Radiation from Circular Waveguide Aperture Fields Using Complex Conical Wave Objects

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Abstract

This paper demonstrates the application of a new kind of conical wave-objects to the computation of fields radiated by circular apertures. The radiation integral is transformed into a summation of complex wave-objects by means of the Fourier series expansion and Generalized Pencil of Functions (GPoF) method, which reduce the starting integral to a form that can be evaluated in a closed form. The results obtained via the new approach are compared to those calculated via direct integration of the starting radiation integral. It is shown that a very low number of wave-objects is required to represent the field of a circular waveguide mode.

1. Introduction

The analysis of beam waveguides and/or reflector systems at mm-wave frequencies is a very large-scale problem, being that the electrical dimensions of each element under analysis are typically of the order of hundred wavelengths or more. This brings out the need to develop new analysis methods particularly suited for that frequency and size range, since the regular approaches become prohibitively slow. Unlike classical numerical methods such as the Method of Moments or even Physical Optics, which use discretisation in order to numerically solve the starting integral equation and determine the field irradiated at a certain point, the methods developed for this area strive towards expressing the field in terms of a relatively low number of "beams", which are generally speaking wave-objects of higher complexity, but whose propagation, reflection, and diffraction if possible, can be resolved at least asymptotically in a closed form. The driving idea, illustrated in Fig. 1, is to expand the starting field into beams, propagate each beam independently to the element under consideration (e.g. a reflector), obtain the reflected field of each beam in a closed form and superimpose all the reflected beams to get the total reflected field. In the next step, the reflected total field can be re-expanded into a new sum of beams, and the process can be repeated, thus enabling the analysis of a complex and electrically large reflector system such as the beam waveguide.

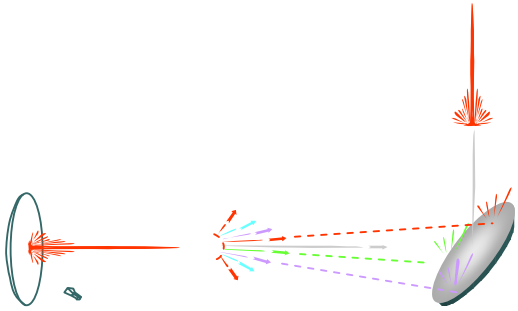


Figure 1. The illustration of the "beam" approach to the analysis of propagation and reflection.

A number of different objects have been introduced that are more or less suitable to the exposed analysis approach. These are the Gaussian Beams [1, 2], higher-order Gauss-Laguerre or Gauss-Hermite modes [3, 4], Complex Source Points [5], etc. Recently, the authors of this article presented a new kind of wave objects [6], introducing a formulation that has potential advantages over other approaches in that it satisfies two important aspects: 1) the wave objects respect the wave equation in all space where they are valid, and 2) the expansion of the field into wave objects is

done in a straightforward fashion. Moreover, the new wave objects possess analytical expressions in both spatial and spectral domain. They also have a directivity property on a cone, which is governed by varying a complex parameter associated with the z-coordinate, similar to the complex point source of a spherical wave. In this paper, following a brief summary of how these beams are generated, we will show how they can be used to reconstruct, in the near-field zone, the field radiated by an aperture with the field distribution of a circular waveguide mode.

2. Formulation

The starting position for our formulation is the spectral-domain radiation integral [7], expressed in cylindrical coordinates for convenience:

$$I(\rho,\phi,z)=\frac{1}{8\pi^2 j}\int_0^{\infty}\int_0^{2\pi} g\left(k_\rho,\alpha\right)e^{-j\rho k_\rho\cos(\alpha-\phi)}\frac{e^{-jz\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}}k_\rho dk_\rho d\alpha\,. \tag{1}$$

Here, $g\left(k_\rho,\alpha\right)$ is the spectrum of any of the Cartesian components of the electric or magnetic fields, or maybe even of the related vector potentials. The process of beam generation is done in two steps. First, the function $g\left(k_\rho,\alpha\right)$ is expanded in a Fourier series in the angular coordinate: $g\left(k_\rho,\alpha\right)=\sum_{n=-\infty}^{\infty}c_n\left(k_\rho\right)e^{-jn\alpha}$; next, the GPoF expansion of the kind $c_n\left(k_\rho\right)=\sum_{m=-M}^{M}d_{mn}e^{b_m\sqrt{k^2-k_\rho^2}}$ is applied to the Fourier coefficients. The double integral is in this way reduced to the following double summation:

