Existence of Limits for Electromagnetic Parameters of Metamaterials?

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Abstract

This presentation discusses different ways how electric and magnetic material parameter limits can be approached. In addition to the magnitudes of permittivity and permeability, the presentation focuses on magnetoelectric parameters in bi-isotropic and bianisotropic media, and the concept of PEMC material.

1. Introduction

Metamaterials are defined to be media which exhibit new and emergent properties in their macroscopic behavior. Therefore it is typical of metamaterials research that the earlier-accepted limits to material parameters are approached and perhaps even exceeded. Materials in electromagnetics are characterized by the constitutive relations in which different electric and magnetic parameters appear. There are several paths along which to expand the allowed parameter regions and to go towards the limits. The amplitudes of the permittivity and permeability can be studied to come close to infinity or zero, the magnetoelectric parameters have their own role to play, the dispersive character of the medium is important, and the complex-valued properties of the parameters may be of interest. In this presentation, some of these approaches will be discussed.

2. Classes of electromagnetic materials

In metamaterials research, one of the projects has been to push the allowed permittivity (ϵ) and permeability (μ) values into negative regimes. The natural four-field classification [1] of ϵ and μ being both positive (DPS), both negative (DNG), or only one of them negative (ENG and MNG), gives a natural framework in the study of backward-wave, negative-index, and Veselago media [2].

This classification is clear for isotropic materials. However, if anisotropy of bianisotropy is allowed, the classes have to be redefined. Isotropic DPS materials are positive definite and DNG media negative definite (meaning that their material matrix—the relation between electric and magnetic flux densities and fields—is definite, either positive or negative). However, for anisotropic or bianisotropic media the matrix is no longer a multiple of a unit matrix, and its eigenvalues do not need all to be of the same sign. In *indefinite media*, for example, waves may be backward if propagating in a certain direction, and forward in another direction. Only in positive definite media all are waves forward and in negative definite media all backward [3].

2.1 Magnitudes of the parameters

Aside from the negative-valued regimes for ϵ and μ, another regime where the limits of the allowed parameter values are of special interest is the plain isotropic DPS region. The "ordinary" materials have "reasonable" permittivity and permeability values. But one may start to increase or decrease these values and ask how large or how small these parameters in magnitude may become. And what kind of material classes are found at these limits.

		ϵ	μ	n	η
PEC	*perfect electric conductor*	large	small	undefined	very small
PMC	*perfect magnetic conductor*	small	large	undefined	very large
ZIM	*zero-index material*	small	small	very small	undefined
IIM	*infinite-index material*	large	large	very large	undefined
ZEM	*zero-electric material*	very small	undefined	small	large
ZMM	*zero-magnetic material*	undefined	very small	small	small
IEM	*infinite-electric material*	very large	undefined	large	small
IMM	*infinite-magnetic material*	undefined	very large	large	large

Table 1: Extreme-parameter materials classified by the magnitude of the four electro-magnetic parameters ϵ, μ, n, η.

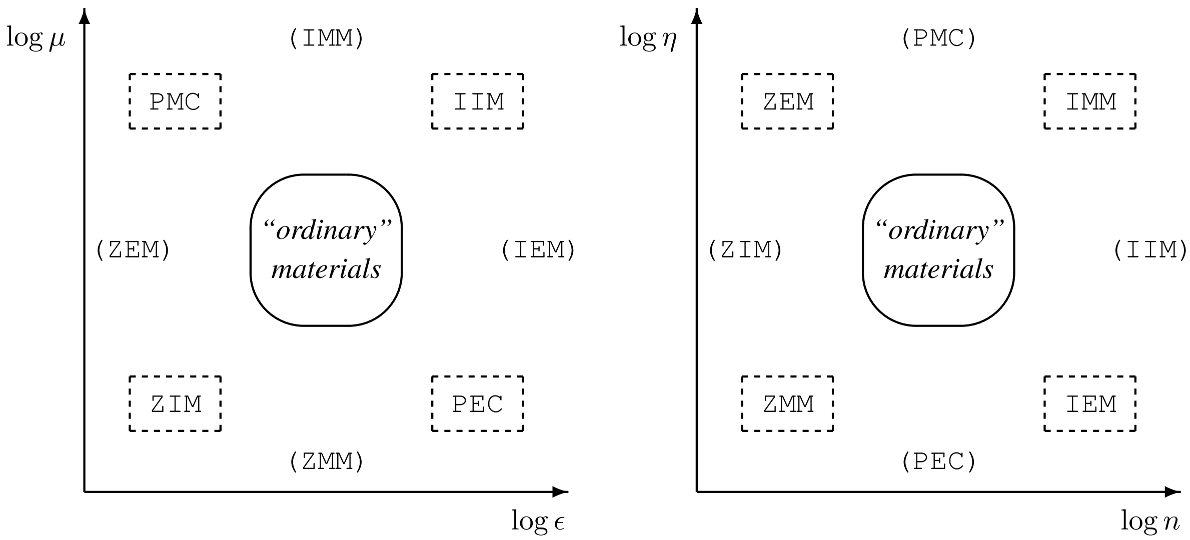

Figure 1: The classification of the materials in the $\epsilon\mu$-plane and in the $n\eta$-plane.

