Resonant Transmission of a Class of Sub-wavelength Apertures in Thin Conducting Screen

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Abstract

Resonant transmission of a class of sub-wavelength apertures in thin conducting screen is investigated in this paper. The relationship between the transmitted power of resonant electrically small apertures with various shapes and the transmission cross section of them is carefully examined, and the admittance of the apertures is calculated to validate the resonant behavior of them. It is found that the transmitted power of an electrically small aperture at a resonance converges to the transmission cross section of it independent of its shape and size, which is $3/4\pi$ square wavelengths.

1. Introduction

Over the past decades the problem of electromagnetic coupling through electrically small holes or apertures has been attracting many researchers in microwave areas and optics. Bethe [1] was first treated this problem using the aperture polarizability concept and it was found that the transmittance normalized to the area scales as $(a/\lambda)^4$ where a is the hole radius and λ is the wavelength. According to this theory, the transmitted power through a small hole in terms of wavelength is very low and the transmission efficiency is poor. Harrington [2] extended this problem by introducing radiation terms in the equivalent circuit and suggested an easy way of improving the poor transmittance by satisfying resonance condition. The resonance condition can be realized by loading the small aperture with a capacitive lumped element or a capacitive structure. He also applied the generalized admittance concept to narrow, infinitely long slots in a conducting plane of finite thickness [3]. Many researchers in optics have been extensively investigating a nano-scale optical probe with a perturbed aperture shape based on similar concept to achieve very high transmission efficiency with a view of enhancing near-field optical applications such as optical data storage, nano-lithography, and nano-microscopy [4]. Recently, Kim *et. al* [5] have analyzed a small circular aperture with a ridged structure in microwave band to find out the condition for maximum transmission through the sub-wavelength aperture and the maximum transmission condition has been represented by using the equivalent circuit concept.

In this paper, the resonant transmission of an electrically small circular aperture with different shapes in thin conducting plane is analyzed The relationship between the transmitted power of a resonant sub-wavelength apertures and the transmission cross section of them is examined by using the Method of Moments (MoM), and aperture impedance and admittance are evaluated to check out the resonant behavior of them.

2. Analysis of a Class of Sub-wavelength Apertures in Thin Conducting Screen

The problem of transmitted power enhancement of an electrically small circular aperture by satisfying a resonance condition is first examined among various resonant aperture configurations. In this case, a small circular aperture with a ridge in an infinite Perfect Electric Conductor (PEC) screen shown in Fig. 1 is considered. The diameter of the circular aperture D is 10mm, the width of the ridge W is 3 mm, and the gap between ridge edges g is 0.478 mm. We note that the dimensions of the circular aperture and the ridge are chosen to resonate at 7.442 GHz (λ = 40 mm). An in-house MoM code using RWG basis functions and Galerkin testing scheme is used to analyze the

aperture. A plane wave polarized parallel to the ridge axis (x-axis) is incident upon normally and the power density of the plane wave is assumed to be P_{inc} = 1 W/m^2.

The transmitted power of the circular aperture with and without the ridge is compared. Figure 2 presents the comparison of the transmitted power through the circular aperture with and without the ridge. We see that the transmitted power through the aperture is significantly enhanced by introducing the ridge in the aperture from 9.7 μW to 390 μW, which is about 40 times higher as compared to that of the original circular aperture. It is worthwhile to mention that the value 390 μW corresponds to the transmission cross section 381.97 μW at the resonant frequency 7.442 GHz, which can be calculated by using the formula $\frac{3\lambda^2}{4\pi}$ [2]. The transmitted power through a small aperture would be the same as the transmission cross section of the aperture as long as the aperture size is electrically small and the resonance condition is satisfied by using a capacitor. The admittance of the ridged circular aperture is calculated by using the equation in [5], and is shown in Fig. 3(a) and (b). It is observed that the aperture susceptance vanishes to zero at a resonance, and, therefore, maximum transmission occurs. Figure 4 (a) shows the transmitted power vs. resonant frequency as the ridge gap increases from 0.5 mm to 3.5 mm with a fixed aperture diameter D = 10mm and the transmission cross section corresponding to each resonant frequency is compared in the same plot. The variation of the ridge gap *vs.* resonant frequency is presented in Fig. 4 (b). We see that the MoM-calculated transmitted power values are very close to those of transmission cross section. Another interesting observation is that the difference between the transmitted power and the transmission cross section becomes small as the frequency decreases because the aperture size in terms of the wavelength reduces accordingly. This means that the transmitted power through the aperture converges to the transmission cross section of it when the aperture size is sufficiently small compared to the wavelength.

