Off-Axis Beam Efficiencies of Parabolic Reflectors Fed by Focal Plane Arrays Comprising Hard Rectangular Waveguides

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Abstract

The efficiencies of off-axis beams of paraboloidal reflector antennas fed by focal plane arrays (FPA) are investigated. The FPA considered here comprises hard rectangular waveguides. The focal plane field, which the FPA samples, is synthesized by physical-optics (PO) integration of the electric currents over the reflector surface caused by the off-axis incident plane wave arriving at that incidence angle of interest. Full mutual coupling analysis has been performed in the FPA sampling, thereby taking into account mutual coupling losses in the arrays. The fields over the tilted elliptical aperture of off-axis beams needed for calculation of the aperture efficiency are obtained by projecting the focal plane fields to this tilted aperture using geometrical optics (GO). Results show that the total efficiency of the FPA-fed reflector decreases with increasing beam angle, and increases with larger number of FPA elements. It is also found that the maximum directive gain of the reflector radiation patterns falls with beam angle.

1. Introduction

There have been recent studies on the use of dense focal plane arrays (FPA) comprising electrically small elements as the feeds for parabolic reflector antennas [1], [2]. One of the main features of such FPA feeds is their ability to adapt to the deformed and skewed focal plane field rings of off-axis beams (as compared with those of axial ones), and thus correct for errors which would have otherwise been suffered by conventional cluster horn feeds. In addition to that, closely separated overlapping beams may be achieved, thus widening the field of view (FOV) of telescopes when used for radio astronomy, or increasing the size of the footprint on the target area when used in satellite communications.

Nevertheless, the foregoing strengths offered by FPA feeds are thus far only imagery and conceptual, but have not yet been rigorously investigated for their actual viability. At least three major factors are of concern when dealing with a more thorough study: a) the coupling efficiency of the FPA feed when it samples the focal plane fields corresponding to off-axis or overlapping beams, b) the aperture efficiency of those beams, comprising the usual well-known reflector subefficiencies (spillover, illumination, polarization, phase, and blockage), and c) the far-field radiation pattern of the reflector. It is thereby the objective of this work to look into these aspects.

In the groundwork of [1] and [2], only axial beams were considered, and the FPA-sampled focal plane fields associated with on-axis plane wave incidence were approximated by the theoretical Airy pattern in closed form. However, there is no likewise simple analytical function for the focal plane field distribution corresponding to off-axis incidence. Rigorous analysis would be required instead. In this work, the focal plane fields which the FPA samples are synthesized by integration of the physical optics (PO) electric currents flowing over the metal surface of the parabolic reflector, arising from an incident plane waves arriving at any incidence angle measured from the reflector axis. The elements of the FPA are then excited according to this field distribution, in a discretized and truncated manner, in the same way as had been done in [1]. Also as in [1], full mutual coupling analysis is performed in the FPA sampling, thereby taking into account all mutual coupling losses in the array. The same type of array element engaged in [1] shall be reused here: the hard rectangular waveguide (HRW) element [3].

The well-known approach of obtaining the aperture efficiency of a parabolic reflector radiating an axial beam is by using geometrical optics (GO) to project the fields radiated by the feed and observed at the reflector surface up to the focal plane aperture, thereby acquiring the focal plane fields expressed in terms of the feed radiation pattern, which can then be used to calculate the various subefficiencies [5]. The present state of the literature on this technique so far, only considers axial beams, thus corresponding to the focal plane aperture on which the feed-radiated fields are to be GO-projected. However, when off-axis beams are concerned, the aperture onto which the fields of the feed are to be projected is no longer the circular focal plane, but rather, a tilted elliptical aperture plane. This is due to the projection of the circular focal plane aperture onto a plane that is perpendicular to the direction of the off-axis beam, thus resulting in a tilted and reduced effective aperture, as required and well known. Therefore, the formerly accustomed GO-projected feed fields on the focal plane must be further multiplied by an appropriate phase shift term, to account for the additional ray path traveled from the focal plane to the new elliptical aperture. Numerical results generated by computer codes entirely developed by us shall be presented.

2. Description of the Theoretical Formulation

In order to treat an off-axis beam to be radiated by an FPA-fed parabolic reflector on transmit mode, the focal plane fields arising from an incident plane wave arriving from that beam direction on receive mode have to be first synthesized. Using physical optics (PO), the electric currents J flowing over the metallic reflector surface are related to the impingent magnetic fields H of that plane wave by the well-known $J = 2n \times H$, where n is the unit normal vector of the parabolic surface. This current may then be used in the well-known radiation field integral to obtain the fields radiated by the PO currents onto the focal plane. The latter integration spans over the entire paraboloidal surface. Thus, the field at any location is composed of radiation contributions from PO electric current sources over the whole reflector surface.

