Polarimetric Kronecker Separability in an Urban Macrocellular Environment

Kriangsak Sivasondhivat [(1)] **Jun-ichi Takada** [(1)] **Ichirou Ida** [(2)]
Yasuyuki Oishi [(2)]

[(1)] ***Tokyo Institute of Technology***
S6-4, 2-12-1, O-okayama, Meguro-ku, Tokyo 152-8552, Japan.
Email: sivasondhivat.kriangsak@gmail.com, takada@ide.titech.ac.jp

[(2)] ***Fujitsu Limited***
Yokosuka-shi, 239-0847, Japan.
Email: ida.ichirou@jp.fujitsu.com

Abstract

This paper investigates the Kronecker separability of the joint polarimetric angular power spectrum density (PSD) between the base station (BS) and mobile station (MS) of double directional measurements in an urban macrocell in Tokyo. A general form of the sum of channel polarization pair-wise Kronecker product approximation is proposed to be used to model a full channel correlation matrix. Its validity is compared with some recently proposed Kronecker product approximations. **Keywords:** Polarimetric Kronecker separability

1. Introduction

For single polarization transmission, the conventional Kronecker product approximation of a full channel correlation matrix was experimentally shown to well predict the ergodic mutual information and ergodic capacity of Multiple Input Multiple Output (MIMO) systems in [1][2]. However, it causes a performance error, when antennas with different element responses are used at either or both ends of the MIMO transmissions. This is the case for MIMO systems with dual-polarized antennas. Thus, the performance assessment is necessary when applying the conventional Kronecker product to multipolarization MIMO systems. Furthermore, more accurate Kronecker product approximations are needed to better predict the performance of polarimetric MIMO systems. In this paper, based on the full-polarimetric double directional measurements in an urban macrocell in Tokyo, the Kronecker separability of the joint polarimetric angular power spectrum density (PSD) between the base station (BS) and mobile station (MS) is investigated. By using the similar concept of analytical work on the Kronecker product of 2-D Spatial Channel Model (SCM) [3], the authors propose a general form of the sum of channel polarization pair-wise Kronecker product approximation, which is shortly called “sum of Kronecker products" herewith, to model the separability of the joint polarimetric angular PSD. It validity is compared with the recently proposed Kronecker product approximations.

2. Measurement and Channel Parameters Extraction

The measurements were carried out in a residential urban area in Tokyo [5]. The measurement site consists of 4 streets, which are Streets I (NS), II (WE), III (SN), and IV (EW). The Medav RUSK Fujitsu channel sounder was used in the measurement. The test signal was a periodic multitone signal with a center frequency of 4.5 GHz and bandwidth of 120 MHz. Multipath parameters are extracted and each modeled by the azimuth at BS (ABS), elevation at BS (EBS), azimuth at MS (AMS), elevation at MS (EMS), delay, and a matrix of polarimetric complex path weights, respectively. For the kth multipath, it is modeled by

$$\begin{bmatrix} \gamma_{\mathrm{VV},k} & \gamma_{\mathrm{VH},k} \\ \gamma_{\mathrm{HV},k} & \gamma_{\mathrm{HH},k} \end{bmatrix} \quad \delta(\phi^{\mathrm{BS}} - \phi_k^{\mathrm{BS}})\delta(\vartheta^{\mathrm{BS}} - \vartheta_k^{\mathrm{BS}})\delta(\phi^{\mathrm{MS}} - \phi_k^{\mathrm{MS}})\delta(\vartheta^{\mathrm{MS}} - \vartheta_k^{\mathrm{MS}})\delta(\tau - \tau_k), \qquad (1)$$

where $\gamma_{\mathrm{VV},k}$, $\gamma_{\mathrm{HV},k}$, $\gamma_{\mathrm{VH},k}$ and $\gamma_{\mathrm{HH},k}$ are the polarimetric complex path weights. The first and the second subscripts show the MS and BS, respectively. The quantities ϕ_k^{BS}, $\vartheta_k^{\mathrm{BS}}$, ϕ_k^{MS}, $\vartheta_k^{\mathrm{MS}}$, and τ_k denote the azimuth and elevation directions at BS and MS and delay, respectively. Their definitions can be found in [5]. The measurement site is mostly characterized by non-line-of-sight (NLOS) conditions. For some line-of-sight (LOS) measurement snapshots, since their LOS paths are deterministic, they are removed from the extracted multipaths, so that the considered channel becomes zero-mean complex circularly symmetric Rayleigh.

