Mean Reflection Coefficient of a Stack of Layered Random Media with Rough Interfaces — A Combined Small Perturbation Method

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Abstract

Scattering of waves from a stack of layered random media with rough interfaces is analyzed using a combined small perturbation method assuming that the volumetric and surface fluctuations are small, stationary, and independent of each other. Employing an integral equation formulation statistical averaging is performed, first with respect to volumetric fluctuations and then with respect to surface fluctuations. Using the properties of the mean propagation constants the wave functions in the layer structure are expressed in terms of surface scattering operators. By imposing boundary conditions on these wave functions and performing elimination of scattering operators the reflection coefficient for the system is obtained as a recurrence relation.

1. Introduction

The model of layered random media with rough interfaces has several applications in various disciplines, yet we do not have a satisfactory rigorous method for studying multiple scattering from this system. For the case of layered random medium with planar boundaries there exist comprehensive methods of analyses [1]. Although the problem of scattering from a single rough surface has been extensively studied [2] there are no simple satisfactory methodologies when we have layered structures with rough interfaces. Using the small perturbation method Elson [3] has carried out a detailed study of this problem. Even under this approximation the procedure is fairly complicated. Ohlidal [4] has obtained a more simple solution to this problem only at the cost of many more approximations. Neither of these papers treats the problem where the layered medium is randomly inhomogeneous as well. For this general problem the radiative transfer approach has been successfully employed [5]. However this approach deals entirely with intensities and hence cannot account for wave phenomena such as diffraction and interference which are very important in certain applications, for example, optical devices. With this in mind we present in this paper a wave approach to study scattering for this multi-layer system and derive an expression for the mean reflection coefficient in the form of a recurrence relation.

2. Description of the Problem

We have an N-layer random medium with rough interfaces. The domain of the j-th layer is denoted as Ω_j whose permittivity is $\epsilon_j + \tilde{\epsilon}_j(\mathbf{r})$ where ϵ_j is the deterministic part and $\tilde{\epsilon}_j$ is the randomly fluctuating part. The rough interfaces are given as $z = z_j + h_j(\mathbf{r}_\perp)$. We denote these interfaces as $\Sigma_j, \quad j = 0, 1, \ldots, N$. The fluctuations of the problem viz., $\tilde{\epsilon}_j$ and h_j are zero mean, isotropic, stationary, random processes independent of each other. Thus the ensemble averages of the interfaces are parallel planes. Let $z_0 = 0$ and let d_j be the thickness of the j-th layer. The medium parameters of the regions above and below the N-layer stack are respectively given by ϵ_0, k_0 and ϵ_{N+1}, k_{N+1}. Suppose a monochromatic plane wave is incident on the the layered stack from above. We are interested in computing the mean reflection coefficient.

3. Formulation

The waves in each layer are governed by $\nabla^2\psi_j + k_j^2\psi_j = -q_j\psi_j$, $j = 1, 2, \cdots, N$ where $q_j = \omega^2\mu\tilde{\epsilon}(\mathbf{r})$ and $k_j^2 = \omega^2\mu\epsilon_j$. The corresponding equations for the regions above and below the layered stack are $\nabla^2\psi_0 + k_0^2\psi_0 = 0$ and $\nabla^2\psi_{N+1} + k_{N+1}^2\psi_{N+1} = 0$. The boundary conditions at the interfaces are given as $\psi_j(\mathbf{r}_\perp, h_j) = \psi_{j+1}(\mathbf{r}_\perp, h_j)$ and $\epsilon_{j+1}\partial_n\psi_j(\mathbf{r}_\perp, h_j) = \epsilon_j\partial_n\psi_{j+1}(\mathbf{r}_\perp, h_j)$ where ∂_n is the normal derivative on the j-th interface pointing into the j-th layer. We assume that the waves satisfy the usual radiation conditions. The monochromatic plane wave incident on this layered medium is given as $\exp[i\mathbf{k}_\perp \cdot \mathbf{r} - ik_{0zi}z]$. The subscript i is used to denote the incident direction. Our primary interest is to compute $\langle\psi_0\rangle$.

