MUSIC-type imaging of a small dielectric sphere buried in a half space from exact and asymptotic data

S.Gdoura[1]***, D. Lesselier***[1]***, P. C. Chaumet***[2]***, G. Perrusson***[1]

[1]Département de Recherche en Electromagnétisme
Laboratoire des Signaux et Systèmes (CNRS-SUPÉLEC-Univ. Paris Sud 11)
3, rue Joliot-Curie - 91192 Gif-sur-Yvette cedex - France - E-mail: name@lss.supelec.fr

[2]Institut Fresnel, UMR 6133 (CNRS-Univ. Aix-Marseille)
Campus de Saint Jérome, case 162, 13397 Marseille cedex, France - E-mail: name@fresnel.fr

Abstract

A small dielectric sphere is buried within the lower half of a two half-space medium and illuminated from the upper one by a dipole array. The Multi-Static Response Matrix is computed either by the Coupled Dipole Method or by an asymptotic formulation taking into account the coupling between the sphere and the interface. Comparisons between field data and between images of the inclusion are discussed. It is shown that whenever the inclusion is far from the interface, simplifying the Generalized Polarization Tensor to the one of an inclusion in free space is still reliable.

1. Introduction

MUSIC-type imaging of volumetric inclusions in 3D free and half space from single-frequency scattered field data is developed in a series of contributions by the authors and colleagues, refer to [1], [7]. In this paper, the work focuses on a spherical inclusion buried within a half space, illuminated and seen from the other half space. In contrast with [7], the fact that this sphere might be at shallow depth is accounted for in the asymptotic field formulation when collecting the data. This involves the rigorous calculation of a Generalized Polarization Tensor (GPT) [6] which takes into account coupling of the inclusion to the interface, via closed forms developed within a bispherical coordinate system attached to it [2], [5]. That being done and illustrated, one is able to study the effect of this coupling both on the MSR matrix and the imaging thereof, and to investigate the reliability of an imaging based on an asymptotic theory when true data are input like those provided by the Coupled Dipole Method (CDM) [3]. In addition, the use of the Leading Order Approximation (LOA) of the Green dyad [4] as a mean to considerably speed up the imaging with possibly little consequence on its accuracy is investigated.

2. Direct problem

Let us consider two homogeneous dielectric half spaces with permeability μ_0 having a horizontal planar interface at $z = 0$. The upper half space is characterized by permittivity ε_0, the lower one by permittivity ε_-. A spherical dielectric inclusion with radius a, volume V and permittivity ε_* is buried in the lower half space, centered at $\mathbf{x}_*$. The distance from the center of the inclusion to the interface is d. The inclusion is illuminated by a planar source array, symmetric with respect to the z-axis, placed in the upper half space at distance h from the interface; this array is made of $M \times M$ ideal electric dipoles orientated parallel to the vertical z-axis, and operated at frequency ω —time-dependence $\exp(-i\omega t)$ is henceforth implied. This array is also assumed to operate as a receiver array. The incident electric field radiated at point $\mathbf{r}$ by the n^{th} dipole of applied current I_n and location $\mathbf{r_n}$ is given by:

$$\mathbf{E}_{\mathbf{0}}^{(\mathbf{n})}(\mathbf{r}) = i\omega\mu_0\, \mathbf{G}^{\mathbf{ee}}(\mathbf{r}, \mathbf{r}_{\mathbf{n}})\;\; \hat{\mathbf{z}}\, I_n . \tag{1}$$

If the radius a is much smaller than the wavelength in the embedding medium (λ_-), we can apply the asymptotic formula of the scattered field [1] as follows:

$$\mathbf{E}_{\mathbf{s}}^{(\mathbf{n})}(\mathbf{r}) = \frac{\mu_0}{i\omega}\, \mathbf{G}^{\mathbf{ee}}(\mathbf{r}, \mathbf{x}_*)\;\; \mathbf{M}^{\varepsilon}\, \mathbf{E}_{\mathbf{0}}^{(\mathbf{n})}(\mathbf{x}_*) + \mathrm{O}(a^5). \tag{2}$$

Since the inclusion lies fully within the lower half space and since receiver and source arrays are coincident in the upper one, only the transmitted part of the Green dyad is needed [7]. As for $\mathbf{M}^{\boldsymbol{\varepsilon}}$, it is the so-called Generalized Polarization Tensor (GPT),

$$\mathbf{M}^{\varepsilon} = a^3 i\omega^3 (\varepsilon_* - \varepsilon_-)\mathbf{M}(\varepsilon_* / \varepsilon_-, V), \tag{3}$$

where $\mathbf{M}(\varepsilon_*/\varepsilon_-, V)$ is the polarization tensor of the inclusion in the configuration. In [7], the same configuration is studied already, but the polarization tensor of an isolated sphere is used as if the sphere was in free space. In the present paper one calculates a more realistic one, which accounts for electromagnetic coupling between the inclusion and the interface. To calculate this tensor, one specializes (G. Dassios, *private communication* [2006]) the analytical derivation of [5], carried out for two dielectric spheres in free space with the help of the bispherical coordinate system [8].