For axial beams, the aperture efficiency comprising the illumination, phase, and polarization subefficiencies may be determined directly from the focal plane fields, which are expressible in terms of the feed radiation pattern [4]. The latter, for the present case, is obtained upon solving the FPA via mutual coupling analysis. However, for off-axis beams, the relevant aperture for calculating the aperture efficiency is no longer the focal plane, but rather, a tilted elliptical plane that is perpendicular to the beam direction. Hence by geometrical optics (GO), the field at every point on the focal plane has to be multiplied by a correction phase term dictated by the distance from that point on the focal plane to its projected point on the tilted aperture. This distance is in terms of the reflector diameter, the incidence angle, and of course the point of interest on the focal plane. The radial coordinate ρ_{ell} of the ellipse is related to that of the circular focal plane via the incidence angle, and both the circular focal plane and the tilted elliptical plane share the same azimuthal coordinate φ. As such, the elemental area of the ellipse $\rho_{ell}d\rho_{ell}d\varphi$ may be expressed in terms of the radial and azimuthal coordinates (ρ, φ) of the focal plane, and thus, of the feed angular coordinates (θ, φ). Subsequently, the field integration over the elliptical aperture with respect to the elliptical coordinates involved in the subefficiency formulas [4] may be transformed into one that is with respect to the feed angular coordinates. Hence, the subefficiencies of the reflector emitting an off-axis beam may be computed by just knowing the feed radiation pattern.

3. Numerical Results and Discussion

For a parabolic reflector with 5-meter diameter and 60 deg half-subtended angle, and illuminated by a y-polarized incident plane wave at 10 GHz, the amplitude of the co and cross polar components (y and x components) of the focal plane E-field synthesized by PO integration for various incidence angles (0.5 deg, 1deg, 1.5 deg, and 2 deg) are shown in Fig. 1. As can be seen, as the incidence angle increases, the co-polar field pattern (comprising a central main lobe surrounded by concentric rings) gets more skewed from the focal point of the paraboloid (being the center point of the plots), as expected. As also observed, the cross-polar field levels are null at the centers of the field patterns (the foci of the incident plane waves), but are maximum along the diagonal 45deg azimuthal lines with respect to those foci (which are different from the center of the plots, being the focal point of the paraboloid).

When this parabolic reflector is fed by a HRW FPA on transmit mode, the grid points of the mesh field plots of Fig. 1 then represent the discrete sampling points of the FPA comprising 31x31 HRW elements each of size 10mm x 10mm, which are excited according to these focal field patterns. The resultant variations of the various subefficiencies with beam angles are shown in Fig. 2, for 10 GHz frequency (note now that "beam angle" instead of "incidence angle" is used, for transmit mode). Evidently, as the beam angle increases, the aperture efficiency falls. This is due to the degradation of the phase, spillover, and illumination subefficiencies with increased obliquity of the beam. However, the coupling efficiency of the FPA does not seem to be affected by the changes in the beam angle, or correspondingly, variations in the array excitation function. This is in concurrence with earlier findings [6], which asserted that the coupling efficiency is only affected by the electrical element separation, but is independent of the array excitation function. It is thus found that as the beam angle increases, the total efficiency of the FPA-fed paraboloid decreases.

The far field radiation patterns of the parabolic reflector at 10 GHz for various beam angles are presented in Fig. 3. They are calculated by Fourier integration of the focal plane aperture fields, which are related to the computed feed patterns. As can be seen, the main lobes of the patterns are achieved perfectly at their respective beam angles. The maximum gains towards the various beam directions are also seen to be fairly equal at around 53 dBi and the first sidelobe levels are about 38 dBi. This comparable performance is chiefly due to the fairly small beam angles considered and the large number of elements used to compose the FPA (31x31). This aforementioned axial gain level is close to the maximum directivity of 54.4 dB of the reflector aperture with 5-meter diameter, being around 167λ at the 10 GHz frequency. Note that since the beam angles are small, the elliptical aperture is still very close to a circular one. It is also observed that the –3dB beamwidth is about 0.36 deg for all the investigated beam angles. This value is very close to the theoretically predicted one of λ/D = 0.344 deg for large D and small beam angles, where D is the dish diameter.