For our approach it is convenient to represent the problem as a system of integral equations. In this context we shall use the rough interface Green's functions (denoted by superscript s). These are the Green's functions

of the problem when the volumetric fluctuations vanish (roughness of the interfaces is the only inhomogeneity). Employing these together with the wave functions in the Green's theorem we arrive at

$$\psi_j(\mathbf{r}) = \psi_j^s(\mathbf{r}) + \sum_{k=1}^{N} \int_{\Omega_k} d\mathbf{r}' G_{jk}^s(\mathbf{r},\mathbf{r}') q_k(\mathbf{r}') \psi_k(\mathbf{r}') \qquad j = 0, 1, \cdots, N+1 \tag{1}$$

Essential for arriving at the above results are the radiation conditions and the symmetry properties of the Green's functions.

4. Equations for the Mean Fields

We first average (1) with respect to volumetric fluctuations,

$$\langle \psi_j(\mathbf{r}) \rangle_v = \psi_j^s(\mathbf{r}) + \sum_{k=1}^{N} \sum_{l=1}^{N} \int_{\Omega_k} d\mathbf{r}' \int_{\Omega_l} d\mathbf{r}'' G_{jk}^s(\mathbf{r},\mathbf{r}') G_{kl}^s(\mathbf{r}',\mathbf{r}'') \langle q_k(\mathbf{r}') q_l(\mathbf{r}'') \rangle \langle \psi_l(\mathbf{r}'') \rangle_v. \tag{2}$$

Operating (2) by $\nabla^2 + k_j^2$ and noting that $\langle q_k(\mathbf{r}') q_l(\mathbf{r}'') \rangle = \delta_{kl} C_k^v(\mathbf{r}' - \mathbf{r}'')$, we obtain

$$(\nabla^2 + k_j^2) \langle \psi_j(\mathbf{r}) \rangle_v = -\int_{\Omega_j} d\mathbf{r}' G_{jj}^s(\mathbf{r},\mathbf{r}') C_j^v(\mathbf{r} - \mathbf{r}') \langle \psi_j(\mathbf{r}') \rangle_v. \tag{3}$$

We next average (3) with respect to surface fluctuations and make the approximation $\langle G_{jj}^s(\mathbf{r},\mathbf{r}') C_j^v(\mathbf{r}-\mathbf{r}') \langle \psi_j(\mathbf{r}') \rangle_v \rangle_s \simeq \langle G_{jj}^s(\mathbf{r},\mathbf{r}') \rangle_s C_j^v(\mathbf{r} - \mathbf{r}') \langle \psi_j(\mathbf{r}') \rangle_{vs}$ to obtain

$$(\nabla^2 + k_j^2) \langle \psi_j(\mathbf{r}) \rangle_{vs} \simeq -\int_{\Omega_j} d\mathbf{r}' G_{jj}^o(\mathbf{r},\mathbf{r}') C_j^v(\mathbf{r} - \mathbf{r}') \langle \psi_j(\mathbf{r}') \rangle_{vs}, \tag{4}$$

where G_{jj}^o is the unperturbed Green's function. We have taken into consideration the fact that the surface fluctuations are small and hence (4) is correct to order ε^2. On writing it as $(\nabla^2 + k_j^2 + \mathcal{L}) \langle \psi_j \rangle = 0$, where $\mathcal{L}$ denotes the integral operator $\int_{\Omega_j} d\mathbf{r}' G_{jj}^o(\mathbf{r},\mathbf{r}') C_j^v(\mathbf{r}-\mathbf{r}')$, we infer that $\sqrt{(k_j^2 + \mathcal{L})}$ represents the mean propagation constant in Ω_j. We see that it is (to the lowest order) independent of surface fluctuations and depends only on the fluctuations of permittivity in Ω_j. It also follows that the mean propagation constants in Region 1 and Region $N+1$ are unaffected by the fluctuations of the problem. We employ a multiple scale analysis to obtain explicit expressions for the mean propagation constants.