Next, we turn to a narrow rectangular slot in thin conducting screen, which is another example of resonant sub-wavelength aperture. The geometry of a narrow rectangular slot in infinitely thin conducting screen is shown in Fig. 5. The narrow rectangular aperture has sides of a (slot length) and b (slot width) in x- and y- directions, respectively, where $a >> b$. The characteristic of the transmitted power as a function of frequency looks similar to that of the ridged circular aperture. Figure 6(a) shows the transmitted power vs. resonant frequency as the slot length a decreases from 35 mm to 15 mm with a fixed slot width b = 1mm and the transmission cross section corresponding to each resonant frequency is plotted in the same graph, and Fig. 6(b) presents the variation of the slot length as a function of resonant frequency. We observe that the maximum transmission occurs when the slot length is slightly smaller than a half of a free space wavelength and it corresponds to the cutoff frequency of TE_{10} mode of a rectangular waveguide. It is also seen that the transmitted power values and the transmission cross section values are very close each other. Although not shown here, the admittance of the narrow rectangular slot shows similar behavior to that of the ridged circular aperture.

Conclusions

In this paper, we have examined the resonant transmission of a class of sub-wavelength apertures such as a ridged circular aperture and a narrow rectangular slot in thin infinite conducting screen. The transmitted power for varying the ridge gap and the slot length for these apertures, respectively, has been compared with the transmission cross section of the apertures to check out the relationship between them, and the admittance of the apertures have been computed to validate the resonant behavior of them. It turns out that the transmitted power of an electrically small aperture at a resonance converges to the transmission cross section of 3/4π square wavelengths independent of its shape and size as long as the aperture size is sufficiently small compared to the wavelength.

References

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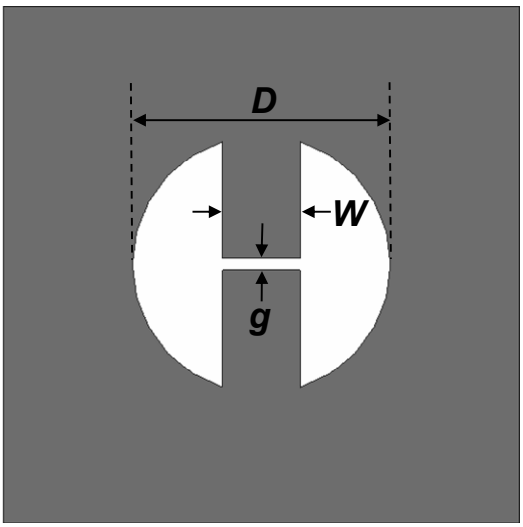


Fig. 1. Geometry of a small circular aperture with a ridge.

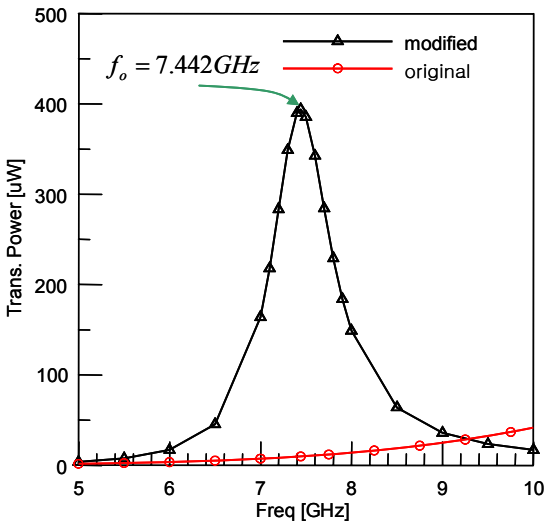
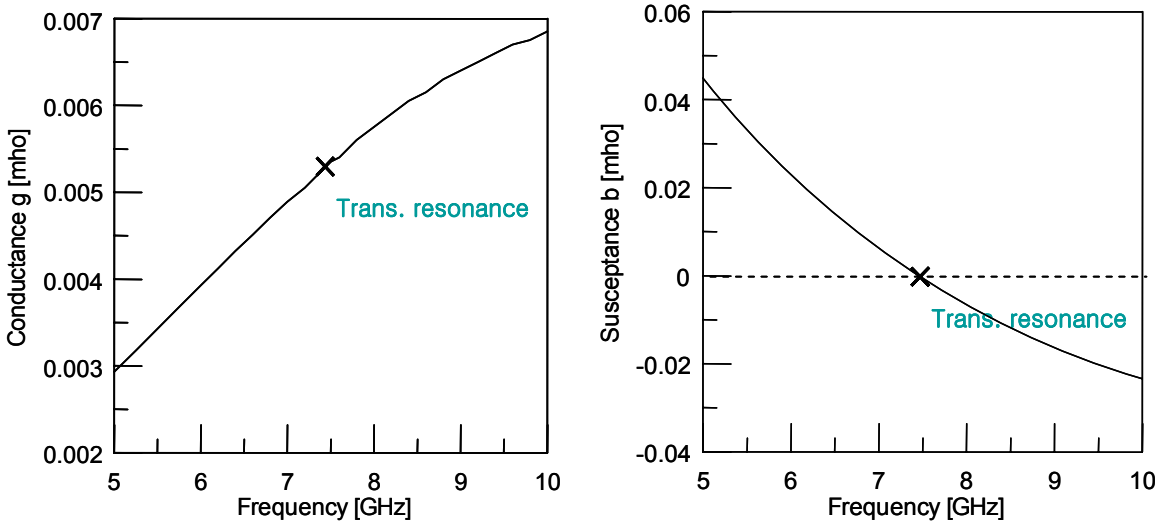
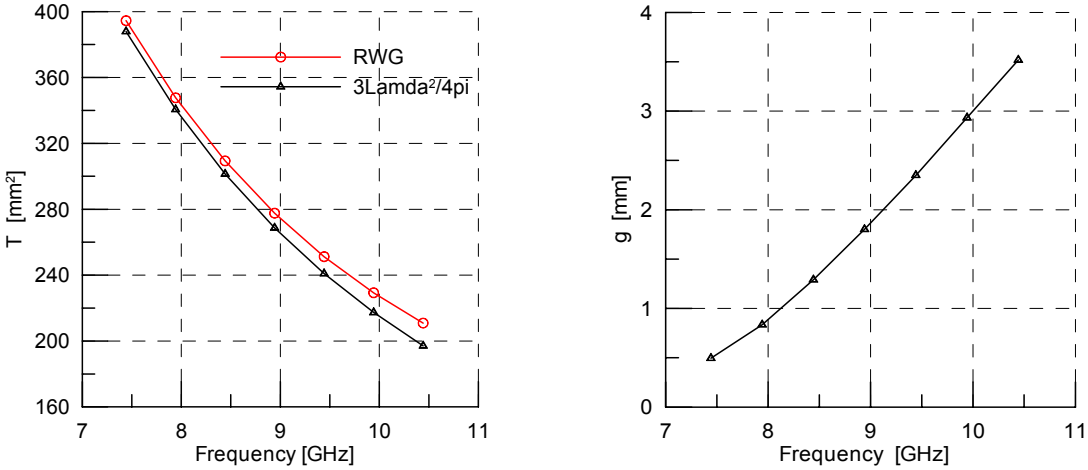


