Three-Dimensional Linear Sampling Applied to Microwave Breast Imaging

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Abstract

Microwave imaging is a promising technique in biomedical imaging. Full-wave quantitative inversion has much potential but is computationally very demanding due to the non-linearity of the problem while qualitative approaches mostly use approximations that do not comply with contrasted biomedical scatterers. In contrast, the linear sampling method is capable of detecting anomalies in a known background by solving only a linear problem. Although only a qualitative image is obtained in the form of an indicator function that assumes low values inside and large values outside the anomaly, it may be suitable for use in breast cancer imaging. This paper reports a preliminary study of the feasibility of linear sampling for this application.

1. Introduction

Microwave imaging is gaining interest in the field of biomedical imaging as an alternative or supplement to X-ray imaging, mainly because of the non-ionizing nature of microwaves and because the interaction of microwaves with biological tissues is governed by other material parameters. It is however known that the most suited form of microwave imaging for biomedical applications, namely quantitative microwave imaging which attempts to solve the non-linear and fully vectorial inverse scattering problem for the point values of the complex permittivity in an investigation domain [1], is computationally very demanding. It invariable requires an iterative solution method and appropriate regularization strategies for non linear problems, which are not as well understood as regularization of linear problems.

The recently developed linear sampling method [2,3] can be used to detect scattering objects in a much cheaper way. It combines the advantages of solving a linear problem without introducing simplifications in the derivations of its equations and a very simple implementation. However, with the linear sampling method, it is only possible to infer information about the shape of the object and the image is only qualitative. It can however be useful when one is only interested in detecting anomalies against a known background. This could be the case when detecting breast cancer, where the tumor is the anomaly. In this contribution we report the application of the linear sampling method to breast imaging. Since we work in the frequency domain, the time dependence $e^{j\omega t}$ of the fields, with ω the angular frequency, will be implicity assumed, as well as the ω-dependence of the complex permittivities.

2. Problem Formulation

The investigated scattering configuration consists of a free-standing breast model, immersed in a homogeneous matching medium characterized by its complex permittivity ϵ_b. The breast is represented by its complex permittivity distribution $\epsilon(\boldsymbol{r})$ and it may contain a tumor, which is the anomaly that has to be detected. We will consider a bounded domain $\mathcal{D}$ that completely includes the breast. Keeping in mind that the breast is only accessible from the front-side of the body, $\mathcal{D}$ is surrounded by a number N^A of antennas located in the points r_i, $i = 1, ..., N^A$. These antennas are modelled in this paper as non-interacting elementary dipoles. N^P (≤ 3) different polarization directions $\hat{\boldsymbol{u}}_{i,p}$, $p = 1, \ldots, N^P$ for the elementary dipoles are assigned to each antenna position. To acquire the scattering data, the breast is successively illuminated with the dipole fields generated by the elementary dipoles. The electric fields due to these excitations in the background, i.e. the breast without anomaly, are defined as the incident fields and denoted $\boldsymbol{E}^{inc}_{i,p} = [\boldsymbol{E}^{inc}_{i,p,x}, \boldsymbol{E}^{inc}_{i,p,y}, \boldsymbol{E}^{inc}_{i,p,z}]$, with $i = 1, ..., N^A$ and $p = 1, \ldots, N^P$. These fields cannot be measured in practice, but they can be computed. For every illumination (i, p), the corresponding total electric field components $\boldsymbol{E}_{i,p}(\boldsymbol{r}_j) \cdot \hat{\boldsymbol{u}}_{j,q}$, for every $j = 1, \ldots, N^A$ and $q = 1, \ldots, N^P$, are measured and subsequently the scattered field, defined as $\boldsymbol{E}^{scat}_{i,p} = \boldsymbol{E}_{i,p} - \boldsymbol{E}^{inc}_{i,p}$, is calculated.