3. Polarimetric MIMO Channel Matrix and Reference Scenarios Considered

For wideband MIMO systems having N_{BS} and N_{MS} antennas at the BS and MS, respectively, where $n_{\mathrm{MS}} = 1, \ldots, N_{\mathrm{MS}}$ and $n_{\mathrm{BS}} = 1, \ldots, N_{\mathrm{BS}}$, the $(n_{\mathrm{MS}}, n_{\mathrm{BS}})$ element of a downlink MIMO channel matrix at the frequency f, $\mathbf{H}(f)$, can be expressed as a sum of channel responses of all polarization-pairs, i. e.

$$[\mathbf{H}(f)]_{n_{\mathrm{MS}} n_{\mathrm{BS}}} = \sum_{\alpha,\beta=\{\mathrm{V,H}\}} [\mathbf{H}_{\beta\alpha}(f)]_{n_{\mathrm{MS}} n_{\mathrm{BS}}}, \tag{2}$$

where $[\mathbf{H}_{\beta\alpha}(f)]_{n_{\mathrm{MS}} n_{\mathrm{BS}}}$ denotes the $(n_{\mathrm{MS}}, n_{\mathrm{BS}})$ element of single polarization $\mathbf{H}(f)$ of a $\{\beta\alpha\}$ polarization-pair in the downlink transmission. Note that β and α show the channel polarization at the MS and BS, respectively. By using the extracted multipaths, $[\mathbf{H}_{\beta\alpha}(f)]_{n_{\mathrm{MS}} n_{\mathrm{BS}}}$ can be expressed as the superposition of all multipaths between the BS and MS as follows.

$$\sum_{k=1}^{K} q_{\alpha,k}^{n_{\mathrm{BS}}} \gamma_{\beta\alpha,k} q_{\beta,k}^{n_{\mathrm{MS}}} \sqrt{G_{n_{\mathrm{BS}}}^{\mathrm{BS}}(\phi_k^{\mathrm{BS}}, \vartheta_k^{\mathrm{BS}}) G_{n_{\mathrm{MS}}}^{\mathrm{MS}}(\phi_k^{\mathrm{MS}}, \vartheta_k^{\mathrm{MS}})}\, a_k^{\mathrm{BS,MS}} \exp(-j2\pi f \acute{\tau}_k + j\nu_k^{\beta\alpha}), \tag{3}$$

where K is the number of extracted multipaths, $q_{\alpha,k}^{n_{\mathrm{BS}}}$ is a polarization mismatch coefficient at the BS, $q_{\alpha,k}^{n_{\mathrm{MS}}}$ is a polarization mismatch coefficient at the MS, $G_{n_{\mathrm{BS}}}^{\mathrm{BS}}(\cdot)$ is the antenna pattern of the n_{BS}th element, $G_{n_{\mathrm{MS}}}^{\mathrm{MS}}(\cdot)$ is the antenna pattern of the n_{MS}th element, $a_k^{\mathrm{BS,MS}}$ is an array factor due to antenna configurations at the BS and MS and directions of the kth path, $\acute{\tau}$ is the excess delay, $\nu_k^{\beta\alpha}$ is a uniformly distributed random phase from 0 to 2π, and λ is a wavelength. Since the validity of polarimetric Kronecker separability of the measured channel depends not only on the characteristics of the channel, but also on the polarized antennas, some standard polarized antennas at the BS and MS have to be assumed in the investigation. As reference antennas, the standard antenna configurations of a 3rd Generation Partnership Project Long-term Evolution (3GPP LTE) channel model are used (see Annex C. of [4]). For the BS, an antenna configuration with 4 antenna elements, where 2 antenna elements are dual at a slanted of $\pm 45^{\circ}$, is assumed. For the MS antenna, the authors assume 3 types of antenna configurations. There are *Laptop*, *Handheld-data*, and *Handheld-talk* scenarios.