As for the Coupled Dipole Method (CDM), it is based on the discretization of the inclusion into L cubical subunits. If their size is small enough compared to the wavelength of the embedding medium, it can be assumed that the field is uniform over any given subunit. Hence, the field in it can be written as

$$\mathbf{E}^{(\mathbf{n})}(\mathbf{r_i}) = \mathbf{E_0^{(n)}}(\mathbf{r_i}) + \sum_{j=1\,,\,j\neq i}^{L} \omega^2 \mu_0\ \varepsilon_- \mathbf{G}^{\mathbf{ee}}(\mathbf{r_i},\ \mathbf{r_j})\ \alpha_j \mathbf{E}^{(\mathbf{n})}(\mathbf{r_j}), \tag{4}$$

where $\alpha_j = 3V_j(\varepsilon_j - \varepsilon_-)/(\varepsilon_j + 2\varepsilon_-)$ is the associated polarizability, and V_j is the volume of each subunit. The field in each subunit is obtained by solving the linear system above. Then, the scattered field at each position of observation $\mathbf{r}$ is computed as

$$\mathbf{E_s^{(n)}}(\mathbf{r}\ ) = \sum_{j=1}^{L} \omega^2 \mu_0\ \varepsilon_- \mathbf{G}^{\mathbf{ee}}(\mathbf{r},\ \mathbf{r_j})\ \ \alpha_j\ \mathbf{E}^{(\mathbf{n})}(\mathbf{r_j}). \tag{5}$$

3. Inverse problem: non-iterative imaging by MUSIC-algorithm

From the scattered field calculated either by CDM or asymptotic formula, we calculate the Multi-Static Response matrix (MSR). The singular value decomposition of this matrix shows three significant singular values. MUSIC-type imaging algorithm, as explained in [1], consists of considering a cubic search box and calculating at each of its points an estimator which is a projection operator of the so-called Green vector from each transmitter and/or receiver on the noise subspace. This estimator is expected to peak at the center of the inclusion $\mathbf{x}_*$. At this stage, one should notice that the computation time of the transmitted part of the Green dyad could become prohibitive since one has to calculate for all points of the search box its value from each transmitter to each receiver ($M \times M$ transmitters and $M \times M$ receivers). The Leading Order Approximation (LOA) by [4] then enables us to reduce the computation time of the estimator.

4. Numerical simulations

In the simulations, the spherical dielectric inclusion is taken of radius a = 0.05m and of permittivity $\varepsilon_* = 12\varepsilon_0$. The half space of burial is of permittivity $\varepsilon_- = 4\varepsilon_0$. The operation frequency is 300 MHz, wavelengths in the upper and lower half spaces being $\lambda_+ = 1$ and $\lambda_- = 0.5$, resp., all dimensions from now being given in meters. One considers that the inclusion center is slightly shifted with respect to the axis of symmetry of the array, at $x_* = 0.15$ and $y_* = 0.23$, its depth z_* being possibly varied and denoted as $-\,d$. Different values d = 1, 0.2, 0.1 are considered thereafter.

The diagonal elements of the polarization tensor taking in account coupling to the interface and calculated via the bispherical system and those of the one in free space are compared in figure 1. They are plotted vs. ratio d/a. It appears that, as soon as d is beyond about $3a$, the two tensors are identical, and the sphere behaves as if it was isolated in free space. When the sphere gets closer and closer to the interface (without contact), the coefficients separate more and more, yet overall differ not that much.

Now, one is setting one vertical dipole transmitter at location (-1.75, 1.75, 0.5), and computing within the plane (y = 0), for different depths d of the sphere, the difference of magnitude between the vertical (z) components of the scattered field obtained via the asymptotic formula with coupling (the polarization tensor is the true one) and without coupling (the polarization tensor is the one of an isolated sphere). This is illustrated in figure 2, taking as distance of reference the wavelength λ_+ in the upper medium. The influence of coupling appears strong whenever the field is observed close to the interface, at a distance of about $0.2\lambda_+$ and below. When the sphere is close to the interface at, as an average, λ_- or less, this effect is emphasized.

