On Extracting the Effective Propagation Constant of a Cut-Wire Array

Yang Li and Hao Ling
Department of Electrical and Computer Engineering
The University of Texas, Austin, TX 78712

Introduction

The effective medium model has been widely used for studying radio wave propagation in forested environments below 200MHz [1]. The effective permittivity of a forest is generally assumed to be slightly greater than that of free space, leading to the guidance of lateral waves along the forest-air interface. Recently, Buhl and Rogers reported on a measurement technique to determine the forest effective permittivity [2]. Results of the measurements were quite surprising, as it was found that the effective permittivity of the forest for the vertical polarization can be less than that of free space. Since actual measurements in such a large-scale, naturally occurring environment are difficult to control, we are motivated to further investigate this finding from a simulation perspective, where controlled parameter variations can be easily carried out.

It is well known that a periodic array of long metallic wires exhibits negative permittivity at sufficiently low frequencies [3, 4]. The so-called "cut-wire" array due to an array of finite-length wires has also been investigated recently in the metamaterials community [5, 6]. For an array of finite-length wires, it is shown that the plasma-like phenomenon is complicated by an additional half-wavelength, cut-wire resonance.

In this paper, we present a simulation and extraction method analogous to that described in [2] to determine the effective propagation constant of a cut-wire array. The array is excited by an embedded transmitting antenna and a receiver is placed in the array at different distances away from the transmitter. The phase of the received voltage versus distance are simulated and analyzed. It is found that a linear phase progression as a function of distance is clearly observed at most frequencies except when the wire is resonant. The effective propagation constant is then extracted and the plasma-like cutoff phenomenon is observed. The plasma frequency is found to be higher than that given in [4] for the infinite-length wire case. In addition, both the half-wavelength and 3/2-wavelength wire resonances can be seen. The frequency behavior of the propagation constant is compared to an approximate formula.

Simulation Setup

The Numerical Electromagnetic Code (NEC) is used to simulate a cut-wire array with both the transmitter and receiver embedded inside the array. For computation savings, an infinite PEC plane is used to image the geometry. The simulation setup is shown in Fig. 1. Each element of the wire array is a vertical wire with length equal to 1.75m. The wire radius is 0.002m. The spacing between the elements is 1.2m in both the x and y directions. Both the transmitter and the receiver are short vertical monopoles embedded in the cut-wire array. The array size used is 6 along the x direction and 36 along the y direction. Further increase in the array size does not change the results significantly. The frequency of the simulation ranges from 10MHz to 180MHz. To extract the effective medium parameters, the voltage at the receiver is modeled as follows:

$$V_{Rx} = A \frac{e^{-j\beta r} e^{-\alpha r}}{r} \qquad (1)$$

where r is the distance between the transmitter and the receiver, β is the propagation constant and α is the decay constant. β and α are related to the effective permittivity by the dispersion relationship. Once the received voltages are collected at different positions and frequencies, β and α can be determined by fitting the simulation data to (1).

Results

Fig. 2(a) shows the phase of the received field versus distance and frequency. In the plot, the phase has been unwrapped versus distance and the color scale reflects the phase delay between the transmitter and the receiver. It is seen that the phase delay increases linearly with distance across almost all frequencies. The only exceptions occur at 40MHz and 130MHz. We also observe an abrupt phase transition at 60MHz. The unwrapped phase delay at 40MHz and 130MHz are plotted in Fig. 2(b) for a closer examination. The phase delay at 85MHz is shown in the figure for comparison. The unwrapped phase functions at 40MHz and 130MHz do not vary linearly versus distance. While not shown here, it is also found that the amplitude versus distance shows significant fluctuation and does not reveal any clear trends.

Based on the above observations, the phase data is fitted using linear least squares (except at 40MHz and 130MHz) to arrive at the propagation constant β for different frequencies. The results are shown as red circles in Fig. 3. To help explain the behavior of the propagation constant versus frequency, we utilize the formula for the effective permittivity for a cut-wire array reported in [5]. However, to account for the higher order resonance that we observe, an additional term is added in the expression as follows:

$$\varepsilon_r(\omega) = 1 - \frac{\omega_{ep}^2 - \omega_{eo1}^2}{\omega^2 - \omega_{eo1}^2 - j\gamma_1\omega} - k\,\frac{\omega_{ep}^2 - \omega_{eo2}^2}{\omega^2 - \omega_{eo2}^2 - j\gamma_2\omega} \qquad (2)$$

