FFT & Equivalently Tapered Aperiodic Arrays

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Abstract

We present a novel approach to the synthesis of equivalently tapered, aperiodic linear arrays, i.e. uniformly excited arrays matching the requirements on the power pattern by acting only on the element positions. The algorithm is computationally efficient as the array factor is computed by routines whose computational burden is comparable to that of standard FFT routines. The approach adopts also an effective representation of the positions to limit the design parameters and to accommodate requirements concerning superdirectivity, mutual coupling and array size.

The computational effectiveness is discussed and the results concerning equivalent binomial and exponential tapers are presented.

1. Introduction

Non-uniform arrays exhibit several interesting features if compared to uniform ones [1]. Indeed, they allow a more efficient power handling, if uniformly excited in amplitude [1], and permit improving the bandwidth performance [2], while also reducing the overall number of elements and mitigating the effects of the grating lobes [3].

The design of aperiodic arrays is a hard task since many design parameters are involved and complexly interrelated and the efficient use of fast algorithms to calculate the array factor (Fast Fourier Transform - FFT) is complicated by an irregular lattice.

The lack of the FFT significantly burdens the computation of the far-field pattern, especially when 2D or 3D systems are considered and/or arrays with a large number of elements are dealt with. Unfortunately, the available approaches do not attempt to face this crucial problem. Nonetheless, the mathematical complexity of the problem reveals itself into multimodal functionals which should be optimized using global techniques for a successful search [4,5]. As a result, the computational burden of the developed synthesis algorithms becomes unaffordable as long as the number of unknown parameters grows.

Finally, satisfying the design specifications can ask, from the one hand side, for interelement spacings significantly lower than $\lambda/2$ with detrimental effects on superdirectivity and mutual coupling, while, on the other hand side, can lead to antennas with unacceptable overall dimensions. Accordingly, constrained optimization becomes mandatory, which further burdens the computation [6].

The aim of this paper is to present a novel, computationally inexpensive, and effective approach to the synthesis of equivalently tapered, aperiodic linear arrays. The algorithm explicitly and efficiently embeds:

- an evaluation of the array factor with computational burden comparable to that of the standard FFT [7,8];
- an effective and flexible representation of the element positions limiting the number of the design parameters;
- the requirements on the minimum allowable interelement spacings, as dictated by superdirectivity and mutual coupling;
- requirements related to the maximum allowable overall dimensions.

The paper is organized as follows. In Section 2, the problem is formulated and the requirements are inserted in the mathematical framework. In Section 3, the approach is presented and discussed. Section 4 contains the results achieved in two cases of practical interest concerning the synthesis of a binomial and of an exponential equivalently tapered pattern. Conclusions are finally drawn in Section 5 along with the foreseen future developments.

2. Statement of the problem

Let us consider a linear, non-uniform array made of N, arbitrarily spaced elements, located on the x axis, having positions $\underline{x} = (x_0,\ldots,x_{N-1})$, with $x_0=0$ (see Fig. 1). The interelement spacings are denoted with $\underline{d} = (d_1,\ldots,d_{N-1})$, with $d_i = x_i - x_{i-1}$. Both $\underline{x}$ and $\underline{d}$ are normalized to $\lambda/2$. The complex excitation coefficients are denoted with $\underline{a} = (a_0,\ldots,a_{N-1})$ and are assumed fixed and uniform both in amplitude and phase, while the array is supposed to radiate a broadside beam described by the only use of the array factor, say F.

The problem amounts at finding the array positions $x_1,\ldots,x_{N-1}$ matching the design specifications.

Formally speaking, by restricting our interest to the half-plane $y = 0$ and $z \geq 0$, we introduce the operator:

$$F(u) = \sum_{i=0}^{N-1} a_i \exp(jux_i), \tag{1}$$

with $u = \pi \sin\theta$ (see Fig. 1), and two non-negative mask functions $M_l(u)$ and $M_u(u)$ representing the design specifications on the pattern as lower and upper bounds of the desired $|F(u)|^2$, respectively [8].

