A Gaussian beam re-expansion scheme for fast physical simulations in large environments.

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Abstract

This paper illustrates the ability of Gaussian beam shooting techniques to accommodate large propagation problems in multipath contexts. Frame based decomposition of fields provides a rigorous and flexible tool to perform beam launching. Paraxial beam tracking leads then to efficient algorithms, relying on the simultaneous spatial-spectral localization of paraxial Gaussian beams.

The spatial divergence of beams however degrades the accuracy of beam field tracking formulas, especially when beams interact with localized obstacles after long range propagation. A beam re-expansion algorithm is proposed to address the problems of beam widening. It makes use of both spatially narrow/wide window frames. This algorithm allows for fast re-expansion of any paraxial beam onto a frame of wide windows radiating in the form of paraxial beams. The formulation is presented here for the 2D case, but will be presented in the context of 3D problems at the conference.

1. Introduction

Frame based beam shooting has been proposed as an alternative to ray based methods, better suited for propagation in dense environments involving many localized reflecting obstacles. Gaussian beam shooting (GBS) takes advantage of the localization of paraxial Gaussian beams (GB) in phase space, i.e. in space and spectrum (directions of propagation). Thanks to this localisation, paraxial transformation operators are used, which only have to be calculated along beam axes [1].

In section **2** the basics of frame based GBS are outlined. Section **3** then proposes a new algorithm to accommodate GB divergence. In this paper, we present a 2D formulation. The 3D formulation will be presented at the conference.

2. Basics of frame based GBS

Source fields in a frame based GBS algorithm are described as a summation of GB, radiated by a set of Gaussian windows chosen so as to form a "frame" [2].

To describe a linear source distribution, frame windows $\{\psi_\mu\}$ are defined on $\mathbb{R}$, translated along a spatial coordinate denoted by x in the following, and along its spectral counterpart denoted by $k_x = k\xi$ (k is the wavenumber):

$$\psi_\mu(x) = \psi(x - m\bar{x})e^{ink\bar{\xi}(x-m\bar{x})},\ \mu = (m,n) \in \mathbb{Z}^2 \tag{1}$$

$\bar{x}$ and $k\bar{\xi}$ are respectively the spatial and spectral translation steps. The Gaussian window is defined as:

$$\psi(x) = \sqrt{\frac{\sqrt{2}}{L}} e^{-\pi \frac{x^2}{L^2}} = \left(\frac{k}{\pi b}\right)^{1/4} e^{-kx^2/2b} \tag{2}$$

L is the Gaussian window width, b its collimation distance. The condition for $\{\psi_\mu\}$ to be a frame of Gaussian windows is : $\bar{x}k\bar{\xi} = 2\pi\nu$, with $\nu < 1$ (ν is called the oversampling factor). The frame is the "snuggest" one, for a given value of ν, if $\bar{x} = \sqrt{\nu}L$ and hence $\bar{k}_x = k\bar{\xi} = \sqrt{\nu}(2\pi/L)$. The grid of beam origins and beam axis directions which are launched from the source distribution is defined by $\bar{x}$ and $\bar{\xi}$, thus entirely determined by the choice of L and ν.

Source beams should propagate along the distances required for successive reflexions within a given environment, and keep yet reasonable spatial localization. Their beamwidth L must be chosen large enough to insure good collimation. It was observed, in the case of indoor channel simulations, that only paths up to 2000λ long were correctly enough accounted for, with a source beamwidth $L = 6\lambda$ and obstacles of typical linear dimensions 200λ (1m at 60GHz). As a consequence, at 3GHz, paths lengths larger than 200m would not be correctly taken into account in the presence of obstacles with linear dimensions less than 20m. This illustrates the need for "beam re-expansion".