In (2), ω_{ep} is the electrical plasma frequency, ω_{eo1} and ω_{eo2} are the resonance frequencies of the cut-wire, γ_1 and γ_2 represent the damping losses of the cut-wire resonances, and k is a scale factor to represent the relative strengths of the resonances. We choose ω_{ep} to be 60MHz based on the observation that the propagation constant approaches zero as the frequency is decreased to 60MHz. We choose ω_{eo1} and ω_{eo2} to be 40 and 130 MHz. These two frequencies correspond closely to the *1/2* and *3/2* wavelength resonance frequencies of the cut-wire. γ_1 and γ_2 are chosen as 1MHz and 30MHz, and k is chosen as 0.05. Given these parameters, we can generate the theoretical propagation constant versus frequency curve. As seen in Fig. 3, the theoretical curve based on (2) agrees fairly well with the numerically extracted results.

To further verify the cut-wire resonance, we plot the averaged current distribution on the cut-wire at 40MHz and 130MHz in Figs. 4(a) and (b). From the two figures, it is clear that the cut-wire is at half-wavelength resonance at 40MHz and 3/2-wavelength resonance at 130MHz. The one-wavelength resonance is not observed due to the symmetry of the excitation. To confirm the cutoff phenomenon below the plasma frequency, we examine the amplitude of the received signal and find that the signal decays very quickly as a function of distance from 45MHz to 60MHz. The current distribution on the entire cut-wire array at 45MHz is plotted in Fig. 4(c). It is seen that only wire rods very close to the transmitter are excited. Given the wire spacing and wire radius in our simulation, Pendry's formula in [4] gives a plasma frequency of 40MHz. However, the formula is based on the assumption that the wires are infinitely long. We find here that the plasma frequency is shifted up to 60MHz due to the finite length of the wires.

Conclusion

To better understand the effective medium parameters of a cut-wire array, a numerical simulation is conducted and the results are fitted to a model to extract the effective propagation constant of the array. The phase of the signal versus distance is linear over almost all the frequencies and can be welll fitted to a linear phase model. The propagation constant outside the resonant region is successfully extracted using this method and agrees well with the theoretical formula. Both the cut-off phenomenon and the cut-wire resonance phenomenon are observed and explained. The finite length of the cut-wire introduces multiple resonances and shifts up the plasma frequency in comparison with the infinite-length wire array.

Acknowledgments

This work is supported by the National Science Foundation under grant ECCS-0725729. We would like to thank Prof. G. Shvets of the Univ. of Texas Physics Department for helpful discussions.

References

[1] T. Tamir, "On radio-wave propagation in forested environments," *IEEE Trans. Antennas Propagat.*, vol. 15, pp. 806-817, Nov. 1967.

[2] D. Buhl and R. L. Rogers, "A technique for characterizing forest as a medium for HF radio propagation," to be presented at the Int. IEEE AP-S Symposium, July 2008.

[3] W. Rotman, "Plasma simulation by artificial dielectrics and parallel-plate media," *IRE Trans. Antennas Propagat.,* vol. 10, pp.82-95, Jan 1962.

[4] J. B. Pendry, A. J. Holden, W. J. Stewart and I. Youngs, "Extremely low frequency plasmons in metallic mesostructures," *Phys. Rev. Lett.*, vol. 76, pp.4773-4776, June 1996.

[5] R. A. Shelby, D. R. Smith and S. Schultz, "Experimental verification of a negative index of refraction," *Science*, vol. 292, pp.77-79, 2001.

[6] T. Koschny, M. Kafesaki, E. N. Economou and C. M. Soukoulis, "Effective medium theory of left-handed materials," *Phys. Rev. Lett.*, vol. 93, 107402, pp.1-4, Sept. 2004.

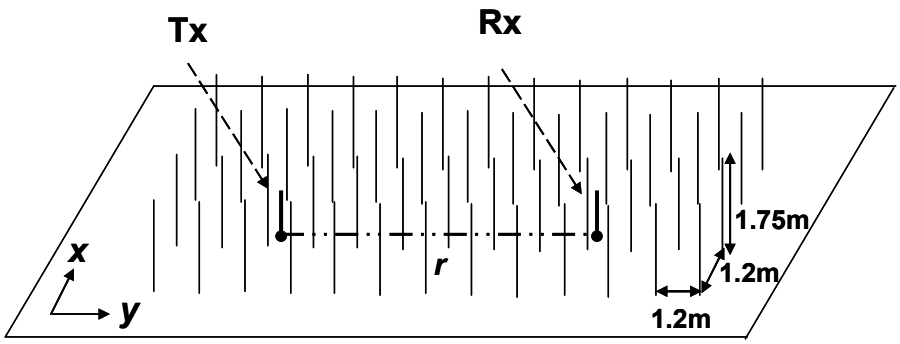


Figure 1. Cut-wire array on a PEC ground plane.

