Waves Scattering Dependence on the Statistical Parameters of Classical and Fractal Rough Surfaces

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Abstract

It has been numerically studied in this work how the statistical parameters of irregularities of classical and fractal rough surfaces affect the radio waves diffuse scattering.

1. Introduction

There are a lot of scientific and engineering problems, which can be successfully solved only with deep understanding of wave scattering characteristics for statistically rough surface. Also there are many works devoted to mathematical analysis of wave interactions with statistically rough surface (see for example [1 - 3] and references).

Currently there are two general approaches of scattering on the statistically rough surface: method of small perturbation (SP) and Kirchhoff approach (tangent plane method (TPM)). These methods relate to two extreme cases of very small flat irregularities or smooth and large irregularities respectively. Two-scale scattering model becomes a generalization of these methods. The model is combination of small ripple (computations using SP) and large irregularities (computations using TPM).

Thus before the present diffraction problems for the statistically rough surfaces took into account irregularities of only single scale. Soon it had been realized that multiscale surfaces lead to better fitting. Currently by taking into account results of our works in Institute of Radio Engineering and Electronics of the RAS [1-5] we can state that physical meaning of diffraction theory, which includes multiscale surfaces, becomes clearer with fractal approach and choosing of fractal dimension (fractal signature) D as a parameter. Moreover as we have found out [1, 2] fractality accounting makes theoretical and experimental scattering patterns for earth cover in microwaves range closer. This fact always be interpreted (and has been being interpreted now) as results of pure instrumental errors.

2. Correlation Functions

Effects of the rough surfaces statistical parameters on the wave diffuse scattering angular distributions were investigated in [6, 7]. The general formulation of such a problem is presented in [8]. Aim of our work is to calculate numerically the effects of classical and fractal [2] rough surfaces statistics on the angular dependences of millimeter and centimeter waves forward diffuse scattering. The investigations have been performed for three kinds of correlation function of irregularities $\rho(\tau)$ and for different root-mean-square heights of irregularities σ_ξ:

$$\rho(\tau) = e^{-\tau^2/T^2} \tag{1}$$

$$\rho(\tau) = \left[1 + (\tau/T)^2\right]^{-3/2} \tag{2}$$

$$K_1(\tau/T) - (\tau/T)^2 K_0(\tau/T) \;\; \rho(\tau) = \left[1 + (\tau^2 \,|\, 8T^2)\right](\tau/T) \tag{3}$$

$$\tilde{\rho}(\tau) = \langle \rho(\tau) \rangle_s = \left[\frac{(1-q^{2(D-3)})}{(1-q^{2(D-3)N})}\right] \sum_{n=0}^{N-1} q^{2(D-3)n} J_0(Kq^n\tau) \tag{4}$$

where D – fractal dimension, q – main space frequency, M and N- harmonics, J_0 – first kind Bessel function of zero order.

Physical optics asymptotic solution for the surface with Gaussian heights distribution can be written as forward scattering dependence on roughness [7]:

$$J_G = 2k^2 \int_0^\infty J_0(v_x \tau) \{\exp\left[-\Sigma^2 (1 - c(\tau))\right] - \exp\left[-\Sigma^2\right]\} \tau d\tau \tag{5}$$

where Σ – Raleigh parameter, and J_G is related to scattering cross-section as::

$$\sigma^0 = \left|\beta_{pq}\right|^2 J_G \tag{6}$$

β_{pq} - linear polarization states matrix, which depends on surface dielectric constant and does not depend on roughness.

3. Results and discussion

On Fig.1 there are scattering patterns for various values of the parameters entering in (5). Calculations were performed for incident angle of 85 degree.

It is essential for fractal rough surfaces to know relationship between fractal parameters and standard statistical values. The following dependences were obtained using (4) (See fig.) On fig. there are average correlation length *Γ* dependences on fractal dimension *D* and on main space surface frequency *q*.

