MULTIFREQUENCY DATA EXTRAPOLATION THROUGH MINIMIZATION OF TIME SIGNAL DURATION

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Abstract

Method of signal extrapolation in frequency domain for improvement of resolution in time domain is proposed. The main idea of this method is use of concept of duration of signal for formulation of non-energy constraints, when there is a priori information about finiteness of signal support in time domain. Unlike traditional extrapolation techniques no strict model of extrapolated data is required, as well as no exact information about signal support in Fourier conjugate domain must be provided. Developed approach is investigated through numerical simulations and applied for processing of multifrequency data of complex reflection coefficient of dielectric layered structure.

1. Introduction

The problem of superresolution is rather general. Accurate recovering of the scattering center locations for purposes of radar target image reconstruction [1], determination of distribution of discontinuities in transmission lines [2], probing of layered dielectric structures [3] via spectral analysis of frequency domain experimental data can be mentioned among traditional applications of superresolution methods. Inverse Fourier transform is rather fast and stable, but it is limited in resolution and dynamic range that is accompanied by widening and overlapping of informative peaks in time domain signal. This fact implies reducing the accuracy of estimation and even impossibility of estimation. Some improvement is possible through extrapolation of multifrequency data into lower and higher range of frequency. Subspace eigenanalysis based methods, such as multiple signal classification (MUSIC) or estimation of signal parameters by rotational invariance techniques (ESPRIT) [1], and methods based on general eigenvalue approach, such as matrix pencil method [2], are very popular among superresolution methods. All these methods directly or indirectly use a strict model of sum of exponentials, but sometimes this model is not completely correct, for instance for disperse loss layered dielectric structures.

In this paper we propose to use the so-called method of minimum of duration (MMD) for processing of experimental data, that shows encouraging results when a priori information about finiteness of desired solution is present. It is not based on the model of sum of exponentials and can be used under more general conditions. Here we give brief theoretical basis for MMD and proposed extrapolation technique and demonstrate our approach through numerical simulations on theoretical example, and afterwards discuss results of application of developed approach for extrapolation of experimental multifrequency data.

2. MMD and Proposed Extrapolation Technique

Method of minimum of duration was originally introduced for the problems of finite spectrum signal restoration [4]. Its main idea is use of concept "duration" as non-energy measure of signals. For the purpose of time description of signal $s(t)$ specific function χ is introduced in the following form: $\chi[s(t)]=1$ if $s(t)\neq 0$ else $\chi[s(t)]=0$. Result of application of such function gives so-called "strict duration" of considered signal, measure of its non-zero values, thus giving possibility to formalize the term "finite signal". But use of function χ is not welcome for at least two reasons: 1) in presence of noise with arbitrarily small dispersion χ will never have zero values; 2) function χ is not suitable for optimization purposes because of its discontinuity. These difficulties have been resolved by use of approximation of χ function for practical calculations in the following form [1]: $\psi[s(t)]=[|s(t)/\lambda|^2+\alpha^2]^{\beta}-\alpha^{2\beta}$; $0<\lambda<\infty$, $\beta=1/k$, $k>2$, with parameters α, β to control its behavior. Integral of function ψ over time domain gives "quasiduration" of signal, and allows overcoming mentioned above problems. Numerical simulations have shown that quasiduration value in general does not coincide with true duration of signal, but has the same dynamics. Thus, minimization of quasiduration gives the same result as minimization of true duration, but results are more robust.

We propose to use the concepts of MMD for the purpose of extrapolation of available multifrequency data in order to improve results of peak parameter estimation in time domain using data in frequency domain and inverse Fourier transformation . According to physical properties of the process, desired time signal must be finite. Thus we formulate extrapolation problem in following terms – such function $F(f)$ must be found, that: 1) its inverse Fourier transform $f(t)$ is of finite duration; 2) estimated $F(f)$ differs from measured multifrequency data $G(f)$ for less than some threshold value σ (in this way we consider possible inaccurateness of data). Appropriate minimization problem is stated as:

$$D_{\alpha,\beta}(f(t))=\int_{-T/2}^{T/2}\{[|f(t)|^2+\alpha^2]^\beta-\alpha^{2\beta}\}dt\to\min_{f(t)},\ \left\|F(f)-G(f)\right\|_{L_2}\Big|_{f_1}^{f_2}\le\sigma\ , \tag{1}$$

with frequency range of the measured data of $[f_1;f_2]$. Alternative unconstrained formulation can be introduced in the following form:

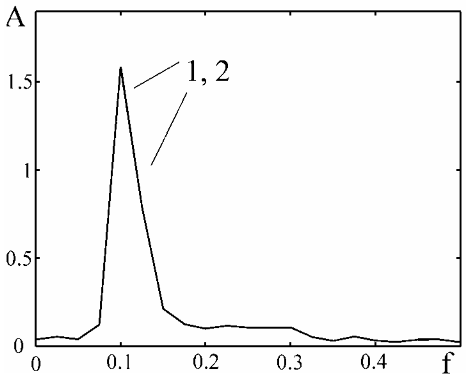
$$\int_{f_1}^{f_2}[F(f)-G(f)]^2df+\gamma D_{\alpha,\beta}\left(\int_{f_{e1}}^{f_{e2}}F(f)e^{j2\pi ft}df\right)\to\min_{F(f)}\ , \tag{2}$$

where $[f_{e1};f_{e2}]$ is the frequency range of extrapolated data.