As previously, a vertical dipole transmitter is placed at location (– 1.75, 1.75, 0.5). One is interested in the E_z components of the scattered field at the position of a 8 × 8 dipole array, the array spacing being $\lambda_+/2$, its height being $h = \lambda_+/2$. In figure 3, one shows a comparison of this scattered field computed according to the asymptotic method and to CDM, for several distances d to the interface. In the case of CDM, the sphere is discretized into 4224 cubical elements. The figure shows that the agreement is quite remarkable, being emphasized that the sphere radius is small compared to the wavelength λ_- ($a = \lambda_-/10$), whereas its dielectric permittivity of 12, compared to 4 in the lower half space, means an electrical size 1.7 times larger. Let us notice that the full polarization tensor is used, and not the one of an isolated sphere —this would have however no visible effects down to about $d = 0.15$ ($= 3a$).

The above being kept in mind, let us now assume that the MSR matrix is made available for the previously introduced $M \times M$ array either by the asymptotic method or by CDM. After singular value decomposition of the matrix, the MUSIC algorithm is applied in order to locate the position of the sphere center within a search box centered on the z-axis finely spanned, 21 × 21 × 21 nodes, $\lambda_-/10$ stepped, being chosen to calculate the MUSIC estimator. Results (not shown here for lack of place) show that both CDM and asymptotic MSR matrices make the algorithm being able to detect the exact position of the sphere center. Figure 4 then illustrates, in the case $d = 1$, what happens from CDM data when the Green dyad is exact or is calculated via the LOA —so doing reduces the computation time by a factor of more than 50. Noise was added with 10dB signal to noise ratio. It is easily observed that the quality of the imaging is not the same, the LOA tending to significantly blur the image into the longitudinal direction, the exact calculation on the contrary providing in contrast excellent sub-wavelength resolution.

5. Conclusion

Asymptotic formulas of the field scattered by a small spherical inclusion buried in a half space yield fields closely resembling to those computed by the full-field CDM method, even at very shallow depths, for a small inclusion as considered herein. This involves the calculation of the polarization tensor in a bispherical coordinate system in order to model the coupling between the inclusion and the interface. The imaging itself appears rather robust, be it carried out from asymptotic or exact MSR matrices, and to a lesser degree when speeding up the computation of the Green dyad by a first-order approximation (LOA). It is shown that sub-wavelength localization can be achieved, the discrimination of two close spheres (in order to assess potential sub-wavelength resolution) being one of the items of forthcoming studies.

6. References

1. H. Ammari, E. Iakovleva, D. Lesselier, and G. Perrusson, "MUSIC-Type Electromagnetic Imaging of a Collection of Small Three-Dimensional Bounded Inclusions," *SIAM J. Scientif. Comput.*, 2007, **29**, pp. 674-709.

2. A. Charalambopoulos, G. Dassios, M. Hadjinicolaou, "An Analytic Solution for Low-Frequency Scattering by Soft Spheres," *SIAM J. Appl. Math*., 1998, **58**, pp. 370-386.

3. P. C. Chaumet, A. Sentenac, and A. Rahmani, "Coupled Dipole Method for Scatterers with Large Permittivity," *Physical Rev. E*, 2004, **70**, pp. 036606-1-6.

4. T. L. Cui and W. C. Chew, "Fast Evaluation of Sommerfeld Integrals for Electromagnetic Scattering and Radiation by Three-Dimensional Buried Objects," *IEEE Trans. Geoscie. Remote Sens*., 1999, **37**, pp. 887-900.

5. G. Dassios, M. Hadjinicolaou, G. Kamvyssas, and A. N. Kandili, "On the Polarizability Potential for Two Spheres," *Int. J. Engineering Science*, 2006, **44**, pp. 1520-1533, 2006.

6. G. Dassios and R. Kleinman, *Low Frequency Scattering*, Oxford, Clarendon, 2000.

7. E. Iakovleva, S. Gdoura, D. Lesselier, and G. Perrusson, "Multi-Static Response Matrix of a 3-D Inclusion in Half Space and MUSIC Imaging," *IEEE Trans. Antennas Propagat*., 2007, **55**, pp. 2598-2609.

8. P. Moon and D. Spencer, Eleven Co-Ordinate Systems, *Field Theory Handbook*, Berlin, Springer,1961.

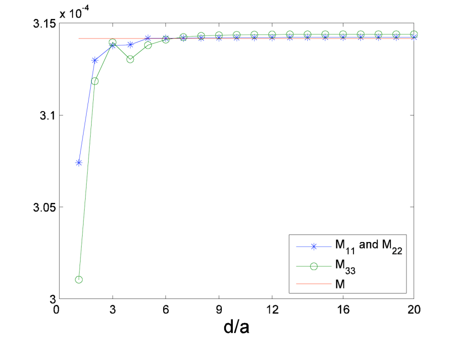

Fig. 1 The diagonal elements of the polarization tensor matrix M_{11} (= M_{22}) and M_{33} computed in the bispherical coordinate system vs. ratio d/a, are compared to the value of M for a isolated sphere in free space.