Much fewer FPA elements (than the former 31x31) are now studied. The graphs of the subefficiencies versus the subarray population for 1 deg and 2 deg beam angles are shown in Fig. 4. The population ranges from 3x3 to 17x17. Here, the element size is now 15mm x 15mm, being half the wavelength at the investigated 10 GHz frequency, which has been found to be optimal [6]. Clearly, for both beam angles, the total efficiency rises as the number of elements increases, due to the increase in the aperture efficiency. The coupling efficiency however is not seen to be significantly affected by the subarray population, so long as it is higher than 5x5. Hence this suggests that the coupling efficiency is negligibly affected by both the array excitation function as well as the extent of this function. Reiterating, it is primarily the electrical element separation, which influences the coupling efficiency [6]. Another observation from Fig. 4 is that, for any one subarray population, as the beam angle increases, the total efficiency falls. This is as expected. It is also evidential that the total efficiency starts to increase rapidly from low subarray population, but then tapers off and saturates at around –1.1dB from about 11x11 or 13x13 onwards, interestingly applying to both beam angles. Hence the improvement in performance diminishes as the subarray population increases. The benefit is thus the greatest when more elements are initially added to a lowly-populated FPA. But as more elements are added, the benefit dwindles. Therefore, a subarray population of around 11x11 or 13x13 provides the best tradeoff between performance and the number of elements (which is proportional to cost and complexity). This consistent optimal-tradeoff subarray population is highly advantageous, since the same number of FPA elements can be reapplied to different beam angles.

4. Conclusion

This work has formulated the theoretical approach for treating off-axis beams of parabolic reflectors fed by FPA feeds. Physical optics (PO) integration of the electric currents over the reflector surface is used to synthesize the focal plane fields arising from any off-axis incident plane wave. For beams radiated along such off-axis directions on transmit mode, the feed-radiated fields reradiated by the reflector surface are integrated over a tilted elliptical aperture for calculations of the subefficiencies. These fields are obtained by projecting the usual focal plane fields onto the tilted aperture using geometrical optics (GO). Results have shown that the total efficiency of off-axis beams radiated by FPA-fed parabolic reflectors falls with increasing beam angle, but rises with increasing number of FPA elements. The benefit of adding FPA elements diminishes though, and a good tradeoff subarray population appears to be around 11x11 or 13x13.

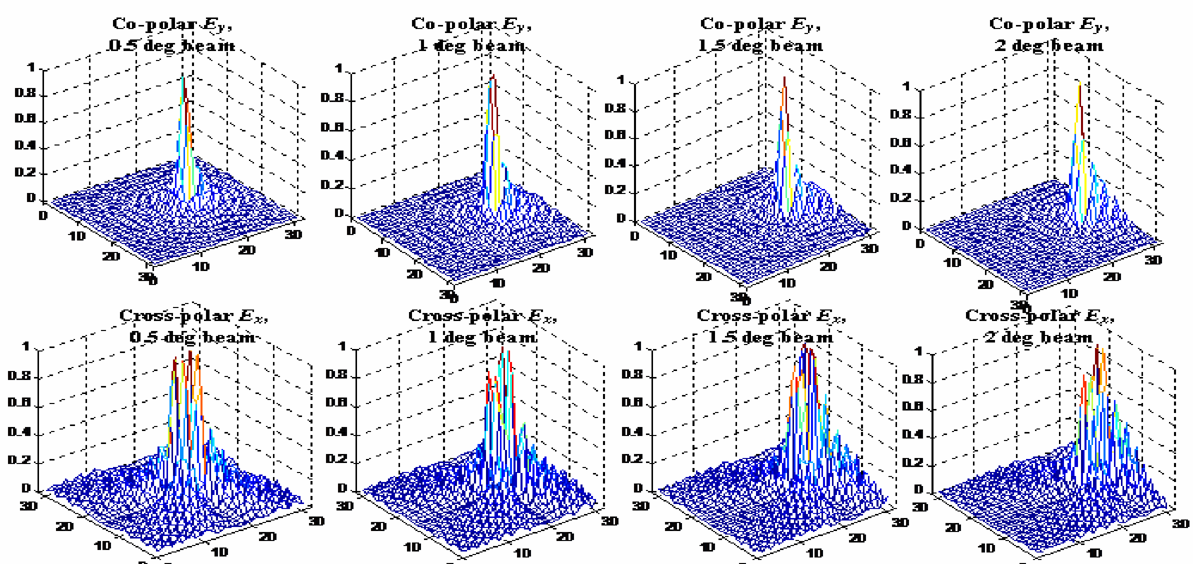


Fig. 1: Amplitude of co and cross polar E-fields on focal plane synthesized by PO integration, for 60deg paraboloid illuminated by y-polarized incident plane waves of various incidence angles at 10 GHz. On transmit mode, the density of the feeding FPA is 31x31, with each element size being 10mm x 10mm.

