Multilevel Time Domain Physical Optics for Non Linear Scattering

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Abstract

Fast time domain physical optics (TD-PO) scheme is proposed for calculating the far field of antennas/scatterers exhibiting passive frequency intermodulations. The TD-PO scheme is based on a domain decomposition of the antenna/scaterer surface into smaller patches. The scheme exploits the fact that, the number of temporal and angular samples for accurate description of the far fields is proportional to the size of the patch. Therefore, the radiation patterns are computed directly only for the smallest subdomains. Subsequently, the radiation patterns are aggregated through a multi-level process comprising temporal and angular interpolations and time-shifts, thus, leading to a reduced computational complexity.

1. Introduction

Large reflector antennas, aircrafts, and many other man-made objects are constructed of a large number of metallic sheets or tiles that are joined with rivets or other fasteners. The imperfect metal-metal contacts give raise to non-linear electromagnetic effects, such as passive intermodulation (PIM), that become significant under high power excitations [1,2]. These non-linear phenomena are best analyzed by time domain (TD) methods. For electrically large smooth geometries, efficient calculation of the radiation patterns or scattering cross sections is often performed via high frequency methods such as the physical optics (PO). In this context, the TD-PO appears as a natural tool for the computation of the radiated fields after they have undergone non linear transformations. The proposed fast TD-PO scheme is based on a hierarchical decomposition of the scattering or radiating surface into smaller patches. This approach has already been successfully applied to frequency domain PO [3].

2. Time Domain Physical Optics

The TD-PO has been introduced recently [4] as a TD tool for large scattering problems. Alike its frequency domain counterpart, it comprises computation of scattered or radiated fields via integrals of equivalent surface currents, which are derived from incident fields through local approximations. Details of TD-PO, especially with regard to the modeling of non linear effects, are discussed below.

2.1 Continuous Formulation

Consider a radiating aperture or scattering surface S over which the equivalent currents are assumed to be known. The far radiated field is given by $\boldsymbol{E}(\boldsymbol{r},t) \sim \boldsymbol{U}(\hat{\boldsymbol{r}},\tau{=}t-r/c)/4\pi r$ where $\boldsymbol{U}(\hat{\boldsymbol{r}},\tau)$ is the TD radiation pattern:

$$\boldsymbol{U}(\hat{\boldsymbol{r}},\tau) = \oiint_S ds'\, \boldsymbol{A}(\hat{\boldsymbol{r}},\boldsymbol{r}',\tau + \tfrac{\hat{\boldsymbol{r}}\bullet\boldsymbol{r}'}{c}), \qquad \text{with} \qquad \boldsymbol{A}(\hat{\boldsymbol{r}},\boldsymbol{r}',t') = -\mu_0 \left[\boldsymbol{I} - \hat{\boldsymbol{r}}\hat{\boldsymbol{r}}\right]\cdot \frac{\partial \boldsymbol{J}_{\mathrm{e}}(\boldsymbol{r}',t')}{\partial t'} + \frac{1}{c}\hat{\boldsymbol{r}} \times \frac{\partial \boldsymbol{J}_{\mathrm{m}}(\boldsymbol{r}',t')}{\partial t'}. \tag{1}$$

Here, $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$ is the unit vector in the direction of observation, c is the speed of light, $\boldsymbol{J}_{\mathrm{e}}(\boldsymbol{r}',\tau)$ and $\boldsymbol{J}_{\mathrm{m}}(\boldsymbol{r}',\tau)$ are the electric and magnetic equivalent surface currents, respectively. In the following, TD-PO algorithms will be presented in the case of radiation pattern evaluation for a reflector antenna, when non linear effects arise along imperfect joints between tiles or metal sheets. Two definitions of equivalent currents have to be used: (*i*) On metallic surfaces considered as perfect conductors, we take as usual: $\boldsymbol{J}_{\mathrm{e}}(\boldsymbol{r}',\tau) = 2\hat{\boldsymbol{n}} \times \boldsymbol{H}^{\mathrm{i}}(\boldsymbol{r}',\tau)$, $\boldsymbol{J}_{\mathrm{m}}(\boldsymbol{r}',\tau) = 0$, with $\boldsymbol{H}^{\mathrm{i}}(\boldsymbol{r}',\tau)$ the incident magnetic field on the reflector; and (*ii*) Along imperfect junctions, non linear equivalent electric and magnetic currents arise, which will be evaluated according to the models proposed in [2].