The first term in (2) is approximation error energy term, and the second one considers “duration” of signal as regularization term. For very small values of γ we get solution without regularization, for very large values of γ solution is over regularized and tends to zero solution. Some optimal value of γ exists, giving minimum to overall functional (2) for “true” function $f(t)$.

3. Numerical Simulations

We consider superresolution problem on simple model of sum of two sinusoids for verification of proposed extrapolation approach on numerical example. Considered sinusoids have amplitudes $A_1=1.5$ and $A_2=1$ with normalized frequencies values of $f_1=0.1$ and $f_2=0.125$ in presence of Gaussian noise with SNR of +5 dB. Spectrum of 4-times extrapolated signal given by minimization of functional (2) along with initial spectrum is shown in figure 1 for following values of parameters: α=0.1, β=1/16 and γ=0.01. Estimated amplitude and frequency values after extrapolation (fig. 1, a) are: $A_1=1.52$, $A_2=0.97$, $f_1=0.1$, $f_2=0.125$, while they are not resolved in results of inverse Fourier transformation of initial data.



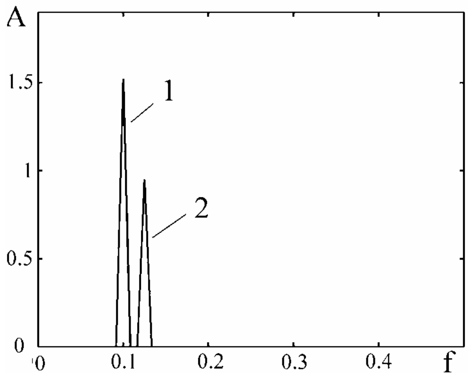
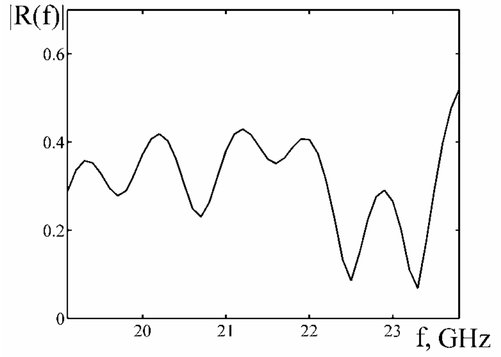


Figure 1. Spectrum of the sum of two sinusoids (1,2 – corresponding spectral components):
a – initial data spectrum; b – extrapolated data spectrum

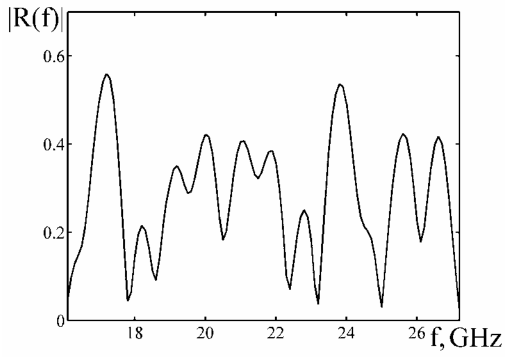
4. Multifrequency Data Processing

Proposed approach is applied for the problem of electrical thickness estimation of dielectric layered structure. We consider 3-layered dielectric structure organic glass 1 – air –organic glass 2 with the purpose of estimation of air layer thickness. Materials have certain losses and illuminated electromagnetic wave can not be considered as plain thus the model of sum of exponentials is not strictly correct. True geometric thicknesses of layers were $d_1=d_2=41$ mm, air layer was 40 mm thick. Measurements are carried out in accordance with scheme, described in detail in [3]. Results of experimental complex reflection coefficient measurement carried out in frequency range of 19 – 23.8 GHz with step of 100 MHz are shown in figure 2. Synthesized time-domain envelope lacks resolution, thus electrical thicknesses can not be estimated accurately enough or even resolved, and thus localization of interfaces is not possible. Result of extrapolation of initial complex reflection coefficient into frequency range of 16-27.3 GHz strongly improves resolution of informative peaks in time domain and they can be observed separately (fig. 3). Estimated value of air layer thickness is 42.1 mm. This result seems to be accurate enough for practical applications.



a) b)

Figure 2. Experimental data processing: a – measured frequency data; b – synthesized time domain signal (1 – reflection from the front interface, 2,3 – reflection from interfaces of air layer, 4 – reflection from last interface)



a) b)

Figure 3. Result of extrapolation: a – extrapolated frequency data; b – synthesized time domain signal (1 – reflection from the front interface, 2,3 – reflection from interfaces of air layer, 4 – reflection from last interface)

5. Conclusion

Developed approach shows good results when a priori information about finiteness of spectrum of extrapolated signal is present. Very important advantage of proposed extrapolation method is that we need no information about signal support in Fourier-conjugate domain, and no strict model of extrapolated data is required, thus allowing to take into account possible measurement errors or distortions. Considered technique was verified through numerical simulations and applied for superresolution problem of dielectric layer parameter estimation through processing of multifrequency data of complex reflection coefficient, strongly improving initial results.

6. References

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