2. Formulation of a beam re-expansion algorithm

We propose a fast algorithm which guarantees the paraxiality of emerging beams. To this end, we make an intensive use of Gabor frame decomposition, playing with the widths of frame windows in a three-step scheme:

1. Decompose the incident field on a frame of (spatially) narrow windows in the plane of decomposition.
2. If some localized transformation of fields occurs in that plane, take it into account in the narrow window frame coefficients.
3. Through a change of frame (analog to a change of basis), compute the frame coefficients for the decomposition of transformed fields on a frame of windows radiating collimated fields.

For the sake of simplicity, we present an application of this scheme in the 2D case of beam re-expansion and in the absence of any obstacle - no step 2 (cf [3] for the case of a beam impinging on an absorbing half-screen).

2.1 Incident field decomposition on a frame of narrow windows

For each tracked beam which contributes to the incident field near the discontinuity, the decomposition coefficients on a frame of narrow windows are obtained in closed form using a paraxial approximation.

Notations: The source distribution of the incident beam is taken along $(O, \hat{x})$, centered at the origin O, and the discontinuity plane, called P', is taken along $(O', \hat{x}')$. The incident Gaussian beam source distribution is given by equations (1,2), with $b = b^i$ the collimation distance; its Fourier transform in the x-plane is denoted by $\widetilde{\psi}^i(\xi)$. The spectra of frame windows in the P' plane are denoted by $\widetilde{\psi'}_{\mu'}(\xi')$. We suppose that fields are polarized along $\hat{y}$, and that they do not vary with y. The vector normals to both planes are respectively $\hat{z}$ and $\hat{z}'$, the oversampling factor of the narrow window frame is ν'. $\hat{\xi} = (\xi, \zeta)$ is the direction of a wave vector in the basis $(\hat{x}, \hat{z})$, (ξ', ζ') in the basis $(\hat{x}', \hat{z}')$.

With these notations, the spectrum of the incident field in the plane $(O', \hat{x}')$ is given by:

$$\widetilde{\psi}^{i,x'}(\xi') = \widetilde{\psi}^{i}(\xi)\frac{\zeta}{\zeta'}e^{ik\hat{\xi}\cdot\overrightarrow{OO'}} \tag{3}$$

The decomposition formula of this incident field spectrum on the narrow window frame is:

$$\widetilde{\psi}^{i,x'}(\xi') = \sum_{\mu'} A^{i}_{\mu'}\widetilde{\psi'}_{\mu'}(\xi') \tag{4}$$

where the $A^i_{\mu'}$ coefficients are obtained by projection of the function $\widetilde{\psi}^{i,x'}(\xi')$ on the dual frame functions $\widetilde{\varphi}_{\mu'}$. We shall use here the approximate dual frame functions, which are valid if ν' is lower than 0.5 : $\widetilde{\varphi}_{\mu'} \sim \nu'\widetilde{\psi'}_{\mu'}(\xi')$ (we take windows with unit L^2-norm) [4]. Consequently:

$$A^{i}_{\mu'} = \int_{-\infty}^{\infty} d\xi' \widetilde{\psi}^{i,x'}(\xi')\widetilde{\psi'}^{\times}_{\mu'}(\xi')\,\nu' \tag{5}$$

If the frame windows in plane P' are narrow enough (spectrally wide), this integral can be evaluated through a paraxial approximation around ξ'^{i}, the x'-component of the direction vector of the incident beam axis, leading to the following result:

$$\begin{aligned} A^{i}_{\mu'} &\sim c_o\sqrt{\tfrac{2\pi}{kg''(\xi'^{i})}}e^{ikg(\xi'^{i})}e^{-\frac{ik}{2}\frac{g'^2(\xi'^{i})}{g''(\xi'^{i})}}e^{i\frac{\pi}{4}} \\ \text{with } c_o &= \nu'\tfrac{2}{k}\sqrt{\pi b^{i}b'},\ f(\xi') = e^{-\frac{kb^{i}}{2}(\xi'-n'\bar{\xi}')^2} \\ g(\xi') &= \tfrac{ib^{i}}{2}(\xi'-n'\bar{\xi}')^2 + \xi'\,m'\bar{x}' + \hat{\xi}\cdot\overrightarrow{OO'} \end{aligned} \tag{6}$$

g' and g'' are the first and second derivatives of g.