2.2 Non Linear Effects

Non-linear effects modify the frequency spectrum of the equivalent currents, as compared to the spectrum of the original excitation (incident field) by introducing inter-modulation products (cf. Fig. 1). To that end, the maximum frequency of the radiated fields can be several times higher than that of the original excitation. Bandpass filter can be applied to the non-linear currents, to limit their frequency spectrum to the band of interest in the radiated fields and to suppress the low frequencies, thus, making the problem amenable to asymptotic methods such as the PO.

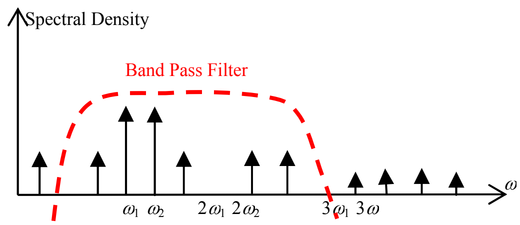
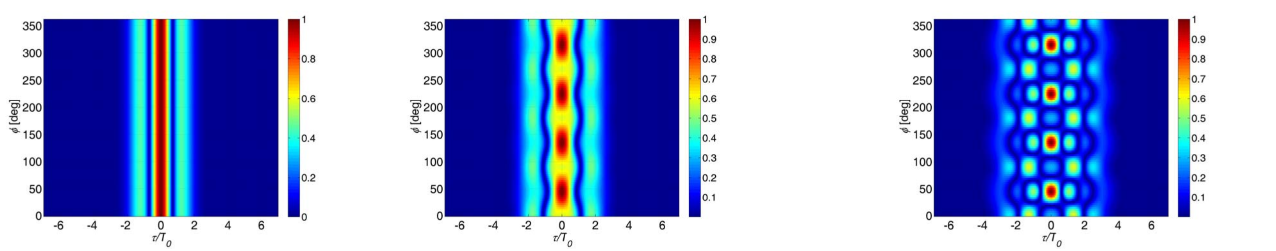


Figure 1: Spectrum of the non-linear currents (for bi-frequency incident fields) and a typical band pass filter profile.

Obviously, the sampling rate in the TD has to satisfy the Nyquist sampling rule. Consequently, non-linear effects lead to increased temporal sampling rates for the TD representations of the currents and fields. Similarly, the sampling rate for each angular variable also has to be proportional to the highest frequency of radiated fields [5]. A full 3D pattern computation in TD with the non-linear effects taken into account up to $\omega_{\max} = q\omega_{\max}^{\mathrm{i}}$, where $\omega_{\max}^{\mathrm{i}}$ is the maximum frequency of the incident source fields, leads to a multiplication of the number of sample points by a factor of q^3.In the following, we shall only consider harmonics of at most second order and inter-modulation products of up to third order. To this end, a band pass filter will be applied to the non-linear currents (cf. Fig. 1).

2.3 Sampling of Time Domain Radiation Pattern

The temporal length of the radiated pulse is upper bounded by $T_{\mathrm{s}} + 2R_{\mathrm{a}}/c$, where T_s is the length of the source pulse and R_{a} is the radius of the smallest sphere circumscribing the antenna. Fig. 2 illustrates the influence of R_{a} on the time duration of the radiated field pulse, for a given pulse source. Fig. 2(a) depicts the pulsed radiation pattern for a z - directed dipole located at the origin and excited by a Gaussian pulse $\left[1-2\left(t/T_0\right)^2\right]e^{-(t/T_0)^2}$. In Figs. 2(b) and 2(c), 5 such dipoles are placed one at the origin and the others at the corners of squares with side lengths of $1.3cT_0$ and $2.6cT_0$, respectively, centered at the origin. As can be observed, the larger the “antenna” (array), the longer is the radiated pulse duration, with more rapid variations vs. the polar angle ϕ in the xy - plane.