$$I=\sum_n c_n\sum_{m=-M}^{M}d_{mn}W_n\left(\rho,\phi,z+jb_{mn}\right), \tag{2}$$

where $W_n\left(\rho,\phi,\tilde{z}\right)$ is the n-th order wave object, evaluated at the point $\left(\rho,\phi,\tilde{z}\right)$ with a complex coordinate $\tilde{z}=z+jb_{mn}$:

$$W_n\left(\rho,\phi,\tilde{z}\right)=e^{-jn\phi}\int_0^{\infty}\frac{e^{-j\tilde{z}\sqrt{k^2-k_\rho^2}}}{\sqrt{k^2-k_\rho^2}}\mathrm{J}_n\left(\rho k_\rho\right)k_\rho dk_\rho\,. \tag{3}$$

As demonstrated in [6], (3) can be evaluated in a closed form, meaning that the starting radiation integral has been reduced to a sum of functions that are well-behaved in the whole upper half-space ($\mathrm{Re}\{\tilde{z}\}>0$).

3. Application to the Radiation of a Waveguide Mode

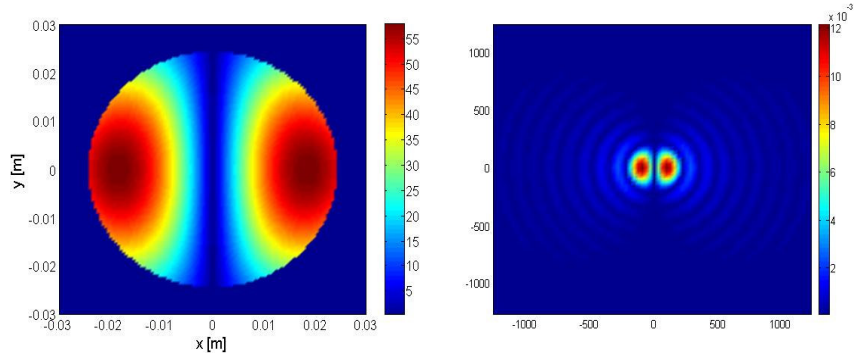
The above process is applied to the radiation from a circular aperture field with a TM circular waveguide mode distribution. This aperture field possesses closed form expressions in both spectral and spatial domain. The variation of the spectrum along the k_ρ spectral radial variable is fairly complex and can easily be varied by simple modification of the mode number, therefore constituting a strong test case for checking the accuracy of the expansion. Furthermore, the Fourier spectrum in the angular variable can be derived analytically which allows observing and testing the behaviour of the automated routine based on fast Fourier transform and its influence on the accuracy of the approximation. A TM_{mn} circular waveguide mode is defined by [9]

$$\mathbf{E}_{mn}\left(\rho,\phi\right)=\alpha_{mn}\mathrm{J}'_m\left(\alpha_{mn}\rho\right)\cos\left(m\phi\right)\hat{\rho}-\frac{m}{\rho}\mathrm{J}_m\left(\alpha_{mn}\rho\right)\sin\left(m\phi\right)\hat{\phi}\,. \tag{4}$$

For simplicity, we will take only its x-component, whose spectrum can be derived in closed form as

$$\tilde{E}_{x,mn}\left(k_\rho,\alpha\right)=j^{m-1}2\pi\frac{\alpha_{mn}r_w}{\alpha_{mn}^2-k_\rho^2}\cdot\left\{\alpha_{mn}\mathrm{J}'_{m+1}\left(\alpha_{mn}r_w\right)\mathrm{J}_{m+1}\left(k_\rho r_w\right)-k_\rho\mathrm{J}_{m+1}\left(\alpha_{mn}r_w\right)\mathrm{J}'_{m+1}\left(k_\rho r_w\right)\right\}\cos\left(m\alpha\right)\cos\left(\alpha\right)\,. \tag{5}$$

Here, J_m is the Bessel function of m-th order, r_w is the waveguide radius, while $\alpha_{mn} = \chi_{mn} / r_w$, where χ_{mn} is the n-th zero of J_m. The spatial and spectral distributions of the x-component of TM_{01} mode are shown in Fig. 2.



a) *b)*

Figure 2. a) Magnitude of the E_x component of a TM_{01} circular waveguide mode aperture field ($r_w = 0.024049$ m).
b) Magnitude of the x-component of the corresponding spectrum $\tilde{E}_x$.