In order to detect an anomaly in the breast, the discretized linear sampling method scans the investigation domain $\mathcal{D}$, i.e. for each point $\boldsymbol{r}_0$ on a testgrid that covers $\mathcal{D}$ the following system of linear equations is solved for the unknowns $g_{i,p,k}(\boldsymbol{r}_0)$:

$$\sum_{i=1}^{N^A}\sum_{p=1}^{N^P}\left(\hat{\boldsymbol{u}}_{j,q}\cdot\boldsymbol{E}_{i,p}^{scat}(\boldsymbol{r}_j)\right)g_{i,p,k}(\boldsymbol{r}_0) = -j\omega\mu_0\hat{\boldsymbol{u}}_{j,q}\cdot\boldsymbol{G}(\boldsymbol{r}_j,\boldsymbol{r}_0)\cdot\hat{\boldsymbol{v}}_k, \quad \forall j,q,k, \tag{1}$$

where $\hat{\boldsymbol{v}}_k$ is the direction of a test dipole which is placed in $\boldsymbol{r}_0$ and $\boldsymbol{G}(\boldsymbol{r}_j,\boldsymbol{r}_0)$ is the Green dyadic of the background with the breast for a source in $\boldsymbol{r}_0$ and a receiver in $\boldsymbol{r}_j$. The index k labels 3 orthogonal directions on the test grid. According to Linear Sampling theory, the indicator function $F(\boldsymbol{r}_0)$, defined as

$$F(\boldsymbol{r}_0) = \sum_{k=1}^{3}\sum_{i=1}^{N^A}\sum_{p=1}^{N^P} g_{i,p,k}^2(\boldsymbol{r}_0), \tag{2}$$

should become very large when $\boldsymbol{r}_0$ approaches the boundary of the anomaly from the inside or lies outside the anomaly.

To compute the incident dipole fields $\boldsymbol{E}_{i,p}^{inc}(\boldsymbol{r}_j) = -j\omega\mu_0\boldsymbol{G}(\boldsymbol{r},\boldsymbol{r}_i)\cdot\hat{\boldsymbol{u}}_{i,p}$ in presence of the breast, a volume integral equation (VIE) technique is used. The VIE is numerically solved by applying a Method of Moments discretization after which the resulting linear system is solved iteratively using the FFT-method to accelerate the iterations and reduce the memory use. Details of this solver can be found in [4]. Such a simulation also yields the incident fields on the grid $\boldsymbol{E}_{j,q}^{inc}(\boldsymbol{r}_0) = -j\omega\mu_0\boldsymbol{G}(\boldsymbol{r}_0,\boldsymbol{r}_j)\cdot\hat{\boldsymbol{u}}_{j,q}$, which can be used in the right hand side of (1) after an application of the reciprocity property

$$\hat{\boldsymbol{v}}_k\cdot\boldsymbol{G}(\boldsymbol{r}_0,\boldsymbol{r}_j)\cdot\hat{\boldsymbol{u}}_{j,q} = \hat{\boldsymbol{u}}_{j,q}\cdot\boldsymbol{G}(\boldsymbol{r}_j,\boldsymbol{r}_0)\cdot\hat{\boldsymbol{v}}_k. \tag{3}$$

3. Implementation

In order to compute the solution of the system (1), we will express it in a matrix notation:

$$\mathbf{E}^{\mathrm{scat}}\mathbf{g} = \mathbf{G}, \tag{4}$$

where the $(N^PN^A \times N^PN^A)$ - matrix $\mathbf{E}^{\mathrm{scat}}$ contains the scattered field for every transmitter-receiver combination, the colums of the matrix $\mathbf{g}$ $(N^PN^A \times 3N^D)$ contain the unknown coefficients $g_{i,p,k}^2(\boldsymbol{r}_0)$ for every one of the N^D test points $\boldsymbol{r}_0$ on the grid and for every test direction $\hat{\boldsymbol{v}}_k$. The matrix $\mathbf{G}$ $(N^A \times 3N^D)$ contains the fields due to the test dipoles at the receiver locations. The system (4) is an ill-posed system. Consider the singular value decomposition (SVD) of the matrix $\mathbf{E}^{\mathrm{scat}} = \mathbf{U}\mathbf{S}\mathbf{V}^H$, where $\mathbf{V}^H\mathbf{V} = \mathbf{I}$ and $\mathbf{U}^H\mathbf{U} = \mathbf{I}$, with $\mathbf{I}$ being the $(N^PN^A \times N^PN^A)$ identity matrix, and $\mathbf{S}$ a diagonal matrix containing the singular values of $\mathbf{E}^{\mathrm{scat}}$. The spectrum of $\mathbf{E}^{\mathrm{scat}}$ rapidly decreases and therefore the smallest singular values and the corresponding singular vectors are corrupted by noise on the measurements. To avoid noise amplification, a Tichonov regularization is used to calculate a regularized solution $\mathbf{g}^\alpha$:

$$\left[\left(\mathbf{E}^{\mathrm{scat}}\right)^H\mathbf{E}^{\mathrm{scat}} + \alpha\mathbf{I}\right]\mathbf{g}^\alpha = \left(\mathbf{E}^{\mathrm{scat}}\right)^H\mathbf{G} \tag{5}$$

