A High Order Space-Time Accurate Time Domain Finite Element Solver Using a New Predictor-Corrector Algorithm

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The recent literature abounds with techniques for constructing spatially high order vector basis functions for frequency domain finite element solvers that result in high spatial accuracy and well-conditioned system matrices. Unfortunately, to date, few of them have been deployed in time domain finite element methods (TD-FEMs). For the most part, modern TD-FEM solvers continue to use relatively low order Newmark and Runge-Kutta ordinary differential equation (ODE) solvers to discretize time. As a result, they require very small time steps and incur large computational expenses when aiming for high accuracy. Of course, any accuracy in the spatial discretization beyond that of the time discretization is unrealized in the final solution, and vice-versa. With advances in high order spatial approximations, there is a need for an ODE solver that delivers high accuracy without sacrificing performance.

A new method for the numerical solution of ODEs [A. Glaser and V. Rokhlin, Yale University, Technical Report A428174, 2007] approximates the solution of a first-order system of ODEs by a linear combination of exponentials. Algorithmically, the technique assumes the same form as traditional predictor-corrector methods, e.g. Adams-Bashforth-Moulton schemes, which approximate the solution by a polynomial. In contrast to existing predictor-corrector methods, the new "k-step $PE(CE)^m$" schemes do not converge in the classical sense. For linear ODEs with constant coefficients, each set of k predictor-corrector coefficients comes with a contour in the complex plane of eigenvalues of the ODE matrix (or corresponding time steps) where the scheme achieves a desired accuracy. For eigenvalues (or corresponding time steps) outside this region, the obtained precision typically decays slowly. Choosing time-steps smaller than the maximum required to ensure the eigenvalues all lie within the contour does not yield any extra accuracy. Needless to say, the performance of a k-step $PE(CE)^m$ scheme designed to deliver 16 digits (double precision) of accuracy is indistinguishable from that of a classically convergent scheme (as long as all calculations are performed in double precision arithmetic).

The new k-step $PE(CE)^m$ methods assume a problem in the form of a first-order ODE with an initial condition. To apply this method in the context of TD-FEM, the second-order vector wave equation in either the electric or magnetic field is spatially discretized using high order elements. The introduction of an auxiliary variable (namely, the time derivative of the field) allows the conversion of the second-order linear system of ODEs to one of first-order. This results in an explicit formulation, as only the mass matrix needs to be inverted at each time-step. Furthermore, the use of generalized mass lumping [A. Fisher, et al, *IEEE Trans. Antennas Propagat*, 9, 2900-2910, 2005] results in a better-conditioned mass matrix with a reduced number of nonzero entries, while introducing an error that is the same order as that of the spatial finite element approximation. The effect of the spectrum of the finite element matrices on the maximum time-step for the predictor-corrector method is similar to that of other methods. Within this setting we successfully applied a variety of k-step $PE(CE)^m$ methods to the analysis of waveguide problems, comparing wherever possible our results with analytical solutions. These comparisons reveal that the proposed method is more efficient than existing ones when high accuracy is required.