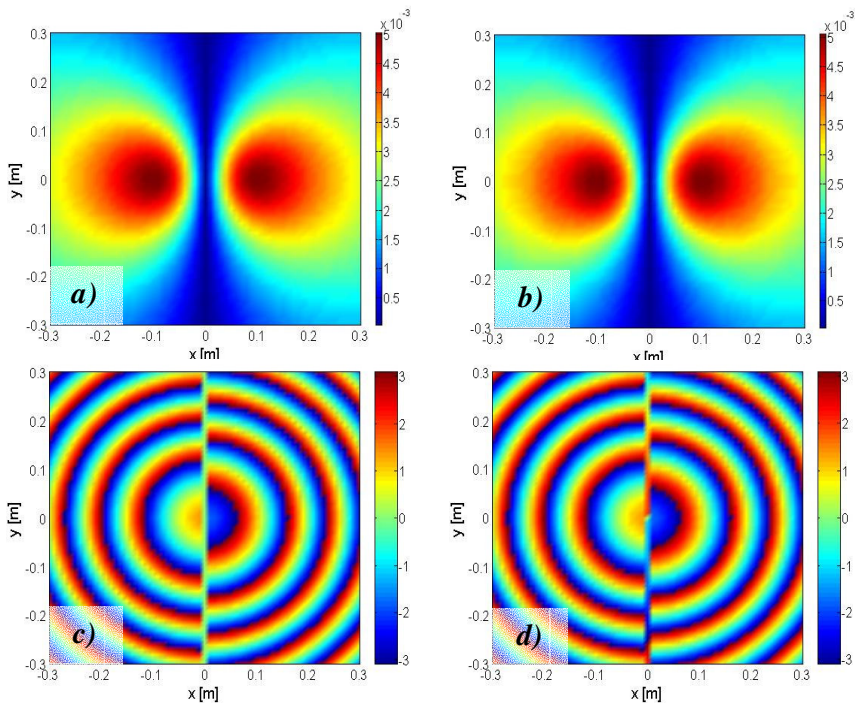


Figure 3. *a*) and *c*) Magnitude and phase computed with direct integration;
b) and *d*) Magnitude and phase computed via expansion in wave-objects.

The validity of the analysis method is established by comparing the results obtained directly via Eq. (1) and via our complex wave-objects expansion (Eqs. (2) and (3)). The wave number was simply chosen $k = 100$. The direct integration was performed analytically in α-coordinate and numerically in k_ρ in Wolfram Mathematica. It should be noted that the direct numerical integration is not an easy task due to the presence of a pole and a branch cut for $k_\rho = k$ and some convergence problems had to be overcome. The results are compared in Fig. 3, which shows the values of the radiation integral computed for the TM_{01} mode over a 5λ by 5λ cut in a plane 2λ away from the waveguide aperture (i.e. for $z = 2\lambda$, while the aperture is at $z = 0$).

As can be seen in Fig. 3, an excellent agreement between the original integral and its wave-objects expansion has been achieved, both in magnitude and in phase. It should be noted that only 16 wave-objects were necessary to completely reconstruct the field of the TM_{01} mode, while in k_ρ plane it was sufficient to sample the spectrum up to $k_\rho = 2k$ for a sufficiently accurate GPOF approximation (the spectrum was sampled in 64 points and represented with 8 GPoF exponents). The automated FFT-GPOF expansion routine, implemented in FORTRAN, determines the appropriate coefficients b_{mn} and d_{mn} almost instantly while the iterative procedure for the computation of wave-objects, set up in Matlab, needed approximately 2 minutes to compute the values in 3721 points on a laptop PC with AMD Turion64x2 1.8GHz processor and 1GB of RAM. This time can be made much shorter if the same routine is transferred to FORTRAN. However, the main advantage of the new representation is the fact that this kind of wave objects satisfies the wave equation in the whole upper half-space (unlike e.g. Gaussian Beams which are only a paraxial solution of the wave equation). Other examples and tests have been successfully performed confirming the usefulness of the new approach, but are omitted here due to space limitations.

4. Conclusion

A novel method for computing the fields radiated by apertures has been presented and its application to the case of circular waveguide openings has been demonstrated. It has been shown that a very small number of new, physically correct wave-objects is sufficient to accurately represent the radiation of a TM_{mn} circular waveguide mode. Practical aspects regarding the sampling in k_ρ plane for the GPoF expansion have been discussed. The vectorialisation of the whole process is currently underway.

5. Acknowledgments

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6. References

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