A General Purpose Inverse Equivalent Current Method Accelerated by the Multilevel Fast Multipole Method

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Abstract

The radiation and scattering behavior of any object in a homogeneous environment can be described by a set of equivalent sources. Different types of equivalent sources are feasible and all of them have their own benefits. Equivalent current methods are especially advantageous for irregular measurement grids of arbitrary shape and if *a priori* information about the object shall be used. Also, they can immediately provide diagnostic information about the object. In this paper, a very flexible equivalent current method is presented, which has been derived from a general purpose boundary integral equation solver. As such the method works with arbitrary triangular surface meshes and Rao-Wilton-Glisson basis functions. High efficiency is achieved since the Multilevel Fast Multipole Method has been adapted to speed-up the inverse solution process.

1. Introduction

In many situations, it is desirable to derive as much information as possible about a radiation or scattering object from near-field measurements. On the one hand, one may want to perform a near-field far-field transformation in order to obtain the far-field scattering or radiation behavior of the object. On the other hand, one may want to gain diagnostic information about the object itself. For this purpose, the object is replaced by a set of equivalent sources, which are often modal expansions in Cartesian, cylindrical or spherical coordinates. An alternative are equivalent current methods [1, 2, 3], which describe the radiation or scattering behavior of the object by equivalent Huygens' currents either over a virtual surface or on the radiating structure itself. The currents of interest are discretized and related to the near-field values by formulating an integral equation, which is solved by a Method of Moments (MoM) like procedure [4]. This approach makes the equivalent current methods attractive for irregular sampling grids or non-closed measurement surfaces as well as for near-field values measured in the close vicinity of the considered object, where modal expansions are no longer applicable. In the following sections, an equivalent current method based on a powerful Boundary Integral (BI) approach working with the well-established Rao-Wilton-Glisson (RWG) basis functions [4] on arbitrary three-dimensional surface meshes is presented. The algorithm efficiency has been optimized by adapting the Multilevel Fast Multipole Method (MLFMM) [5, 6, 7] to the inverse equivalent current technique. In contrast to this, [1] is restricted to planar equivalent current distributions, but has Fast Fourier Transform (FFT) based acceleration, [2] concentrates on rotationally symmetric current expansion surfaces, and [3] has no fast integral acceleration and works with nodal current expansions.

2. Formulation

Core of the formulation is the integral representation

$$\boldsymbol{E}(\boldsymbol{r}) = \iint_A \bar{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}') \cdot \boldsymbol{J}(\boldsymbol{r}') da', \tag{1}$$

for the electric field in an arbitrary observation point $\boldsymbol{r}$ computed from the equivalent surface current distribution $\boldsymbol{J}(\boldsymbol{r}')$ representing the object to be characterized. $\bar{\boldsymbol{G}}(\boldsymbol{r},\boldsymbol{r}')$ is the dyadic Green's function of free space and A is the surface on which the surface currents are presumed. The discretized surface currents

$$\boldsymbol{J}(\boldsymbol{r}') = \sum_n J_n \boldsymbol{\beta}_n(\boldsymbol{r}') \tag{2}$$

can be assumed on almost any virtual surface, e.g. a sphere or a box enclosing the object, a plane in front of the object, or if some *a priori* information is given, directly on the structure of the object. $\boldsymbol{\beta}_n(\boldsymbol{r}')$ are RWG basis functions [4] and the expansion coefficients J_n are determined in the inverse solution process so that the calculated currents reproduce the given electric field values. The integral expression in (1) is efficiently evaluated using an adapted form of the MLFMM, which is similar to the procedure described in [7]. Observation points close to the object are treated with numerical quadrature in the conventional way. For all other observation points, the fields due to the currents are evaluated by near-field and far-field translations on the various MLFMM levels. Once the current expansion coefficients J_n have been obtained by the inverse solution process, all post-processing functionalities of the original BI-MLFMM code can be utilized, including near-field and far-field computations.

3. Numerical Results

In the following section, numerical results of a horn antenna and of an offset-fed parabolic reflector with triple feed horn are presented.