Requirements are also needed to avoid too small spacings and to limit the overall dimension of the antenna. Indeed, both small and large spacings are expected to be exploited by the synthesis algorithm to outperform uniform arrays. Two constraints must be then introduced to correctly formulate the problem, i.e. a tight one concerning the minimum allowable spacing $d_{\min}$ and a loose one related to the maximum allowable spacing $d_{\max}$.

3. The proposed FFT-based approach

Let us first address the issue of the effective computation of the array factor.

The discrete values F_k of F at $u = k\Delta u$, with $k = -M,\ldots,M$ and $\Delta u = 2\pi/(2M+1)$, cannot be calculated by a standard FFT routine (exhibiting an asymptotic $N \log N$ behavior) due to the expression of the kernel matrix $\underline{\underline{B}} = \{\exp(jk\Delta u x_i)\}$, such that

$$F_k = \sum_{i=0}^{N-1} B_{ki} a_i \tag{2}$$

Actually, the product $\underline{\underline{B}}.\underline{a}$ can be computed by exploiting fast algorithms for matrix multiplication, outperforming trivial summation procedures. Nevertheless, the efficient calculation of $\underline{\underline{B}}.\underline{a}$ can be arranged in procedures whose computational complexity outperforms "smart" matrix products since the latter becomes comparable to that of standard FFT routines. The efficient evaluation of the array factor is deemed as a key tool to make the numerical handling of large arrays possible and opens the way to the use of sophisticated algorithms [4,5] to search for the solution of the problem. In this paper, the array factor is efficiently computed according to FFT-based routines according to those in [7,8].

The aim of the algorithm is to provide N-1 parameters defining the array positions $\underline{x}$. To make the representation flexible and to efficiently enforce the above mentioned constraints, a modal representation is introduced. Such a choice allows also the tuning of the number of unknowns, which is crucial point to handle the computational complexity. The representation relies on the use of a smooth function $f(\xi)$, continuously defined on the interval $\xi \in (0,(N\text{-}1)d_{\min})$, so that $x_i = f(\xi_i)$, with $\xi_i = i\, d_{\min}$. The function f is expressed in terms of an auxiliary function $f_0(\xi)$ so that enforcing the minimum prescribed and allowable interelement spacing to $d_{\min}$ can be obtained as follows (f'_0 being the derivative of f_0):

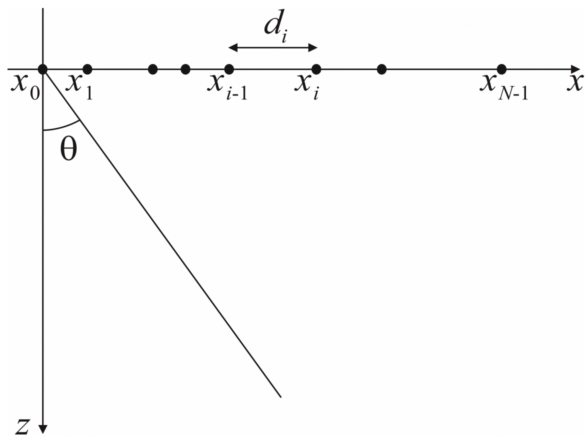

Fig. 1: Geometry of the aperiodic array.

$$f(\xi) = f_0(\xi) + (1 - \min_{\xi}(f_0'))\xi \cdot \tag{3}$$

In turn, the auxiliary function $f_0(\xi)$ is expressed by a modal expansion in terms of Chebyshev polynomials T_l

$$f_0(\xi) = \sum_{l=0}^{L} c_l T_l(\xi), \tag{4}$$

so that the unknown to be searched for by the algorithm turns to be the vector $\underline{c} = (c_1,\ldots,c_L)$. The convenience of the Chebyshev polynomials amounts to their nearly optimal for the interpolation of the samples of f_0 [9] and to the capability of providing satisfactory interpolation accuracies even when the degree L of f_0 is significantly smaller than the number of function samples.