5. Mean Reflection Coefficient

As mentioned earlier, our primary interest is in computing the mean reflection coefficient of the system of random medium layers. In the last section we found that the mean wave in the j-th layer travels with propagation constant $\chi_j \equiv \sqrt{(\mathbf{k}_\perp^2 + \chi_{jz}^2)}$. Thus we can formulate the waves averaged with respect to permittivity fluctuations as

$$\langle \psi_j(\mathbf{r}) \rangle_v = \frac{1}{4\pi^2} \int d\mathbf{k}_\perp e^{i\mathbf{k}_\perp \cdot \mathbf{r}} \Big\{ A_j(\mathbf{k}_\perp, \mathbf{k}_{\perp i}) e^{i\chi_{jz} z} + B_j(\mathbf{k}_\perp, \mathbf{k}_{\perp i}) e^{-i\chi_{jz} z} \Big\}, \qquad j = 1, 2, \cdots, N \tag{5}$$

$$\langle \psi_0(\mathbf{r}) \rangle_v = e^{i\mathbf{k}_{\perp i} \cdot \mathbf{r} - ik_{0zi} z} + \frac{1}{4\pi^2} \int d\mathbf{k}_\perp e^{i\mathbf{k}_\perp \cdot \mathbf{r}} R(\mathbf{k}_\perp, \mathbf{k}_{\perp i}) e^{ik_{0z} z}, \tag{6}$$

and

$$\langle \psi_{N+1}(\mathbf{r}) \rangle_v = \frac{1}{4\pi^2} \int d\mathbf{k}_\perp e^{i\mathbf{k}_\perp \cdot \mathbf{r}} T(\mathbf{k}_\perp, \mathbf{k}_{\perp i}) e^{-ik_{(N+1)z} z}. \tag{7}$$

The boundary conditions at the interfaces are $\langle \psi_j(\mathbf{r}_\perp, h_j) \rangle_v = \langle \psi_{j+1}(\mathbf{r}_\perp, h_j) \rangle_v$ and $\chi_{j+1}^2 \partial_n \langle \psi_j(\mathbf{r}_\perp, h_j) \rangle_v = \chi_j^2 \partial_n \langle \psi_{j+1}(\mathbf{r}_\perp, h_j) \rangle_v$, $j = 1, 2, \cdots, N$. We are interested in finding $\langle R \rangle_{vs}$. To this end we may substitute (5), (6), (7) in the boundary conditions and use the small perturbation method. Although in principle it is straightforward to carry out this procedure, it is very cumbersome in practice. Instead we focus attention on the first layer and seek to represent the reflection coefficient of Σ_0 in terms of the reflection coefficient of Σ_1. Voronovich [2] has given a formula for this in terms of scattering amplitudes. A similar formula can be used to express the reflection coefficient of Σ_1 in terms of the reflection coefficient of Σ_2 and so on. The main difficulty we face in implementing this procedure is in the evaluation of an inverse operator involved in these formulas. We shall see that our procedure does not have such difficulties.

First we impose the boundary conditions on Σ_0, operate by $\int d\mathbf{r}_\perp k_0^2 \big(\chi_{1z} - \nabla h_0 \cdot \mathbf{k}_\perp\big) \exp[-i\mathbf{k}_\perp \cdot \mathbf{r} - i\chi_{1z} h_0]$ and by $\int d\mathbf{r}_\perp \exp[-i\mathbf{k}_\perp \cdot \mathbf{r} - i\chi_{1z} h_0]$ to obtain the following.

$$\begin{aligned}(\chi_1^2 - k_0^2) \int d\mathbf{k}'_\perp \frac{k'_{0z}\chi_{1z} + \mathbf{k}_\perp \cdot \mathbf{k}'_\perp}{\chi_{1z} - k'_{0z}} R(\mathbf{k}'_\perp, \mathbf{k}_{\perp i}) \mathcal{F}\Big(\exp[-i(\chi_{1z} - k'_{0z}) h_0] \Big)_{\mathbf{k}_\perp - \mathbf{k}'_\perp} \\ = 2k_0^2 \chi_{1z} A_1(\mathbf{k}_\perp, \mathbf{k}_{\perp i}) + (\chi_1^2 - k_0^2) \frac{k_{0zi}\chi_{1z} - \mathbf{k}_\perp \cdot \mathbf{k}_{\perp i}}{\chi_{1z} + k_{0zi}} \mathcal{F}\Big(\exp[-i(\chi_{1z} + k'_{0zi}) h_0] \Big)_{\mathbf{k}_\perp - \mathbf{k}_{\perp i}}\end{aligned} \tag{8}$$

where $\mathcal{F}$ stands for the Fourier transformation of the quantity in the brackets. The subscript is the transform variable. We next operate by $\int d\mathbf{r}_\perp k_0^2\big(-\chi_{1z}-\nabla h_0\cdot\mathbf{k}_\perp\big)\exp[-i\mathbf{k}_\perp\cdot\mathbf{r}+i\chi_{1z}h_0]$ and by $\int d\mathbf{r}_\perp \exp[-i\mathbf{k}_\perp\cdot\mathbf{r}+i\chi_{1z}h_0]$ to obtain the following relation between R and B_1 .