3.1 Horn Antenna

Electric field values of a horn antenna have been calculated at 10 GHz. In the inverse process, the current distribution on a triangular mesh of the horn antenna as well as on a rectangular box enclosing the antenna have been computed using 36326 and 30537 unknown current expansion coefficients, respectively. Fig. 1 shows the current distributions on the triangular mesh of the horn antenna for the MoM reference solution and the equivalent current method. Since the horn mesh is quite complex taking into account the wall thickness of the horn, the MoM currents of the reference solution differ from the inverse equivalent currents. This was to be expected since the equivalent current distribution cannot be unique under those circumstances. The equivalent currents have been calculated under the condition to reproduce the given field distribution. Fig. 2 shows the equivalent currents on a rectangular box enclosing the horn antenna.

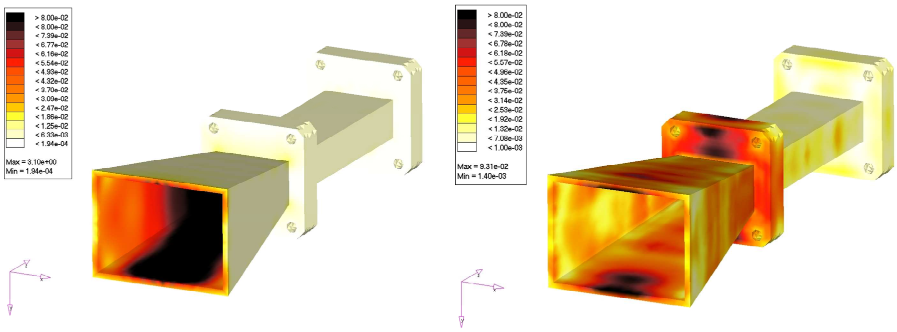

Fig. 1. Magnitude of electric surface currents on horn antenna, MoM reference (left), inverse equivalent currents (right).

The obtained far-field patterns of the three current distributions in an H-plane cut are shown in Fig. 3. The far-field computed from the inverse equivalent currents on the original horn mesh shows a very good agreement with the reference solution in both polarizations. The pattern resulting from the currents on the meshed box shows some fluctuations around the reference pattern, but shows overall also good accuracy. Especially the reproduction of the cross-polarized component appears to be quite satisfactory in view of the low absolute level with respect to the co-polarized component.

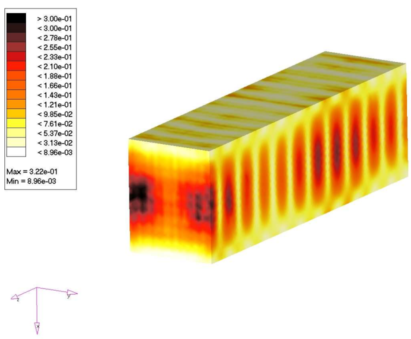

Fig. 2. Magnitude of electric current distribution on rectangular box enclosing the horn.

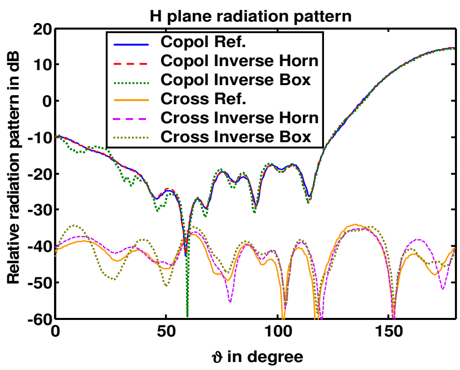

Fig. 3. Far-field pattern of horn antenna for different computation methods.

3.2 Reflector Antenna

As a second example, an offset-fed parabolic reflector with triple feed horn at a frequency of 3.95 GHz is considered. Again equivalent currents are calculated from near-field values, obtained from a MoM solution, using 247326 current expansion coefficients. Fig. 4 shows the MoM reference current distribution as well as the computed inverse equivalent current distribution on the reflector. The similarity is obvious, nevertheless the inverse currents show some jitter compared to the reference. The current distribution of the feed horn structure is shown in Fig. 5. Since the contribution of the feed horns to the totally observed near-field is only minor, the visible discrepancies can probably not be avoided, and uniqueness can again, of course, not be expected.