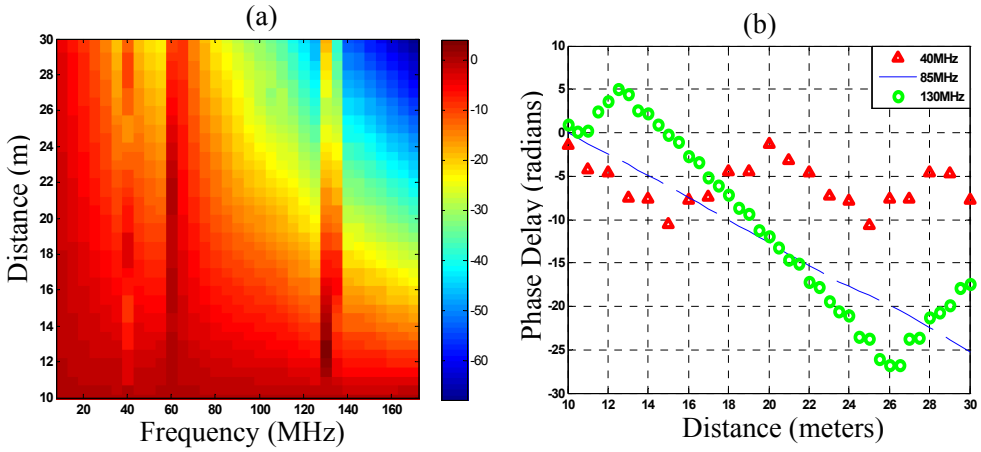


Figure 2 (a) Phase delay at different frequencies and positions
(b) Phase delay at 40MHz, 85MHz and 130MHz

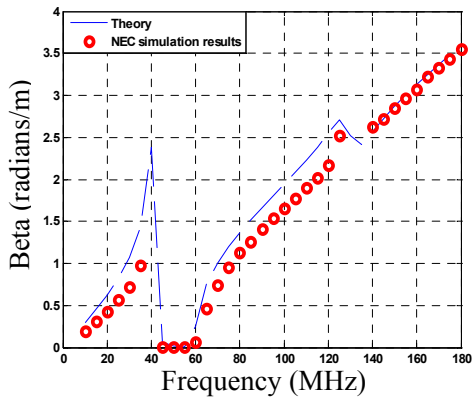


Figure 3 Extracted propagation constant at different frequencies

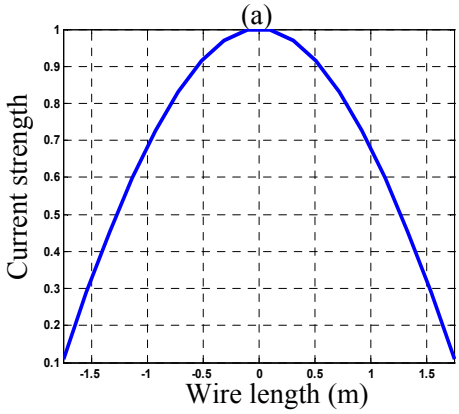
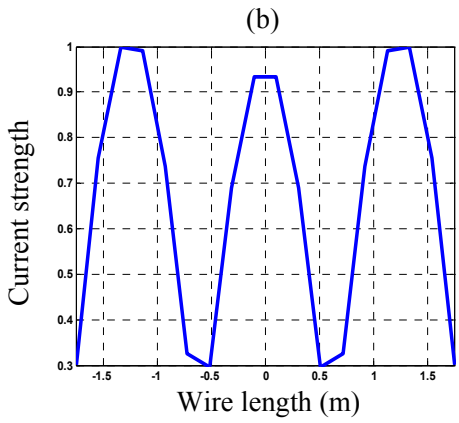


Figure 4(a) Current on the cut-wire at 40MHz



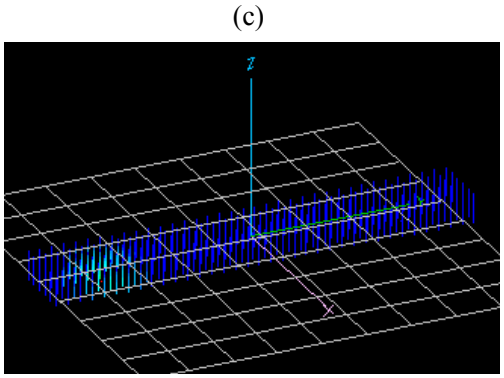


Figure 4 (b) Current on the cut-wire at 130MHz.
(c) Induced current on the array elements at 45MHz.