4. Polarimetric Kronecker Product Approximations

In zero-mean complex circularly symmetric Rayleigh channels, $\mathbf{H}(f)$ is described by a full channel correlation matrix, $\mathbf{R}(f)$, which is $\mathbf{R}(f) = E\left[\mathrm{vec}(\mathbf{H}(f))\mathrm{vec}(\mathbf{H}(f))^H\right]$, where vec$(\cdot)$ stacks the columns of $\mathbf{H}(f)$ into a column vector, while $E(\cdot)$ and $(\cdot)^H$ are the expectation operator and the Hermitian transpose, respectively. The conventional Kronecker product approximation [1] models $\mathbf{R}(f)$ by $\mathbf{R}^{\mathrm{Con}}(f)$, which is the Kronecker product of the BS and MS antenna correlation matrices, i.e. $\mathbf{R}^{\mathrm{BS}}(f)$ and $\mathbf{R}^{\mathrm{MS}}(f)$, respectively. That is

$$\mathbf{R}^{\mathrm{Con}}(f) = \frac{1}{\mathrm{tr}(\mathbf{R}^{\mathrm{MS}}(f))} \mathbf{R}^{\mathrm{BS}}(f) \otimes \mathbf{R}^{\mathrm{MS}}(f), \tag{4}$$

where $\otimes$ denotes the Kronecker product, $\mathbf{R}^{\mathrm{BS}}(f) = E\left[\mathbf{H}(f)^T \cdot \mathbf{H}(f)^*\right]$, and $\mathbf{R}^{\mathrm{MS}}(f) = E\left[\mathbf{H}(f)\mathbf{H}(f)^H\right]$. $(\cdot)^T$ and $(\cdot)^*$ indicate the transpose and the complex conjugate, respectively. Note that the denominator term (i. e. tr$(\mathbf{R}^{\mathrm{MS}}(f))$) is used to equalize the traces of $\mathbf{R}(f)$ and $\mathbf{R}^{\mathrm{Con}}(f)$. Recently, in the framework of 3GPP LTE, the 3GPP LTE Kronecker product approximation has been proposed to model the polarimetric 3GPP LTE channel model [4]. Here, $\mathbf{R}(f)$ is approximated by $\mathbf{R}^{\mathrm{3GPP}}(f)$, which is the Kronecker product of the polarization covariance matrix and the BS and MS spatial correlation matrices as follows.

$$\mathbf{R}^{\mathrm{3GPP}}(f) = \begin{bmatrix} 1 & (\rho^{\mathrm{BS}})^* \\ \rho^{\mathrm{BS}} & 1 \end{bmatrix} \otimes \mathbf{\Lambda}(f) \otimes \begin{bmatrix} 1 & (\rho^{\mathrm{MS}})^* \\ \rho^{\mathrm{MS}} & 1 \end{bmatrix}, \tag{5}$$

where ρ^{BS} and ρ^{MS} are the spatial correlation coefficients between 2 identical omni-directional antenna elements assumed at the BS and MS, respectively, while $\mathbf{\Lambda}(f)$ is the polarization covariance matrix of the co-located polarization antenna elements, $\mathbf{H}_{\mathrm{pol}}(f)$. It is obtained as follows: $\mathbf{\Lambda}(f) = E\left[\mathrm{vec}(\mathbf{H}_{\mathrm{pol}}(f))\mathrm{vec}(\mathbf{H}_{\mathrm{pol}}(f))^H\right]$. Based on an analytical derivation by assuming certain PSD models, the use of the sum of channel polarization pair-wise Kronecker products has been proposed to model the full correlation matrix of the 2-D SCM by Shafi, et al. in [3]. However, its validity has not been verified

or compared with the above mentioned Kronecker product approximations by using real measurement data. Moreover, its extension to 3-D case has not been discussed. By using the similar concept, the authors propose the following general form of the sum of channel polarization pair-wise Kronecker products approximation, which the authors shortly call as the "sum of Kronecker products" herewith.