and using the SVD of $\mathbf{E}^{\mathrm{scat}}$, we obtain

$$\|\mathbf{g}^\alpha\|_m^2 = \sum_{i=1}^{N^PN^A}\frac{\sigma_i^2}{\left(\sigma_i^2+\alpha\right)^2}\|\mathbf{u}_i^H\mathbf{G}\|^2, \tag{6}$$

where σ_i is the i-th singular value of $\mathbf{E}^{\mathrm{scat}}$ and $\mathbf{u}_i$ the i-th right singular vector, where $\|\cdot\|$ represents the two-norm for a vector and where $\|\mathbf{g}\|_m^2 = \sum_k\|\mathbf{g}_k\|^2$, with $\mathbf{g}_k$ the k-th column of $\mathbf{g}$. In order to calculate the regularization parameter α, the 'Generalized Discrepancy Principle' is employed:

$$\left\|\mathbf{E}^{\mathrm{scat}}\mathbf{g}^\alpha - \mathbf{G}\right\|_m^2 = \epsilon^2\left\|\mathbf{g}^\alpha\right\|_m^2 \tag{7}$$

with ϵ the largest singular value of the noise matrix $\mathbf{E}^{\mathrm{scat}} - \mathbf{E}_{clean}^{\mathrm{scat}}$, where $\mathbf{E}_{clean}^{\mathrm{scat}}$ is the ideal, noise free data matrix. (7) can be reformulated using (6) and the SVD factorization, resulting in

$$f(\alpha) = \sum_{i=1}^{N^P N^A} \frac{\alpha^2 - \epsilon^2 \sigma_i^2}{\left(\sigma_i^2 + \alpha\right)^2} \|\mathbf{u}_i^H \mathbf{G}\|^2 = 0. \tag{8}$$

The chosen regularization parameter thus is the root of $f(\alpha)$ and $f(\alpha)$ has a unique root in the interval $\left[0, \epsilon^2 \sigma_1^2\right]$ since $f(0) < 0$, $f(\epsilon^2 \sigma_1^2) > 0$, $f(\alpha)$ is continuous for $\alpha \geq 0$ and $df/d\alpha > 0$ for $\alpha \geq 0$.

4. Example

As an example, consider the scattering configuration from Figure 1. 84 dipoles are positioned around a free-standing breast, which is immersed in a background with $\epsilon_b = (10 - 2j)\epsilon_0$. The realistic numerical breast phantom is phantom 1 from ACR class 1 from the web site of UWCEM [5], and is based on MRI images and an extensive study of the dielectric properties of breast tissues [6]. The simulations are conducted at a frequency of 2 GHz. In order to obtain a feasible number of unknowns in our VIE-based forward scattering software, we deduced two coarser models from the full resolution phantoms through local averaging of the permittivities, one with cell spacing of 5 mm and one with 2.5 mm resolution, and added a tumor with a 1 cm radius and permittivity $(50 - 10j)\epsilon_0$ to both models. Figure 2 shows a slice through the 2.5 mm model.

Two linear sampling solutions were calculated. For the first one, the measurement data (i.e. $\hat{\boldsymbol{u}}_{j,q} \cdot \boldsymbol{E}_{i,p}(\boldsymbol{r}_j)$) were generated with the 5 mm model with tumor, and the incident dipole fields (i.e. $\hat{\boldsymbol{u}}_{j,q} \cdot \boldsymbol{E}_{i,p}^{inc}(\boldsymbol{r}_j)$) as well as the Green dyadic were calculated for the 5 mm model without a tumor. To make this example somewhat more realistic, additive Gaussian noise corresponding to SNR = 30 dB is added to the data, before the scattered field ($\hat{\boldsymbol{u}}_{j,q} \cdot \boldsymbol{E}_{i,p}^{scat}(\boldsymbol{r}_j) = \hat{\boldsymbol{u}}_{j,q} \cdot \boldsymbol{E}_{i,p}(\boldsymbol{r}_j) - \hat{\boldsymbol{u}}_{j,q} \cdot \boldsymbol{E}_{i,p}^{inc}(\boldsymbol{r}_j)$) is calculated. The indicator function F in a slice through the tumor center is depicted in Figure 3(a). The tumor is clearly visible as the region with minimal indicator values. For the second example, the data were generated using the 2.5 mm model with tumor, while the incident fields and the Green dyadic are calculated with the 5 mm model without tumor, thus creating a mismatch between the actual and estimated background. Although the linear sampling image is not as clear as the previous one, the tumor can still be detected, as appears from Figure 3(b).