2.2 Frame change

To express the fields as a summation of collimated beams radiated by wide frame windows, a change of frame is then performed in plane P'. The required matrix can be precomputed, its elements being obtained by projection of the initial frame windows on the dual functions of the final frame. Using the usual approximation for dual frame functions, these coefficients are easily expressed in closed form.

This matrix is denoted by $C^{\mu'}_{\mu}$, with μ' the narrow window indices and μ the window indices in the final frame. The vector $\underline{A}_{\mu}$ of final frame coefficients for the diffracted fields is then given by:

$$\underline{A}_{\mu} = C^{\mu'}_{\mu}\underline{A'}_{\mu'} \quad \text{with } C^{\mu'}_{\mu} = \int_{-\infty}^{\infty} dx\, \psi'_{\mu'}(x)\psi^{\times}_{\mu}(x)\,\nu \tag{7}$$

Introducing $\sigma = L/\sqrt{2\pi}$ and $\sigma' = L'/\sqrt{2\pi}$, where L and L' are respectively the width of the narrow and wide frame windows, the closed form expression of $C^{\mu'}_{\mu}$ is:

$$C^{\mu'}_{\mu} = c_1 e^{d+ig}\sqrt{\frac{\pi}{z}}e^{-\frac{t^2}{4z}} \tag{8}$$

$$\text{with } c_1 = \frac{\nu}{\sqrt{\pi\sigma'\sigma}}, \quad z = \frac{1}{2}\left(\frac{1}{\sigma'^2} + \frac{1}{\sigma^2}\right), \quad t = k\left(n'\bar{\xi}' - n\bar{\xi}\right), \quad d = -\frac{1}{4z}\left(\frac{m'\bar{x'} - m\bar{x}}{\sigma'\sigma}\right)^2$$

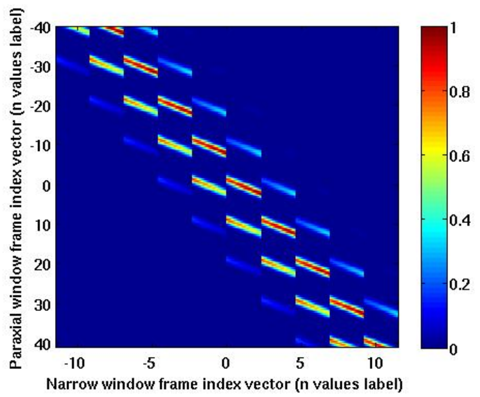

Figure 1: $C_\mu^{\mu'}$ matrix coefficients with $\nu' = 0.0625$, $L' = \lambda$, $\nu = 0.25$ and $L = 20\lambda$.

$$\text{and } \; g = -\frac{1}{2z}\left(m'\bar{x'} - m\bar{x}\right) k \left(\frac{n'\bar{\xi'}}{\sigma^2} + \frac{n\bar{\xi}}{\sigma'^2}\right)$$

The new frame parameters ν and σ can be chosen in such a way that $\bar{x}$ and $\bar{\xi}'$ are respectively integer multiples of $\bar{x}'$ and $\bar{\xi}$, which makes the $C_\mu^{\mu'}$ matrix a block periodic one, as can be seen on Fig. 1.

If $\underline{A}'_{\mu'}$ is the vector of narrow window frame coefficients of the incident beam ($\underline{A}^i_{\mu'}$ given by (6)), then equation (7) gives the vector of frame coefficients of the propagating beam on a new frame of spatially wide windows in the P' plane.

4. Conclusion

In this paper, frame based representation of fields is used as a basis for GBS algorithms. Playing with successive interchanges between wide and narrow frame windows provides an easy and flexible way to address the problems of beam widening and of localized field transformations, for instance by non planar surfaces or by realistic obstacles of finite dimensions. The numerical efficiency as well as the accuracy of the proposed algorithm will be discussed.

References

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