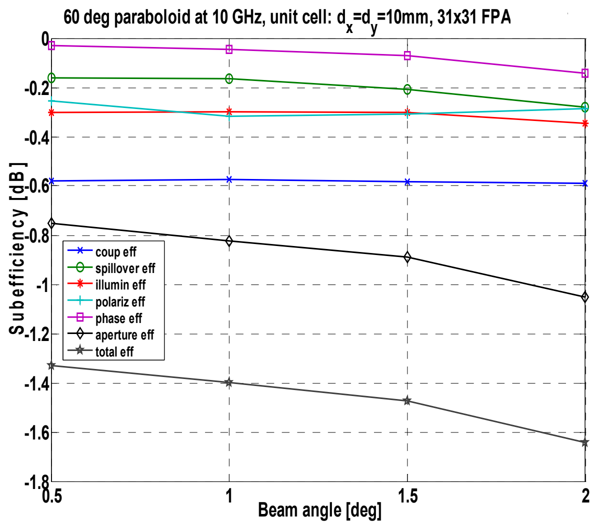


Fig. 2: Variation of subefficiencies with beam angle, for 60deg parabolic reflector at 10 GHz for subarray population of 31x31, and element size 10mm x 10mm.

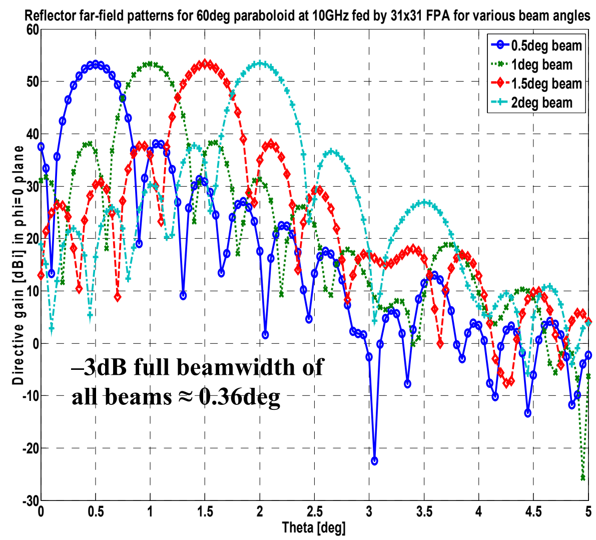
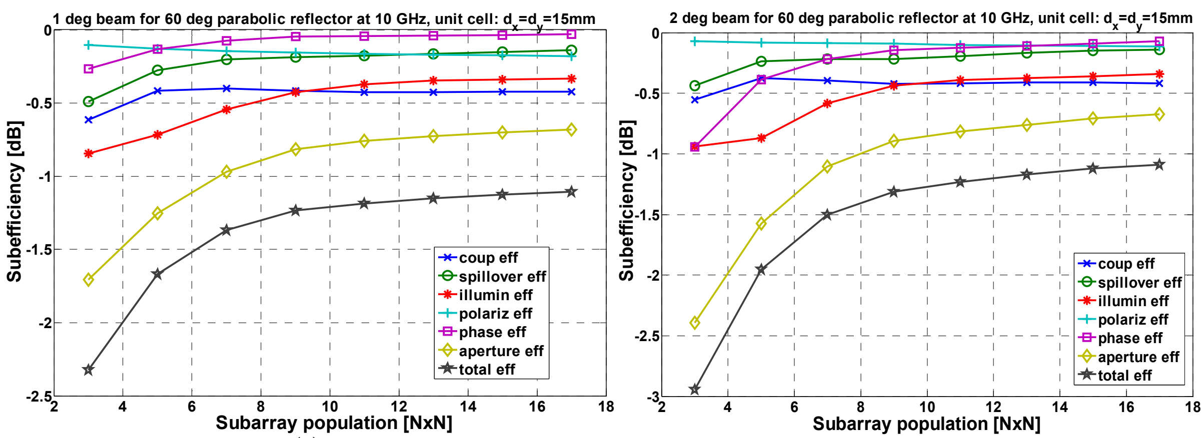


Fig. 3: Far field radiation pattern of 60 deg parabolic reflector at 10 GHz, fed by 31x31 FPA for various beam angles



(a) (b)

Fig. 4: Variation of subefficiencies with subarray population for (a) 1deg beam, and (b) 2 deg beam of 60-deg parabolic reflector at 10 GHz, with unit cell $d_x = d_y = 15$mm.

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