Finally, in order to enforce also requirements about the maximum allowable interelement spacing, eq. (3) is turned to

$$f(\xi) = af_0(\xi) + b\xi, \tag{5}$$

with a = (α-1)/(A-B), b = (A-αB)/(A-B), $A = \min_\xi \{f'_0\}$, $B = \min_\xi \{f'_0\}$ and $\alpha = d_{\max} / d_{\min}$ whenever $f'(\xi)$ becomes larger than $d_{\max} / d_{\min}$. Eq. (5) ensures that f' be always comprised between 1 and $d_{\max} / d_{\min}$. We note that, by these constraints, the array size cannot be larger than $d_{\min}+(N\text{-}2)d_{\max}$.

Regarding the synthesis procedure, it is performed by an iterative, gradient-based procedure aiming at optimizing the functional

$$\Phi(\underline{c}) = \left\| |F|^2 - \mathcal{P}(|F|^2) \right\|^2 \tag{6}$$

where $\mathcal{P}(|F|^2)$ denotes the projection of the array factor $|F|^2$ over the masks [9].

The starting point of the optimization procedure is obtained by exploiting an aperture approach to the power pattern synthesis.

4. Results

Let us begin by pointing out the computational efficiency of the technique. In Fig. 2, the behavior of the computational burden concerning the array factor evaluation is illustrated (in a logarithmic scale), in terms of elementary calculations and against the number of array elements, for two cases: The computation of eq. (2) by an efficient matrix multiplication (dotted line) and by the FFT-based routine employed by the algorithm. As it can be seen, for the same number of array elements N, the burden introduced by the FFT-based routine keeps lower than that of the matrix multiplication and comparable to the $N \log N$ behavior of a standard, uniform FFT routine, which, for the sake of comparison, is also reported. The efficiency issue is even more relevant if we note that the calculation of the array factor along with that of the gradient of Φ requires (L+1) calls of the FFT-based routine.

The first case concerns the equivalent taper of a uniform, binomial array of 25, $\lambda/2$ spaced elements by a non-uniform one synthesized through the proposed algorithm. A number of 15 elements has been considered for the non-uniform case. The lower and upper masks are defined according to the desired array factor of the uniform, binomial array, with the upper mask set to $+\infty$ when the desired array factor becomes larger than -25dB, and set constant to -25dB when the latter becomes lower. The minimum and maximum enforced spacings have been $d_{\min}=\lambda/5$ and $d_{\max}=2\lambda$ and the number of Chebyshev polynomials employed for the representation of the array element positions has been L=6. The minimum and maximum spacings obtained following the synthesis have been $d_{\min}=\lambda/5$ and $d_{\max}=0.54\lambda$ for an overall array dimension of approximately 4λ. The element positions are represented under Fig. 3.a and the synthesized pattern in the visible region is displayed in Fig. 4.a along with the enforced masks. As it can be seen, the algorithm attempts to narrow the array in order to exploit as much as possible the minimum allowable interelement spacing. Accordingly, it is not surprising to enforce a tight constraint on the minimum allowable distance and a loose constraint on the maximum allowable array dimension. As shown in Fig. 4.a, the synthesis algorithm has been able to achieve a good pattern, with a sidelobe level of about 25dB.

The second case concerns the equivalent taper of a uniform, exponentially tapered array of 25, $\lambda/2$ spaced elements by a non-uniform one. A number of 25 elements has been considered for the non-uniform case. The minimum and maximum enforced spacings have been $d_{\min}=\lambda/3$ and $d_{\max}=3\lambda$ and the number of Chebyshev polynomials employed for the representation of the array element positions has been L=10. The minimum and maximum spacings obtained following the synthesis have been $d_{\min}=\lambda/3$ and $d_{\max}=1.95\lambda$ for an overall array dimension of approximately 13.4λ. The element positions are represented under Fig. 3.b and the synthesized pattern in the visible region is displayed in Fig. 4.b along with the enforced masks and the starting point. As shown in Fig. 4.b, the synthesis algorithm has been able to achieve a good pattern, with a sidelobe level of about 25dB.