Fig. 2. Comparison of the transmitted power through the aperture with/without the ridge.



(a) Conductance (b) Susceptance

Fig. 3. Admittance of the ridged circular aperture.



(a) Transmitted power *vs.* resonant frequency (b) Ridge gap *vs.* resonant frequency

Fig. 4. Transmitted power vs. resonant frequency for different ridge gap and corresponding relationship between the ridge gap and the resonant frequency.

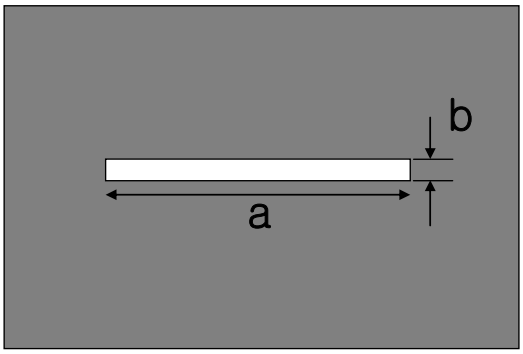
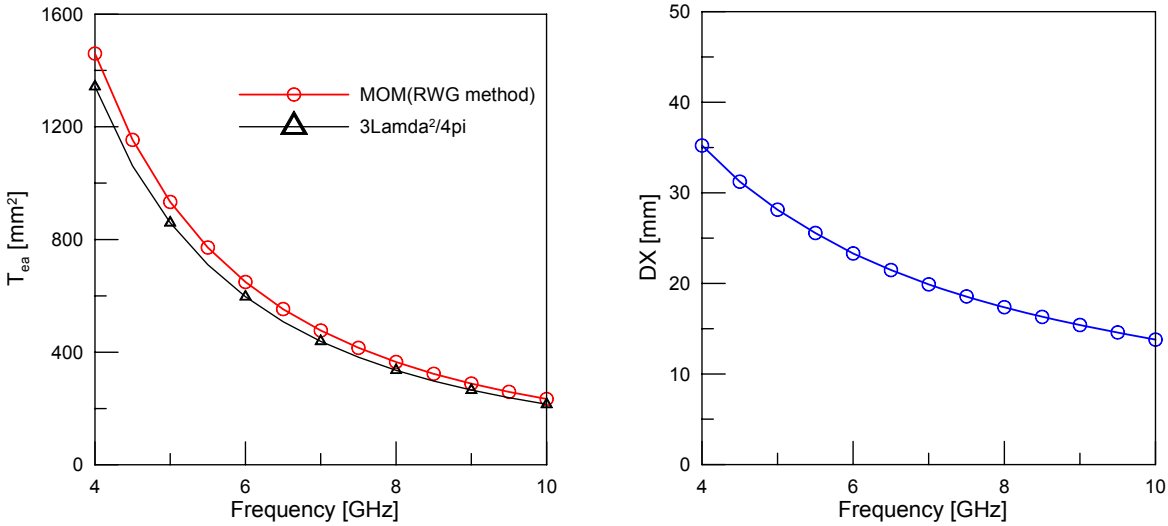


Fig. 5. Geometry of a narrow rectangular slot.



(a) Transmitted power *vs.* resonant frequency (b) Slot length *a vs.* resonant frequency

Fig. 6. Transmitted power vs. resonant frequency for different slot length and corresponding relationship between the slot length and the resonant frequency.