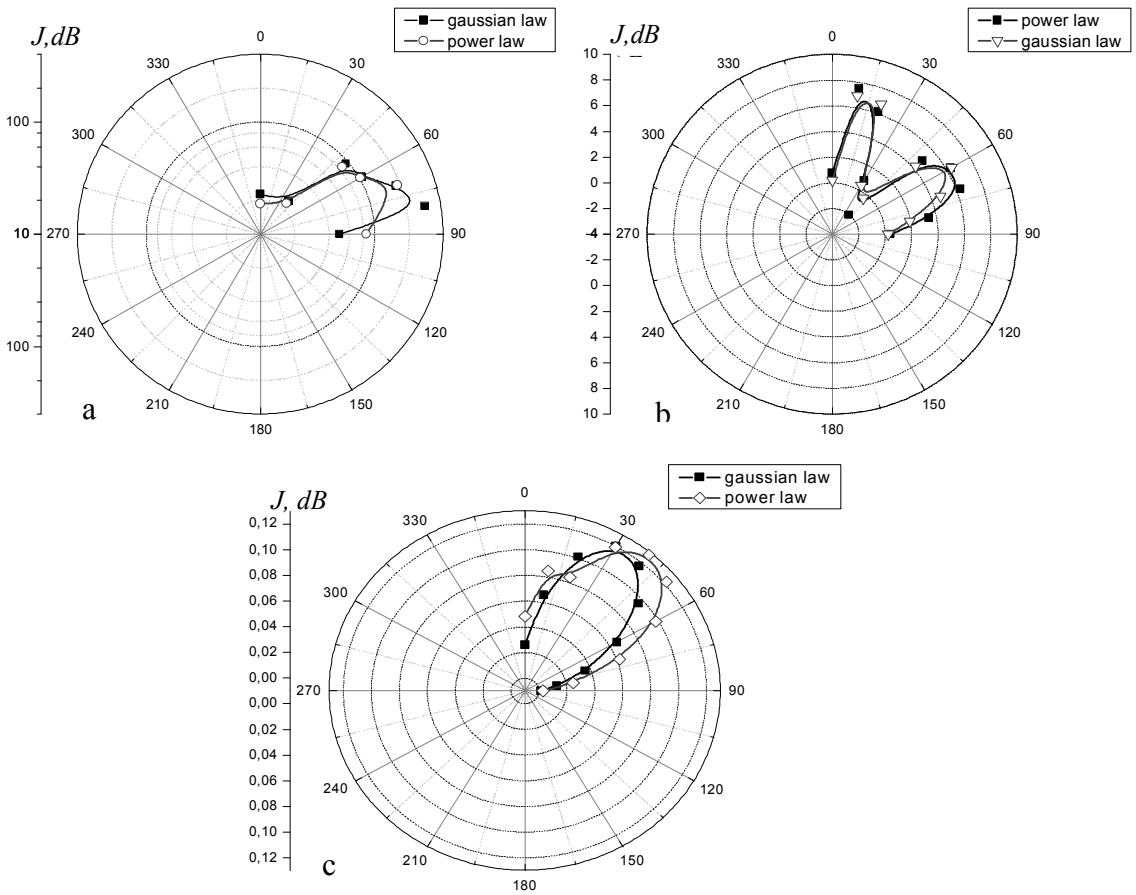

Figure 1. $J(\theta_S)$ dependence; ρ = 1 cm, σ_ξ =1 mm and λ = 2,2 mm (*a*), λ = 8,6 mm (*a*), λ = 3 cm (*c*)

It is essential for fractal rough surfaces to know relationship between fractal parameters and standard statistical values. The following dependences were obtained using (4) (see fig. 2) On fig.2 there are average correlation length *Γ* dependences on fractal dimension *D* and on main space surface frequency *q*.

The following fundamental conclusions can be made. Scattered field value increases when roughness decreases. For surfaces with exponentially distributed heights the complete diffuse scattering angular pattern as a roughness function is similar for three different correlation functions. Also there is no significant difference

between this surface behaviour and Gaussian height statistics surface behaviour. Thus, from the point of physical optics view the diffuse scattering angular distribution dependence on roughness is not sensitive to correlation function, height statistics and incident angle. Diffuse specular scattering depends on height statistics for both the high values of Raleigh parameter and the moderate values (~1). Diffuse scattering does not depend on height distribution only for small values of Σ. Thus two main distinctions from backscattering directions lie in the dependence on slope for small Σ and in the sensitivity to heights statistics for moderate Σ.