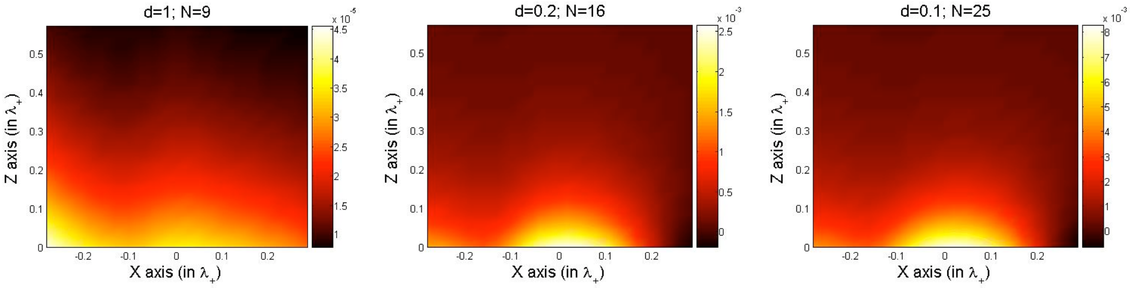

Fig. 2 The magnitude of the differences between the vertical components of the scattered field obtained via the asymptotic formula with coupling (the polarization tensor is the true one) and without (the polarization tensor is the one of an isolated sphere) within the vertical (x, $z > 0$)-plane for various d.

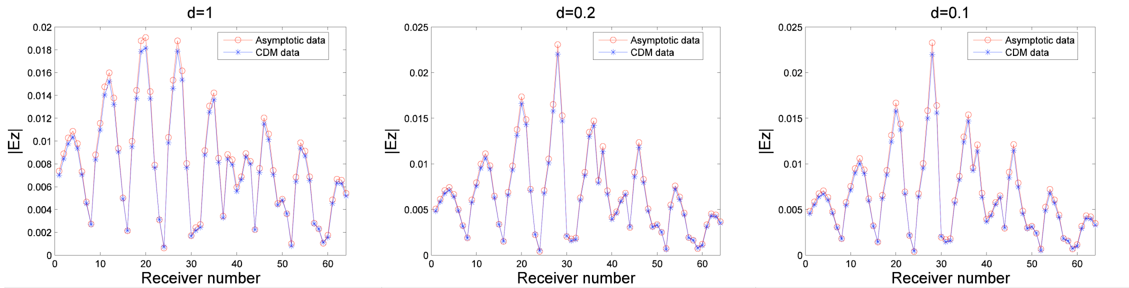

Fig. 3 Comparison of the amplitudes of the vertical E_z-component of the scattered field computed by the asymptotic method with full polarization tensor and by CDM for several d when the (– 1.75, 1.75, 0.5)-dipole is radiating. The receivers are numbered from 1 to 64, by sets of eight, one array line after another.

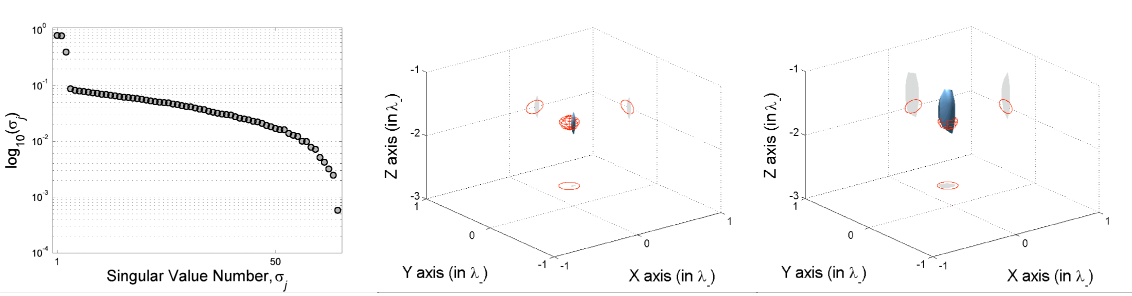

Fig. 4 MUSIC imaging (iso-surface at 80%) of the sphere buried at d = 1 using the three singular values of highest amplitude of the MSR matrix computed by the CDM, with 10 dB added noise. Left: singular spectrum; middle: image obtained via the Green dyad computed by the exact formulation; right: image obtained via the Green dyad computed by the LOA.