$$\mathbf{R}^{\mathrm{Sum}}(f) = \sum_{\alpha,\beta=\{\mathrm{V,H}\}} \frac{1}{\mathrm{tr}(\mathbf{R}^{\mathrm{MS}}_{\beta\alpha}(f))} \mathbf{R}^{\mathrm{BS}}_{\beta\alpha}(f) \otimes \mathbf{R}^{\mathrm{MS}}_{\beta\alpha}(f), \tag{6}$$

where $\mathbf{R}^{\mathrm{BS}}_{\beta\alpha}(f) = E\left[\mathbf{H}_{\beta\alpha}(f)^T \cdot \mathbf{H}_{\beta\alpha}(f)^*\right]$ and $\mathbf{R}^{\mathrm{MS}}_{\beta\alpha}(f) = E\left[\mathbf{H}_{\beta\alpha}(f)\mathbf{H}_{\beta\alpha}(f)^H\right]$. The MIMO channel matrix by using the Kronecker product approximations, $\mathbf{H}^{\mathrm{Kron}}(f)$, can be obtained as

$$\mathrm{vec}(\mathbf{H}^{\mathrm{Kron}}(f)) = \hat{\mathbf{R}}^{\frac{1}{2}}(f)\mathrm{vec}(\mathbf{A}), \tag{7}$$

where $\hat{\mathbf{R}}(f)$ is the approximated full correlation matrix. It is replaced by either $\mathbf{R}^{\mathrm{Con}}(f)$, $\mathbf{R}^{\mathrm{3GPP}}(f)$, or $\mathbf{R}^{\mathrm{Sum}}(f)$ in the equation above. $\mathbf{A}$ is an independent and identically distributed (i.i.d.) random fading matrix with unity-variance, circularly symmetric complex Gaussian entries.

5. Evaluation Process, Criteria, and Results

For each measurement snapshot, the authors synthesize measurement-based random $\mathbf{H}(f)$, according to Eq. (2) by Monte-Carlo simulations. Each channel realization is generated by the random phase method using Eq. (3). The number of the realizations, N_r, is set to 400. The number of frequency bins, N_f, is set to 25 within a bandwidth of 120 MHz. To take into account the change of the antenna orientation during the movement of the MS, the N_a combinations of antenna array orientation are also considered for each measurement snapshot. It is set to 8 with the step of 45°. Next, $\mathbf{R}^{\mathrm{Con}}(f)$, $\mathbf{R}^{\mathrm{3GPP}}(f)$, and $\mathbf{R}^{\mathrm{Sum}}(f)$ are obtained by using Eqs. (4), (5), and (6). Using Eq. (7), then $\mathbf{H}^{\mathrm{Kron}}(f)$ is thus synthesized. The ergodic mutual information, which is an important criteria from the viewpoint of maximum data rate, is used to evaluate the Kronecker product approximations. In case that the total power is equally allocated to each BS antenna assuming the channel state information is only known at the MS, the ergodic mutual information, $\mathcal{I}(n_f, n_a)$, of the n_fth frequency bin, where $n_f = 1 \cdots N_f$, and the n_ath MS orientation, where $n_a = 1 \cdots N_a$, is given by

$$\mathcal{I}(n_f, n_a) = E\left[\log_2 \det\left(\mathbf{I}_{N_{\mathrm{MS}}} + \frac{\mathrm{SNR}}{N_{\mathrm{BS}}} \tilde{\mathbf{H}}(n_f, n_a)\tilde{\mathbf{H}}^H(n_f, n_a)\right)\right], \tag{8}$$

where $\mathbf{I}_{N_{\mathrm{MS}}}$ denotes the identity matrix of size N_{MS}, and SNR is the average signal-to-noise ratio at the MS. The expectation is taken over the N_r realizations of $\tilde{\mathbf{H}}(n_f, n_a)$. For each instantaneous MIMO channel matrix, $\mathbf{H}^{(n_r)}(n_f, n_a)$, where $n_r = 1, \ldots, N_r$, the normalized instantaneous channel matrix, $\tilde{\mathbf{H}}^{(n_r)}(n_f, n_a)$, is obtained as