$$(\chi_1^2-k_0^2)\int d\mathbf{k}'_\perp \frac{k'_{0z}\chi_{1z}-\mathbf{k}_\perp\cdot\mathbf{k}'_\perp}{\chi_{1z}+k'_{0z}}R(\mathbf{k}'_\perp,\mathbf{k}_{\perp i})\mathcal{F}\Big(\exp[i(\chi_{1z}+k'_{0z})h_0]\Big)_{\mathbf{k}_\perp-\mathbf{k}'_\perp}$$
$$=-2k_0^2\chi_{1z}B_1(\mathbf{k}_\perp,\mathbf{k}_{\perp i})+(\chi_1^2-k_0^2)\frac{k_{0zi}\chi_{1z}+\mathbf{k}_\perp\cdot\mathbf{k}_{\perp i}}{\chi_{1z}-k_{0zi}}\mathcal{F}\Big(\exp[i(\chi_{1z}-k_{0zi})h_0]\Big)_{\mathbf{k}_\perp-\mathbf{k}_{\perp i}} \tag{9}$$

However, A_1 and B_1 are related through the reflection amplitude of Σ_1 as $A_1(\mathbf{k}_\perp,\mathbf{k}_{\perp i})=\int d\mathbf{k}'_\perp R_1(\mathbf{k}_\perp,\mathbf{k}'_\perp)$ $\exp[i(\chi_{1z}+\chi'_{1z})d_1]B_1(\mathbf{k}'_\perp,\mathbf{k}_{\perp i})$ Making use of this property in (8) and (9) leads to the following relation between R and R_1.

$$\int d\mathbf{k}'_\perp R_1(\mathbf{k}_\perp,\mathbf{k}'_\perp)E_1\int d\mathbf{k}''_\perp R(\mathbf{k}''_\perp,\mathbf{k}_{\perp i})\big\{\mathbf{k}'_\perp,\mathbf{k}''_\perp\big\}_0^-\mathcal{F}\Big(\exp[i(\chi'_{1z}+k''_{0z})h_0]\Big)_{\mathbf{k}'_\perp-\mathbf{k}''_\perp}$$
$$+\int d\mathbf{k}'_\perp R(\mathbf{k}'_\perp,\mathbf{k}_{\perp i})\big\{\mathbf{k}_\perp,\mathbf{k}'_\perp\big\}_0^+\mathcal{F}\Big(\exp[-i(\chi_{1z}-k'_{0z})h_0]\Big)_{\mathbf{k}_\perp-\mathbf{k}'_\perp}$$
$$=\big\{\mathbf{k}_\perp,\mathbf{k}_{\perp i}\big\}_0^-\mathcal{F}\Big(\exp[-i(\chi_{1z}+k_{0zi})h_0]\Big)_{\mathbf{k}_\perp-\mathbf{k}_{\perp i}}+\int d\mathbf{k}'_\perp R_1(\mathbf{k}_\perp,\mathbf{k}'_\perp)E_1\big\{\mathbf{k}'_\perp,\mathbf{k}_{\perp i}\big\}_0^+\mathcal{F}\Big(\exp[i(\chi'_{1z}-k_{0zi})h_0]\Big)_{\mathbf{k}'_\perp-\mathbf{k}_{\perp i}} \tag{10}$$

where $\qquad E_j=e^{i2\chi_{jz}d_j} \qquad \big\{\mathbf{k}_\perp,\mathbf{k}_{\perp i}\big\}_0^\mp=\dfrac{\chi_{1z}k_{ozi}\mp\mathbf{k}_\perp\cdot\mathbf{k}_{\perp i}}{\chi_{1z}(\chi_{1z}\pm k_{0zi})}$

Similarly R_1 is related to R_2 and so on. We can now employ the small perturbation method to this system by treating h_j as small parameters. Consider (10) and expand R_j as a perturbation series $R_j^{(0)}+R_j^{(1)}+R_j^{(2)}+\cdots$ and equate terms of similar order. In this paper we are interested in the mean reflection coefficient which to the lowest order is $\langle R\rangle\simeq R^{(0)}+\langle R^{(2)}\rangle$. Thus we need to carry out our calculations up to two orders. Noting that $R_{N+1}=0$ we have a complete set of recurrence relations. Results up to two orders are given below.