(a) One source at the origin. (b) Sources in a $1.3cT_0$ side length square. (c) Sources in a $2.6cT_0$ side length square.

Figure 2: Antenna size influence on time duration of the radiated field pulse.

As well-known, full angular representation of the fields radiated by an electrically large antenna with the smallest circumscribing sphere of radius R_a requires for each angular variable a number of samples proportional to $N = k_{\max} R_\mathrm{a}$, where $k_{\max} = \omega_{\max}/c$ [5]. Small patch radiation patterns can thus be described through computation of radiated fields on shorter time intervals and for fewer directions in space than large antenna radiation patterns, all other parameters being the same.

2.4 Complexity of Direct Computation

Here, the complexity of computations will be evaluated for asymptotically large values of the parameter N. As mentioned above, TD description of the radiated fields requires a number of samples in angular domain proportional to N along each angular variable, at a number of instants proportional to N for large values of N (i.e., $cT_\mathrm{s} \ll R_\mathrm{a}$). The computation of these instantaneous field is performed through integrations which require the evaluation of integrand values at a number of points on the antenna proportional to N for each integration variable. Finally, the total number of operations required for the direct computation of radiated fields in the TD is of $O(N^5)$.

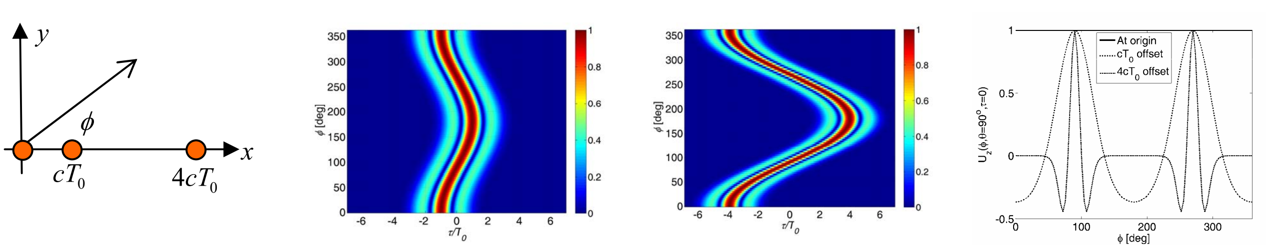
3. Domain Decomposition Approach

Domain decomposition approach takes advantage of the observations presented in Sec. 2.3. Decomposing the large surface of a reflector into small patches allows for fast direct computation of fields radiated by these patches on coarse grids of angular directions and on short time intervals. In this section we address the problem of aggregating these "partial" radiated fields in order to obtain the total field radiated by the entire reflector surface.

3.1 Delay Compensation

Note first that slow angular variations and short temporal duration characterize the TD radiation patterns only of patches that are placed at the origin of the coordinate system used for the radiation pattern calculation. This observation is of utmost importance for further aggregation of partial radiated patterns in the multilevel TDPO algorithms.

Fig. 3 illustrates this observation with a single dipole source with the same pulse excitation as in Sec. 2.3, successively placed at the origin and at two different distances from the origin, along the x - axis, as shown in Fig. 3(a). Figs. 3(b)-3(c) show the level of the z component value of the radiation pattern $\boldsymbol{U}(\hat{\boldsymbol{r}},\tau)$, as a function of the polar angle ϕ in the xy - plane and of the time variable τ. The translated source cases presented in Figs. 3(b) and 3(c) can be compared to the "trivial" case presented in Fig. 2(a) with the source at the origin. Clearly, increased distance between the source and the origin leads to increased time duration, as the pulse does not "arrive" at the same moment for different observation directions. More specifically in Fig. 3(d), the radiation pattern is becoming more rapidly varying function of ϕ (for a given observation instant) as the source gets further from the origin.



(a) Three Sources

(b) $U_z(\phi,\theta=90^o,\tau)$ with source at cT_0 .

(c) $U_z(\phi,\theta=90^o,\tau)$ with source at $4cT_0$.