$$\tilde{\mathbf{H}}^{(n_r)}(n_f, n_a) = \frac{\mathbf{H}^{(n_r)}(n_f, n_a)}{\frac{1}{N_r N_f N_a N_{BS} N_{MS}} \sum_{n_r=1}^{N_r} \sum_{n_a=1}^{N_a} \sum_{n_f=1}^{N_f} \|\mathbf{H}^{(n_r)}(n_f, n_a)\|_F^2}, \tag{9}$$

where $\|\cdot\|_F$ is the Frobenius norm. $\mathbf{H}^{(n_r)}(n_f, n_a)$ is obtained by replacing ϕ_k^{MS} with $\{\phi_k^{\mathrm{MS}} - \phi^{\mathrm{MS}}(n_a)\}$ in Eq. (3), where $\phi^{\mathrm{MS}}(n_a) = 0^\circ, 45^\circ, \ldots, 315^\circ$ for $n_a = 1, \ldots, 8$, respectively. It should be noted that the differences in received power fading among frequency bins and MS antenna orientations are also considered when calculating $\mathcal{I}(n_f, n_a)$ in addition to those realizations. The ergodic mutual information of the Kronecker product approximations, $\mathcal{I}^{\mathrm{Kron}}(n_f, n_a)$, can be obtained by replacing the normalized $\mathbf{H}(n_f, n_a)$ with the normalized $\mathbf{H}^{\mathrm{Kron}}(n_f, n_a)$ in Eq. (8). $\mathbf{H}^{\mathrm{Kron}}(n_f, n_a)$ is a MIMO channel matrix by applying the Kronecker product approximations to the full correlation matrix of $\mathbf{H}(n_f, n_a)$. As an example, Fig. 1 shows $\mathcal{I}(n_f, n_a)$ and $\mathcal{I}^{\mathrm{Kron}}(n_f, n_a)$ at MS8 of the *Laptop* scenario. The variation of $\mathcal{I}(n_f, n_a)$ and $\mathcal{I}^{\mathrm{Kron}}(n_f, n_a)$ with the MS antenna orientation can be clearly seen in the figure. The accuracy investigation of the predicted $\mathcal{I}^{\mathrm{Kron}}(n_f, n_a)$ is done by comparing $\mathcal{I}(n_f, n_a)$ and $\mathcal{I}^{\mathrm{Kron}}(n_f, n_a)$ of the same frequency bin and MS antenna orientation at a measurement snapshot. The absolute percentage of the prediction error is calculated as

$$\varepsilon_{\mathcal{I}^{\mathrm{Kron}}}(n_f, n_a) = \frac{|\mathcal{I}^{\mathrm{Kron}}(n_f, n_a) - \mathcal{I}(n_f, n_a)|}{\mathcal{I}(n_f, n_a)} \times 100 \quad [\%]. \tag{10}$$

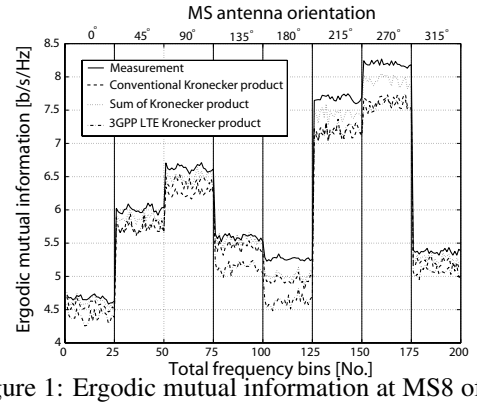

Figure 1: Ergodic mutual information at MS8 of the *Laptop* scenario.

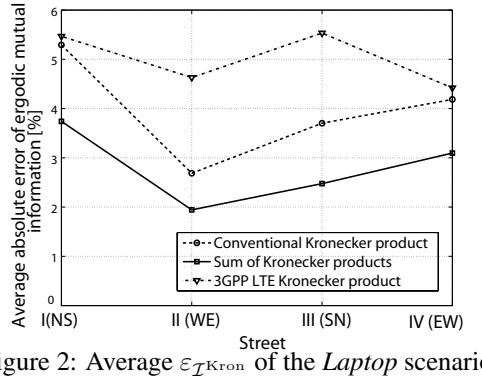

Figure 2: Average $\varepsilon_{\mathcal{I}^{\mathrm{Kron}}}$ of the *Laptop* scenario.