<u>Zeroth Order:</u>

$$R_j^{(0)}(\mathbf{k}_\perp,\mathbf{k}_{\perp i})=\delta(\mathbf{k}_\perp-\mathbf{k}_{\perp i})\frac{\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^-+R_{j+1}^{(0)}(\mathbf{k}_\perp)\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^+E_{j+1}}{\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^++R_{j+1}^{(0)}(\mathbf{k}_\perp)\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^-E_{j+1}} \qquad j=1,2,\cdots(N-1) \tag{11a}$$
$$R_N^{(0)}(\mathbf{k}_\perp,\mathbf{k}_{\perp i})=\delta(\mathbf{k}_\perp-\mathbf{k}_{\perp i})\frac{\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_N^-}{\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_N^+} \tag{11b}$$

<u>First Order:</u>

$$4\pi^2R_j^{(1)}(\mathbf{k}_\perp,\mathbf{k}_{\perp i})\Big[\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^++R_{j+1}^{(0)}(\mathbf{k}_\perp)\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^-E_{j+1}\Big]$$
$$=i\Big[\chi_{(j+1)z}-\chi_{jzi}\Big]\{\mathbf{k}_\perp,\mathbf{k}_{\perp i}\}_j^+h_j(\mathbf{k}_\perp-\mathbf{k}_{\perp i})\Big[R_j^{(0)}(\mathbf{k}_{\perp i})+R_{j+1}^{(0)}(\mathbf{k}_\perp)E_{j+1}\Big]$$
$$-i\Big[\chi_{(j+1)z}+\chi_{jzi}\Big]\{\mathbf{k}_\perp,\mathbf{k}_{\perp i}\}_j^-h_j(\mathbf{k}_\perp-\mathbf{k}_{\perp i})\Big[1+R_{j+1}^{(0)}(\mathbf{k}_\perp)E_{j+1}R_j^{(0)}(\mathbf{k}_{\perp i})\Big]$$
$$+4\pi^2R_{j+1}^{(1)}(\mathbf{k}_\perp,\mathbf{k}_{\perp i})E_{j+1}\Big[\{\mathbf{k}_\perp,\mathbf{k}_{\perp i}\}_j^+-R_j^{(0)}(\mathbf{k}_{\perp i})\{\mathbf{k}_{\perp i},\mathbf{k}_{\perp i}\}_j^-\Big] \qquad j=1,2,\cdots(N-1) \tag{12a}$$

$$4\pi^2R_N^{(1)}(\mathbf{k}_\perp,\mathbf{k}_{\perp i})\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_N^+=i\Big[k_{(N+1)z}-\chi_{Nzi}\Big]\{\mathbf{k}_\perp,\mathbf{k}_{\perp i}\}_N^+h_N(\mathbf{k}_\perp-\mathbf{k}_{\perp i})R_N^{(0)}(\mathbf{k}_{\perp i})$$
$$-i\Big[k_{(N+1)z}+\chi_{Nzi}\Big]\{\mathbf{k}_\perp,\mathbf{k}_{\perp i}\}_N^-h_N(\mathbf{k}_\perp-\mathbf{k}_{\perp i}) \tag{12b}$$

<u>Second Order:</u> $\qquad \langle R_j^{(2)}(\mathbf{k}_\perp,\mathbf{k}_{\perp i})\rangle=\delta(\mathbf{k}_\perp-\mathbf{k}_{\perp i})\langle R_j^{(2)}(\mathbf{k}_\perp)\rangle \tag{13a}$