5. Conclusions and future developments

In this paper, we have presented a novel approach to the synthesis of equivalently tapered, aperiodic linear arrays. The algorithm is computationally efficient as it evaluates, at each iteration step, the array factor with a “smart” FFT-based routine with a computational burden comparable to that of the standard FFTs. In addition, it adopts an effective representation of the element positions to limit the design parameters and explicitly accommodates requirements concerning superdirectivity, mutual coupling and overall array dimensions. The computational effectiveness of the procedure has been pointed out and results concerning non-uniform arrays with binomial and exponential equivalent taper have been presented for a first assessment of the performance of the algorithm.

Future developments of the presented technique will regard improvements of the performance by a phase only synthesis of the excitation coefficients [9], the extension to planar and conformal arrays [11], and applications to reflectarray antennas [12].

6. References

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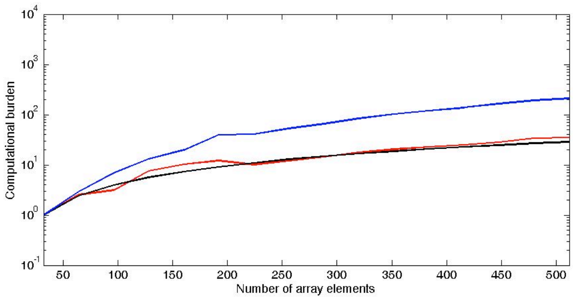

Fig. 2: Computational burden concerning the calculation of eq. (2). Red line: FFT-based routine. Blue line: matrix multiplication. Black line: $N \log N$ behavior.

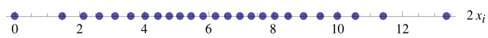

Fig. 3.a: Element positions for the equivalent binomial taper.

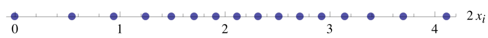

Fig. 3.b: Element positions for the equivalent exponential taper.

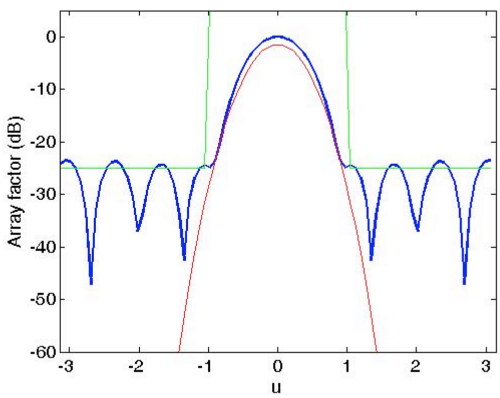

Fig. 4.a: *Equivalent binomial taper*. Blue: synthesized pattern. Green: upper mask. Red: lower mask.

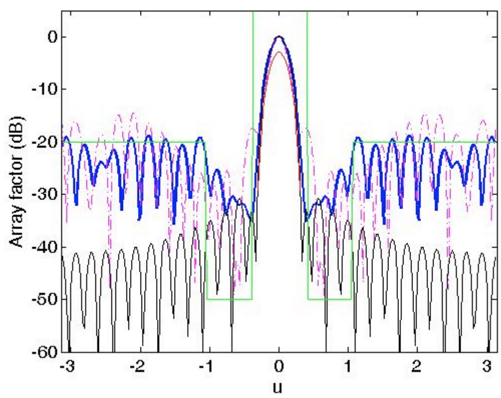

Fig. 4.b: *Equivalent exponential taper*. Blue: synthesized pattern. Black: uniform array factor. Magenta dash-dot: starting point. Green: upper mask. Red: lower mask.