(d) $U_z(\phi,\theta=90^o,\tau=0)$ with sources at cT_0 and $4cT_0$.

Figure 3: Influence of source to origin distance on the TD radiation pattern.

For a small patch approximately centered at point $\overline{\boldsymbol{r}}$, calculating its radiated fields in its own patch-centered coordinate system simply means replacing $\boldsymbol{r}'$ by $\boldsymbol{r}'-\overline{\boldsymbol{r}}$ in the expression of $\boldsymbol{U}(\hat{\boldsymbol{r}},\tau)$ given by Eq. (1). Since the norm $\boldsymbol{r}'-\overline{\boldsymbol{r}}$ is small, this interchange will affect the value of $t' = \tau + \hat{\boldsymbol{r}}\cdot\boldsymbol{r}'/c$ by introducing small variations vs. $\hat{\boldsymbol{r}}$.

$\boldsymbol{U}(\hat{\boldsymbol{r}},\tau)$ will vary slowly with $\hat{\boldsymbol{r}}$ and only a short time duration of the equivalent current pulses, around τ, will contribute to $\boldsymbol{U}(\hat{\boldsymbol{r}},\tau)$.

3.2 Multi-Level Domain Decomposition

In the multi-level algorithm, the reflector surface is recursively decomposed into a hierarchy of patches whose linear sizes are approximately divided by two at each level of decomposition; until obtaining patches of approximately square wavelength size. Only the radiation patterns of the smallest patches (at the highest level of decomposition) are computed. They are then successively aggregated into patterns radiated by lower level patches. At each level, angular interpolation, and change of origin of coordinate systems (hence temporal delay using interpolation) are performed. At the finest level of decomposition, denoted by L, the radiation patterns of all the small patches, indexed by *n*, are computed as:

$$\overline{\boldsymbol{U}}_n^L(\hat{\boldsymbol{r}},\tau) = \int_{\overline{S}_n^L} \boldsymbol{A}(\hat{\boldsymbol{r}},\boldsymbol{r}',\tau + \hat{\boldsymbol{r}}\cdot(\boldsymbol{r}' - \overline{\boldsymbol{r}}_n^L)/c)ds' \,. \tag{2}$$

From any level *l* to the next lower one, level *l*-1, the aggregation is performed to calculate the radiation pattern of the *m*th patch at level *l*-1, as:

$$\overline{\boldsymbol{U}}_m^{l-1}(\hat{\boldsymbol{r}},\tau) = \sum_{n:P^l(n)=m} \overline{\boldsymbol{U}}_n^l(\hat{\boldsymbol{r}},\tau + \hat{\boldsymbol{r}}\cdot(\overline{\boldsymbol{r}}_n^l - \overline{\boldsymbol{r}}_m^{l-1})/c) \,. \tag{3}$$

The *l*th level patches which contribute to this pattern are those whose "parent" is the *m*th patch at level *l*-1. This is expressed in Eq. (3) by the fact that their indices belong to the set: $\{n : P(n) = m\}$.
For this multi-level algorithm, the number of operations required to compute the fields radiated by small patches is bounded, the number of patches is of $O(N^2)$. However, as compared to the frequency domain algorithm, the number of interpolations is not of $O(N^2)$ per level: at each level, the number of patches is divided by 4, the number of angular samples is multiplied by 4, and the temporal duration of the pulse, hence the number of time samples, is multiplied by 2. Finally, the total computational complexity is of $O(N^3)$ for asymptotically large values of *N*. Thus, the multilevel algorithm leads to a reduced computational complexity, as compared to that of the direct TD-PO evaluation.

4. Conclusions

A multilevel TD-PO algorithm for analysis of linear and non-linear TD scattering and radiation by arbitrary shaped smooth surfaces/apertures has been developed. The proposed fast algorithm will be illustrated by its application to the radiated field computation for problems involving non-linear induced currents. A pulsed beam will be used as the primary source. The accuracy of the fast algorithm will be assessed through comparison with the regular TD-PO algorithm and modified frequency domain computations. Accuracy and computation time variation with the size of the problem will be investigated.

5. Acknowledgments

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6. References

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