$$4\pi^2\langle R_j^{(2)}(\mathbf{k}_\perp)\rangle\big[\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^++R_{j+1}^{(0)}(\mathbf{k}_\perp)E_{j+1}\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^-\big]$$
$$=2\pi^2\sigma_j^2\Big[\chi_{(j+1)z}+\chi_{jz}\Big]^2\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^-\big[-1+R_{j+1}^{(0)}(\mathbf{k}_\perp)E_{j+1}R_j^{(0)}(\mathbf{k}_\perp)\big]$$
$$+2\pi^2\sigma_j^2\big[\chi_{(j+1)z}-\chi_{jz}\big]^2\{\mathbf{k}_\perp,\mathbf{k}_\perp\}_j^+\big[-R_{j+1}^{(0)}(\mathbf{k}_\perp)E_{j+1}+R_j^{(0)}(\mathbf{k}_\perp)\big]$$
$$+\int d\mathbf{k}'_\perp W_j(\mathbf{k}'_\perp-\mathbf{k}_\perp)\big[\{\mathbf{k}'_\perp,\mathbf{k}'_\perp\}_j^++R_{j+1}^{(0)}(\mathbf{k}'_\perp)E_{j+1}\{\mathbf{k}'_\perp,\mathbf{k}'_\perp\}_j^-\big]^{-1}$$
$$\cdot\Big\{-(\chi'_{(j+1)z}-\chi_{jz})\{\mathbf{k}'_\perp,\mathbf{k}_\perp\}_j^+\big[R_j^{(0)}(\mathbf{k}_\perp)+R_{j+1}^{(0)}(\mathbf{k}'_\perp)E_{j+1}\big]+(\chi'_{(j+1)z}+\chi_{jz})\{\mathbf{k}'_\perp,\mathbf{k}_\perp\}_j^-\big[1+R_j^{(0)}(\mathbf{k}_\perp)R_{j+1}^{(0)}(\mathbf{k}'_\perp)E_{j+1}\big]\Big\}$$
$$\cdot\Big\{\big[\chi_{(j+1)z}-\chi'_{jz}\big]\{\mathbf{k}_\perp,\mathbf{k}'_\perp\}_j^+-\big[\chi_{(j+1)z}+\chi'_{jz}\big]\{\mathbf{k}'_\perp,\mathbf{k}'_\perp\}_j^-R_{j+1}^{(0)}(\mathbf{k}_\perp)E_{j+1}\Big\}$$
$$-4\pi^2\int d\mathbf{k}'_\perp\{\mathbf{k}'_\perp,\mathbf{k}'_\perp\}_j^-\Big[\{\mathbf{k}'_\perp,\mathbf{k}'_\perp\}_j^++R_{j+1}^{(0)}(\mathbf{k}'_\perp)E_{j+1}\{\mathbf{k}'_\perp,\mathbf{k}'_\perp\}_j^-\Big]^{-1}$$

$$\cdot\Big\langle R^{(1)}_{j+1}(\mathbf{k}_\perp,\mathbf{k}'_\perp)R^{(1)}_{j+1}(\mathbf{k}'_\perp,\mathbf{k}_\perp)\Big\rangle E_1(\mathbf{k}_\perp,\mathbf{k}'_\perp)E_1(\mathbf{k}'_\perp,\mathbf{k}_\perp)\Big[\{\mathbf{k}_\perp,\mathbf{k}_\perp\}^+_j - R^{(0)}_j(\mathbf{k}_\perp)\{\mathbf{k}_\perp,\mathbf{k}_\perp\}^-_j\Big]$$
$$+4\pi^2\Big\langle R^{(2)}_{j+1}(\mathbf{k}_\perp)\Big\rangle E_1^2(\mathbf{k}_\perp,\mathbf{k}_\perp)\Big[\{\mathbf{k}_\perp,\mathbf{k}_\perp\}^+_j - R^{(0)}_j(\mathbf{k}_\perp)\{\mathbf{k}_\perp,\mathbf{k}_\perp\}^-_j\Big] \quad (13b)$$

$$4\pi^2\langle R^{(2)}_N(\mathbf{k}_\perp)\rangle\{\mathbf{k}_\perp,\mathbf{k}_\perp\}^+_N = -2\pi^2\sigma_N^2\Big[k_{(N+1)z}+\chi_{Nz}\Big]^2\{\mathbf{k}_\perp,\mathbf{k}_\perp\}^-_N$$
$$+2\pi^2\sigma_N^2\Big[k_{(N+1)z}-\chi_{Nz}\Big]^2\{\mathbf{k}_\perp,\mathbf{k}_\perp\}^+_N R^{(0)}_N(\mathbf{k}_\perp) + \int d\mathbf{k}'_\perp W_N(\mathbf{k}'_\perp-\mathbf{k}_\perp)\Big[\{\mathbf{k}'_\perp,\mathbf{k}'_\perp\}^+_N\Big]^{-1}$$
$$\cdot\Big\{-\big[k'_{(N+1)z}-\chi_{Nz}\big]\{\mathbf{k}'_\perp,\mathbf{k}_\perp\}^+_N R^{(0)}_N(\mathbf{k}_\perp) + \Big[k'_{(N+1)z}+\chi_{Nz}\Big]\{\mathbf{k}'_\perp,\mathbf{k}_\perp\}^-_N\Big\}\cdot\Big[k_{(N+1)z}-\chi'_{Nz}\Big]\{\mathbf{k}_\perp,\mathbf{k}'_\perp\}^+_N \quad (13c)$$