5. Conclusion

The linear sampling method might provide a computationally cheap means to detect breast cancer detection. However, it requires an estimate of the breast permittivity without a tumor. From the presented example it can be concluded that the background does not need to be rigorously known. An estimate is sufficient, but it remains yet to be investigated how much this estimate can deviate from the actual background for the tumor to be detectable.

6. References

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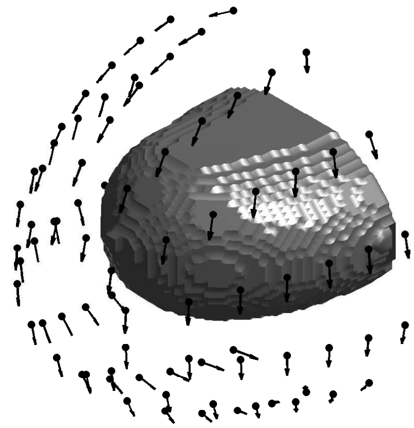
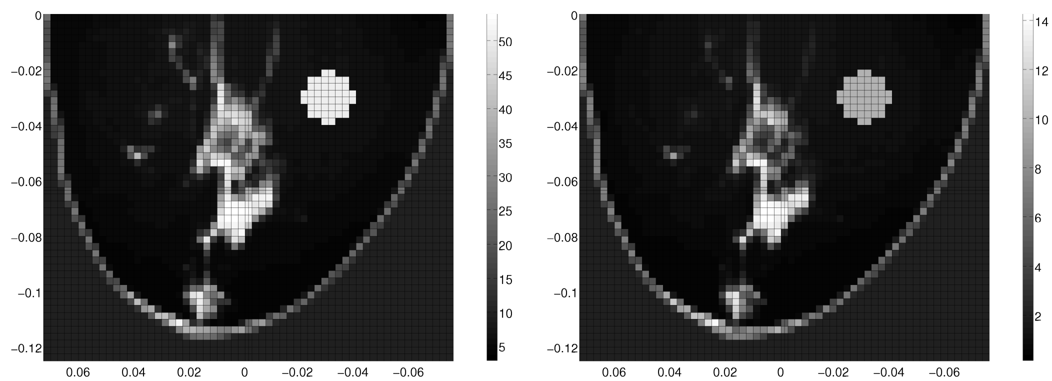


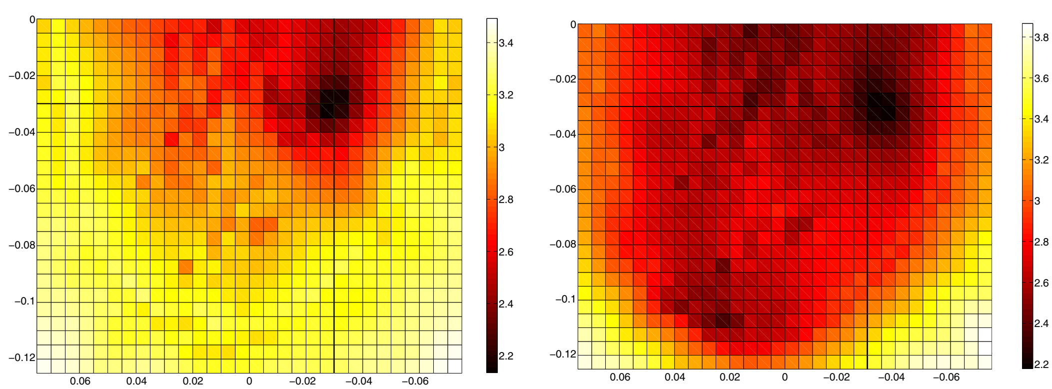
Figure 1: A free standing breast model is surrounded by 84 elementary dipoles to collect the scattering data.



(a) Real part of the relative permittivity.

(b) Imaginary part of the relative permittivity.

Figure 2: A slice through the complex permittivity profile of the 2.5 mm breast phantom. The tumor is located in the upper right corner.



(a)

(b)

Figure 3: A slice through the indicator function F for the two linear sampling reconstructions: (a) for data obtained with the coarse model and using the coarse model for the background, (b) for data obtained with the fine model and using the coarse model for the background. Values are in a logarithmic scale. (Images should be viewed in color)