Γ drops faster when fractal dimension D increases than for the same variations of q. But when q becomes higher than certain value, Γ begins increasing. It means that Γ in general is not sensitive to q. Γ drops monotonically when D increases. Nevertheless Γ does not depend on D for q = 1.01. These results indicate that the fractal surface main parameter defining its features is fractal dimension D.

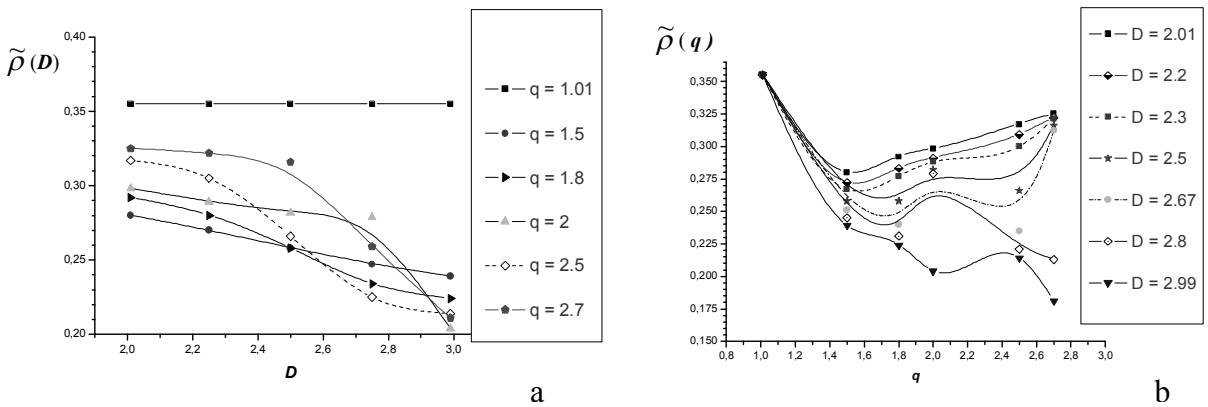

Figure 2. Average correlation length $\tilde{\rho}$ as a function of D for q = 1,01; q = 1,5; q = 1,8; q = 2; q = 2,5; q = 2,7 (*a*) and as a function of q for D = 2,01; D = 2,2; D = 2,3; D = 2,5; D = 2,67; D = 2,8; D = 2,99 (*b*)

4. Conclusion

Now on the basis of large scattering characteristics data array we can do some significant conclusions. When D has small value the main part of energy is scattered in the specular direction. Side lobes appear due to Bragg scattering. Number of side lobes and their intensity increase with increased value of fractal dimension D of the dispersive surface. Angular range of the side lobes also increases with increase of D when higher spatial frequencies become significant. Radio wave that interacts with a fractal can be viewed as a yardstick to probe rough surfaces by means of spatial frequencies selection on the basis of Bragg diffraction conditions [1], [2], [4], [6]. In the case of small D values classical and fractal approaches for scattered field solution coincide with each other. Thus fractal dimension D of the rough surface can be estimated using calculated or measured scattering patterns. In practice sizes of illuminated area must be at least two times greater than the main period of a surface structure in order to obtain fractal parameters information from scattering patterns.

Effectiveness of radio physics investigations can be significantly improved by taking into account fractality of wave phenomena that are progressing at every stage of wave radiation, scattering and propagation in different mediums. Undoubtedly, fractal describing of the wave scattering process [1], [2], [4] will result in establishing of new physical laws in the wave theory. These results can be applied widely for fractal antennas modeling, fractal frequency-selective structures modeling, solid-state physics, physics of nanostructures and for synthesis of nano- and meta-materials.

5. References

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