where σ_j and W_j are respectively the rms fluctuation and the spectral density of the j-th surface. The computational procedure is to first evaluate $R^{(0)}$, then $R^{(1)}$ and then $R^{(2)}$. While computing $R^{(0)}$, first we compute $R^{(0)}_N$, then $R^{(0)}_{N-1}$, etc using the recurrence relation. The result that we get is the well-known expression for the reflection coefficient of stratified media [6]. We follow a similar procedure to compute $R^{(1)}$ and $R^{(2)}$. The advantage of our procedure is that we can carry out computations of higher order terms without too much difficulty. The result for the case $N = 0$ is the well-known SPM result for a single rough surface [2].

6. Discussion

The fluctuations of our problem are statistically homogeneous in azimuth and hence the translational properties of the solution lead to $\langle R(\mathbf{k}_\perp,\mathbf{k}_{\perp i})\rangle = \delta(\mathbf{k}_\perp-\mathbf{k}_{\perp i})\langle R(\mathbf{k}_\perp)\rangle$. Thus terms of all orders of the perturbation expansion of R will display this property. We have obtained a solution for the reflection coefficient up to the second order because the second order term is the lowest order of fluctuation correction in our problem. On the other hand, when intensity or mean reflectivity is calculated the first order term is sufficient. Furthermore, our wave-based solution, as opposed to the radiative transfer approach, is essential for studying the phenomenon of backscattering enhancement. The recurrence relations that we obtained for the reflection coefficients are more simple than the procedure described by Elson [3]. Also the media in our problem are randomly inhomogeneous as well and we have developed a procedure to address the issue of surface scattering and volumetric scattering in a unified manner. We showed that the mean propagation constant, to a first order, is influenced only by the volumetric fluctuations of the medium under consideration. All effects of surface fluctuations are higher order effects. We found that we can employ the concept of effective or mean propagation constant to carry out computations of the mean reflection coefficient and a key condition for that is the weak surface correlation approximation. Our problem can be approached in a different manner from a rigorous numerical standpoint [7]. Given the statistical properties of the random fluctuations of the problem one can generate sample realizations of the random system. The scattered waves corresponding to each realization are calculated by numerically solving the boundary value problem of the scattering system. This procedure is carried out for each realization and hence the characteristics of the scattered fields inferred [8]. As one might expect this is a computationally intensive approach. Although we do not have the limitations here that are inherently present in the approximate analytic methods we do have certain limitations of a different kind. For example, in order to keep the size of the problem computationally manageable only certain configurations are feasible in practice. Nevertheless, for those situations where it can be applied the numerical procedure produces benchmark results which are valuable to better understand the approximate methods. We intend to compare our results obtained in this paper with those of such numerical approaches in future.

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8. References

[1] H.C. Van De Hulst, *Multiple Light Scattering : Tables, Formulas, and Applications*, Academic Press, 1980.
[2] A.G. Voronovich, *Wave Scattering from Rough Surfaces*, 2nd Ed., Spriger-Verlag, 1999.
[3] J.M. Elson, *SPIE* **240**, pp 296-305. 1980
[4] I. Ohlidal, *Opt. Commun.* **71**, pp 323-326, 1989.
[5] F.T. Ulaby, R.K. Moore and A.K. Fung, *Microwave Remote Sensing: Active and Passive*, Volume 3, Artech House, 1986.
[6] W.C. Chew, *Waves and Fields in Inhomogeneous Media*, IEEE Press, 1995.
[7] H. Giovannini, M. Saillard and A. Sentenac, *J. Opt. Soc. Am.* **A15**, pp 1182-1191, 1998.
[8] K. Pak, L. Tsang, L. Li and C.H. Chan, *Radio Sci.* **23**, 331-338, 1993.