Rayleigh fading, i.e. zero mean, complex, circular symmetric Gaussian distribution. To characterize the statistics of $\mathbf{H}$, MIMO fading correlation matrix $\mathbf{R}_{\mathrm{MIMO}}$ is defined as

$$\mathbf{R}_{\mathrm{MIMO}} = E[\mathrm{vec}(\mathbf{H})\mathrm{vec}(\mathbf{H})^H]. \tag{2}$$

Once $\mathbf{R}_{\mathrm{MIMO}}$ is known, a realization of $\mathbf{H}$ is obtained from $\mathbf{h}_{\mathrm{iid}}$, zero mean, unit variance, complex, circular symmetric i.i.d. Gaussian random entries, in the Monte-Carlo simulation as

$$\mathrm{vec}(\mathbf{H}) = \mathbf{R}_{\mathrm{MIMO}}^{\frac{1}{2}} \mathbf{h}_{\mathrm{iid}}. \tag{3}$$

In the macrocellular environment, the fading correlation among the BS array elements and the that among UT array elements are observed to be uncorrelated, at least when BS array elements are vertically-polarized [9]. In the case, the MIMO correlation matrix $\mathbf{R}_{\mathrm{MIMO}}$ is expressed as the Kronecker product between UT correlation matrix $\mathbf{R}_{\mathrm{UT}}$ and BS correlation matrix $\mathbf{R}_{\mathrm{BS}}$ as

$$\mathbf{R}_{\mathrm{MIMO}} = \mathbf{R}_{\mathrm{UT}} \otimes \mathbf{R}_{\mathrm{BS}}, \tag{4}$$

which is known as the Kronecker model.

Different from system level design, a designer of UT array antenna in the manufacturer can not arbitrarily choose BS array configuration. Therefore, the ideal vertically-polarized uncorrelated BS array can be assumed as a typical case. Then, BS correlation matrix is simply expressed as

$$\mathbf{R}_{\mathrm{BS}} = \mathbf{I} \tag{5}$$

The correlation matrix of UT array can be derived in the same manner as the correlation matrix of the diversity antenna [10]. APS for vertical and horizontal polarizations are defined as $p_{\mathrm{UT}\vartheta}(\vartheta, \varphi)$ and $p_{\mathrm{UT}\varphi}(\vartheta, \varphi)$ to satisfy the following normalization condition.

$$\int_0^{2\pi} \int_0^{\pi} \{p_{\mathrm{UT}\vartheta}(\vartheta, \varphi) + p_{\mathrm{UT}\varphi}(\vartheta, \varphi)\} \sin\vartheta \mathrm{d}\vartheta \mathrm{d}\varphi = 1. \tag{6}$$

To represent the UT antenna characteristics, polarimetric complex directivity of i-th user terminal antenna is defined as $e_{\mathrm{UT}\vartheta i}(\vartheta, \varphi)$ and $e_{\mathrm{UT}\varphi i}(\vartheta, \varphi)$ for ϑ and φ also to satisfy the normalized in the same manner as the antenna gain as

$$\frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} \left\{ |e_{\mathrm{UT}\vartheta i}(\vartheta, \varphi)|^2 + |e_{\mathrm{UT}\varphi i}(\vartheta, \varphi)|^2 \right\} \sin\vartheta \mathrm{d}\vartheta \mathrm{d}\varphi = \eta_i, \tag{7}$$

where $\eta_i \leq 1$ is the efficiency of the i-th UT antenna considering the loss and mismatch.

The correlation matrix of UT array is then calculated by using APS and the complex antenna directivity of UT antenna array as

$$[\mathbf{R}_{\mathrm{UT}}]_{ij} = \int_0^{2\pi} \int_0^{\pi} \left\{ e_{\mathrm{UT}\vartheta i}(\vartheta, \varphi) e^*_{\mathrm{UT}\vartheta j}(\vartheta, \varphi) p_{\mathrm{UT}\vartheta}(\vartheta, \varphi) + e_{\mathrm{UT}\varphi i}(\vartheta, \varphi) e^*_{\mathrm{UT}\varphi j}(\vartheta, \varphi) p_{\mathrm{UT}\varphi}(\vartheta, \varphi) \right\} \sin\vartheta \mathrm{d}\vartheta \mathrm{d}\varphi. \tag{8}$$

3. Impact of Azimuth Power Spectrum

Although the formulation of the previous section handles 3-D radiation patterns and 3-D angular power spectra, this section focuses on the influence of concentrated APS together with the user rotation to the performance of the array antennas in 2-D for simple analytical treatment.

Two APS models are considered, i.e. omnidirectional APS $p_{\mathrm{O}}(\varphi)$ and unidirectional APS $p_{\mathrm{U}}(\varphi)$. Satisfying Eq. (6), they are defined as

$$p_{\mathrm{O}}(\varphi) = \frac{1}{2\pi}, \tag{9}$$

$$p_{\mathrm{U}}(\varphi, \varphi_0) = \frac{1 + \cos(\varphi - \varphi_0)}{2\pi}, \tag{10}$$

where φ_0 is the nominal angle of arrival (AoA) varying within $[0, 2\pi)$ according to the rotation of the user terminal.