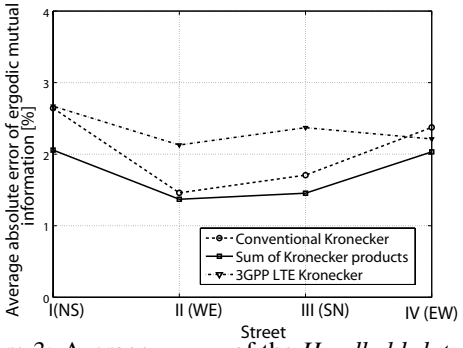

Figure 3: Average $\varepsilon_{\mathcal{I}^{\mathrm{Kron}}}$ of the *Handheld-data* scenario.

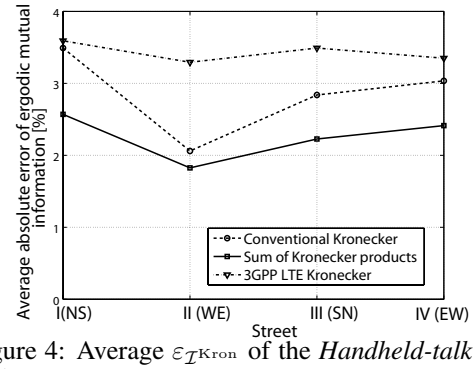

Figure 4: Average $\varepsilon_{\mathcal{I}^{\mathrm{Kron}}}$ of the *Handheld-talk* scenario.

Figures 2, 3, and 4 show the average $\varepsilon_{\mathcal{I}^{\mathrm{Kron}}}$ as a function of streets of the *Laptop*, *Handheld-data*, and *Handheld-talk* scenarios, respectively. As can be seen, the sum of Kronecker products approximation gives the most accurate prediction of the ergodic mutual information as compared to the others for all measurement streets and scenarios. While the 3GPP LTE Kronecker product approximation seems to be the worst, except for Street IV (EW) of *Handheld-data* scenario. This performance degradation could be because of the use of the common correlation coefficients for different co-located polarized antenna elements. Among all streets, Street II (WE), where multiple scattering occurs due to its only NLOS characteristic, seems to be most suitable street for applying the Kronecker product approximations. From the viewpoint of propagation channel, the validity of the sum of Kronecker product implies that the joint angular PSD between the BS and MS can be reasonably modeled as the product of the marginal angular PSDs at the BS and MS when the same single channel polarization-pair is considered.

6. Acknowledgments

This research is supported by the National Institute of Information and Communications Technology of Japan.

References

[1] J. P. Kermoal, L. Schumacher, K. I. Pedersen, P. E. Mogensen, and F. Frederiksen, "A stochastic MIMO radio channel model with experimental validation," IEEE Trans. on Selected Areas in Commun. vol. 20, no. 6, pp. 1211-1226, Aug. 2002.

[2] K. Sivasondhivat, J. Takada, I. Ida, and Y. Oishi, "Verification of Kronecker MIMO Channel Model in a NLOS Macrocellular Environment," Proc. IEICE General Conf. B-1-233, Mar. 2006.

[3] M. Shafi, M. Zhang, A. L. Moustakas, P. J. Smith, A. F. Molisch, F. Tufvesson, S. H. Simon, "Polarized MIMO channels in 3-D: models, measurements and mutual information," IEEE Trans. on Selected Areas in Commun. vol. 24, no. 3, pp. 514-527, March 2006.

[4] 3GPP, R4-060334, "LTE Channel Models and simulations," Feb. 2006.

[5] K. Sivasondhivat, J. Takada, I. Ida, and Y. Oishi, "Experimental Analysis and Site-Specific Modeling of Channel Parameters at Mobile Station in an Urban Macrocellular Environment," accepted for publication on IEICE Trans. Commun. .