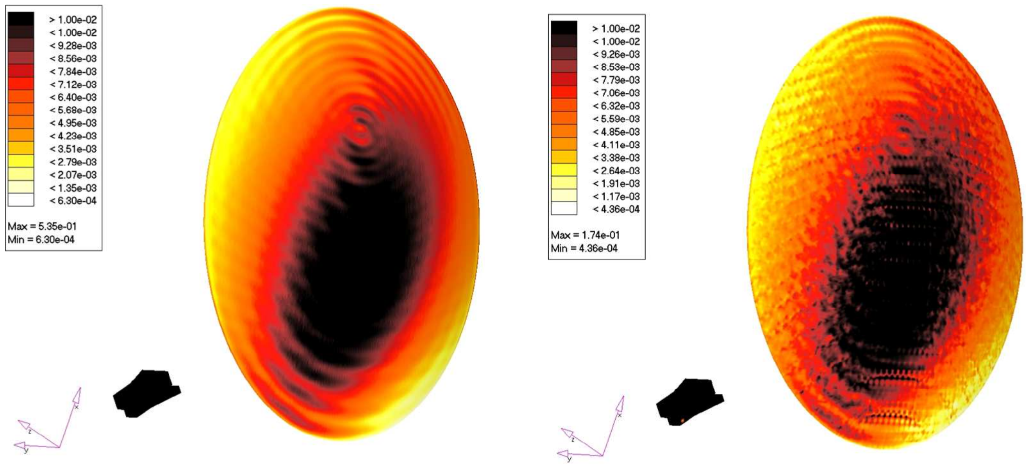

Fig. 4. Magnitude of electric current distribution on reflector, MoM reference (left), inverse currents (right).

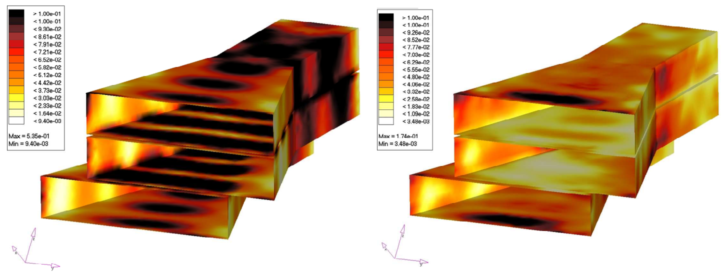

Fig. 5. Magnitude of electric current distribution on feed horns, MoM reference (left), inverse currents (right).

The far-field pattern calculated from the equivalent currents in an E plane cut is shown in Fig. 6. Both, copolar and crosspolar component show an excellent agreement with the reference solution obtained from a MoM simulation.

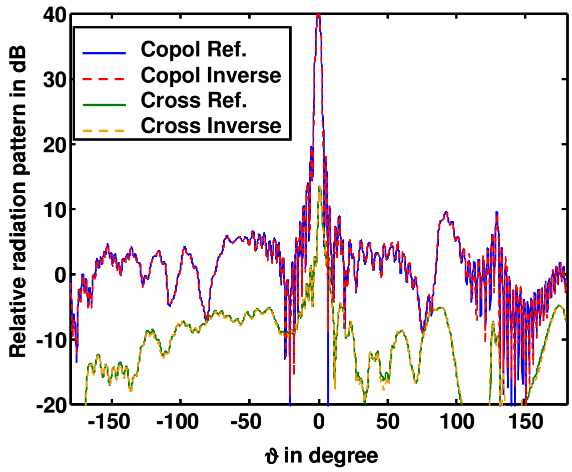

Fig. 6. Far-field pattern of offset-fed parabolic reflector with triple feed horn (E plane cut).

4. Conclusion

An inverse equivalent current method has been presented. Equivalent currents either on a virtual surface or direct on the radiating/scattering structure have been determined from electric near-field values. The discretized currents were related to the electric field values formulating an electric field integral equation, which has been solved very efficiently by adapting the Multilevel Fast Multipole Method to the inverse algorithm. The far-field patterns obtained from the equivalent currents show excellent agreement with the reference solutions. The presented approach is advantageous especially for irregular measurement grids and non-closed measurement surfaces, as well as for near-field values in the close vicinity of the radiating/scattering structure. Also, antenna and scattering object diagnostics using the equivalent current distribution itself or computed near-field values can be done efficiently.

5. References

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