Two types of two-element array antennas are considered, i.e. space diversity array with half-wavelength separated omnidirectional elements, and direction diversity array with unidirectional elements. Satisfying Eq. (7), they are defined as

$$e_{\mathrm{S}1}(\varphi) = \exp\left(-\mathrm{j}\frac{\pi}{2}\cos\varphi\right) \quad , \quad e_{\mathrm{S}2}(\varphi) = \exp\left(+\mathrm{j}\frac{\pi}{2}\cos\varphi\right), \tag{11}$$

$$e_{\mathrm{D}1}(\varphi) = \sqrt{\frac{2}{3}}(1+\cos\varphi) \quad , \quad e_{\mathrm{D}2}(\varphi) = \sqrt{\frac{2}{3}}(1-\cos\varphi). \tag{12}$$

For all four combinations of APS and array antennas, $\mathbf{R}$'s are analytically derived as

$$\mathbf{R}^{\mathrm{OS}} = \begin{bmatrix} 1 & J_0(\pi) \\ J_0(\pi) & 1 \end{bmatrix} \quad , \quad \mathbf{R}^{\mathrm{US}} = \begin{bmatrix} 1 & J_0(\pi) - \mathrm{j}\cos\varphi_0 J_1(\pi) \\ J_0(\pi) + \mathrm{j}\cos\varphi_0 J_1(\pi) & 1 \end{bmatrix}, \tag{13}$$

$$\mathbf{R}^{\mathrm{OD}} = \begin{bmatrix} 1 & \frac{1}{3} \\ \frac{1}{3} & 1 \end{bmatrix} \quad , \quad \mathbf{R}^{\mathrm{UD}} = \begin{bmatrix} 1+\frac{2}{3}\cos\varphi_0 & \frac{1}{3} \\ \frac{1}{3} & 1-\frac{2}{3}\cos\varphi_0 \end{bmatrix}, \tag{14}$$

where $J_n(x)$ is the n-th order Bessel function of first kind.

For two-dimensional matrix, the characteristic equation can be also analytically solved and the eigenvalues are given as

$$\lambda = \frac{1}{2}\left\{(R_{11}+R_{22}) \pm \sqrt{(R_{11}+R_{22})^2 - 4(R_{11}R_{22} - R_{12}R_{21})}\right\}. \tag{15}$$

By substituting Eqs. (13) and (14) into Eq. (15), the following eigenvalues are obtained in these four scenarios:

$$\lambda^{\mathrm{OS}} = 1 \pm |J_0(\pi)| \quad , \quad \lambda^{\mathrm{US}} = 1 \pm \sqrt{J_0^2(\pi) + \cos^2\varphi_0 J_1^2(\pi)}, \tag{16}$$

$$\lambda^{\mathrm{OD}} = 1 \pm \frac{1}{3} \quad , \quad \lambda^{\mathrm{UD}} = 1 \pm \frac{1}{3}\sqrt{4\cos^2\varphi_0 + 1}. \tag{17}$$

Figure 1 shows the variation of eigenvalues for these four scenarios as functions of φ_0. It is obvious that the eigenvalues are fluctuating along φ_0 for the unidirectional APS. The gap of eigenvalues is the biggest when φ_0 is 0 deg and 180 deg in both space and direction diversity configurations. In space diversity configuration, the effective array size becomes maximum for φ_0 of ± 90 deg so that the degrees of freedom of the array can be fully utilized. In direction diversity configuration, the gap of the mean effective gain (MEG) [4] of two antennas, i.e. diagonal components of $\mathbf{R}$, is the biggest when φ_0 is 0 deg and 180 deg.

Once the eigenvalues of the correlation matrix is known, cumulative distribution function (CDF) $F(\gamma \leq x)$ of output SNR γ of the maximum ratio combining (MRC) is also analytically presented by [11]

$$F(\gamma \leq x) = \frac{1}{\lambda_1 - \lambda_2}\left[\lambda_1\left\{1 - \exp\left(-\frac{x}{\lambda_1}\right)\right\} - \lambda_2\left\{1 - \exp\left(-\frac{x}{\lambda_2}\right)\right\}\right]. \tag{18}$$

It is noted that F is averaged over φ_0, instead of the unidirectional APS.

Figure 2 shows the CDF of relative SNR after MRC for these four scenarios. Although forecasted from the eigenvalue distribution, these four CDFs almost coincide. It is noted that the average MEG values are identical in all four scenarios, as presented in Eqs. (13) and (14). As is well known, the diversity effect is insensitive to the correlation unless it exceeds 0.8–0.9, and that is the major reason why the results are almost identical.

4. Conclusion

This paper examined the influence of the concentrated APS together with the user rotation to the performance of the array antennas. The comparison of maximum ratio combining diversity performance for the simple scenarios reveals that the concentrated APS does not give any big impact to the performance. It is known that the correlation has relatively small impact to the performance unless it is below 0.8–0.9, which is well known. In the case, the mean effective gain may have the bigger impact, although it has been kept constant in the present study. The impact to the MIMO mutual information should be examined in the next step.

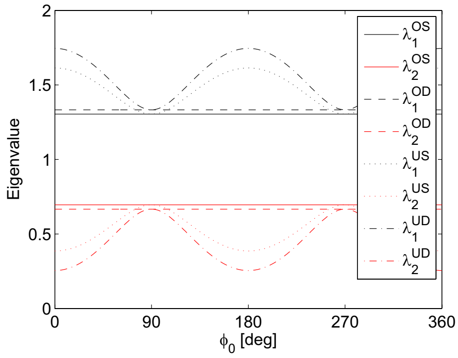

Figure 1: AoA dependence of the eigenvalues of correlation matrices.

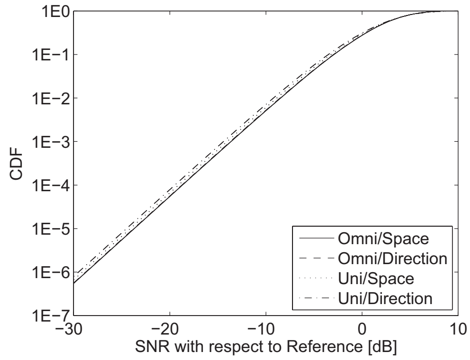

Figure 2: CDF of relative SNR after MRC.

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