Table 1 and Figure 1 present a way to look at these regimes and to attach labels within these regions. There the essential parameters, in addition to the relative permittivity ϵ and relative permeability μ are the refractive index $n = \sqrt{\mu\epsilon}$ and the wave impedance $\eta = \sqrt{\mu/\epsilon}$. Each of these parameters can be either very large or very small, resulting in the given eight classes: there, by "Z" and "I" refering to "zero" and "infinite" we have, for example, "ZMM" meaning "zero-magnetic-material" (for which μ is very small), "IEM" (infinite-electric-material, in other words ϵ very large), or "ZIM" which would be "zero-index-material" (n close to zero).

The natural label for the low-impedance and high-impedance materials are PEC and PMC (perfect electric and magnetic conductors). For example, PEC is characterized by very large ϵ and very small μ, meaning that the impedance $\eta = \sqrt{\mu/\epsilon}$ is extremely small. (And vice versa for PMC: it is a "high-impedance" medium since μ is large and ϵ small.)

2.2 Magnetoelectric coupling

A very relevant dimension in the characterization of materials in electromagnetics and analyzing the parameter limitations is magnetoelectric coupling which is present in bianisotropic materials. The constitutive relations for bi-anisotropic materials include cross-couplings between to the electric field $\mathbf{E}$ and magnetic displacement (flux density) $\mathbf{B}$, and also the magnetic field strength $\mathbf{H}$ and electric displacement $\mathbf{D}$. The relations, in a normalized form, look like

$$\begin{pmatrix} \mathbf{D}/\sqrt{\epsilon_0} \\ \mathbf{B}/\sqrt{\mu_0} \end{pmatrix} = \begin{pmatrix} \overline{\overline{\epsilon}} & \overline{\overline{\xi}} \\ \overline{\overline{\zeta}} & \overline{\overline{\mu}} \end{pmatrix} \cdot \begin{pmatrix} \sqrt{\epsilon_0}\,\mathbf{E} \\ \sqrt{\mu_0}\,\mathbf{H} \end{pmatrix} \tag{1}$$

where the four material parameter dyadics are in addition to the relative permittivity $\overline{\overline{\epsilon}}$ and permeability $\overline{\overline{\mu}}$ dyadics, two dimensionless magnetoelectric dyadics $\overline{\overline{\xi}}$ and $\overline{\overline{\zeta}}$.

The magnetoelectric dyadics are often separated into the chirality and non-reciprocity parts [4]. In isotropic form, the material matrix C containing the four parameters is often written as

$$C = \begin{pmatrix} \epsilon & \chi - \mathrm{j}\kappa \\ \chi + \mathrm{j}\kappa & \mu \end{pmatrix} \tag{2}$$

where the imaginary unit reminds about the time-harmonic dependence $\exp(\mathrm{j}\omega t)$. The handedness-related chirality (Pasteur) parameter is κ, and χ is the non-reciprocity (Tellegen) parameter of the bi-isotropic medium under discussion.

How do physical principles restrict the amplitudes of these new parameters? One way to tackle this question is to look at the wave numbers of the two propagating eigenwaves in bi-isotropic medium:

$$k_\pm = k_0(\sqrt{\mu\epsilon - \chi^2} \pm \kappa), \quad k_0 = \omega\sqrt{\mu_0\epsilon_0} \tag{3}$$

From this relation it can be seen that when either κ or χ grows very large, something changes qualitatively in the propagation characteristics. In fact, the restriction $\chi^2 + \kappa^2 \leq \mu\epsilon$ has been regarded, in earlier times (the past decade, 1990's), as the upper limitation for the magnetoelectric parameters.

However, the present-day focus and interest on metamaterials has made negative refractive index an accepted concept. Therefore it should not cause any problems to allow the chirality parameter κ to be larger than $\sqrt{\mu\epsilon}$. In light of Equation (3), that one of the eigenwaves is a backward wave. The medium is indefinite, since one of the eigenvalues of the matrix C in (2) is positive, and the other negative. And indefinite materials are no longer discriminated in complex materials research. Likewise, one may relax the limitation on the Tellegen parameter χ. It is true that if $\chi > \sqrt{\mu\epsilon}$, the eigenwaves will become attenuated (cf. Equation(3)) but this is by no means a sufficient reason to infer that they are incompatible with electromagnetic waves.

One particular type of magnetoelectric materials that is especially relevent with respect to material parameter limitations is the so-called Perfect ElectroMagnetic Conductor (PEMC) [5]. PEMC is a generalization of PEC and PMC materials, obeying the following unfamiliar constitutive relations between the electric and magnetic fields and flux densities:

$$\mathbf{D} = M\mathbf{B}, \quad \mathbf{H} = -M\mathbf{E} \tag{4}$$

Here M is a real scalar admittance-type quantity (theoretically, it is a pseudoscalar). The special case $1/M = 0$ corresponds to PEC and $M = 0$ to PMC. In four-dimensional electromagnetic formulation, PEMC corresponds to the axion piece [6].

The constitutive relation (4) can be written in a form where flux densities are given as functions of fields. However, then the relations appear in a more complicated form:

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = q \begin{pmatrix} M & 1 \\ 1 & 1/M \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix}, \quad \text{with} \quad q \to \infty \tag{5}$$

These isotropic relations fall into the class of Tellegen media (note that C is symmetric), although the amplitudes of the three parameters ϵ, μ, χ are very large. PEMC has been predicted to be very useful in various engineering problems where manipulation of wave polarization is desirable. Furthermore, PEMC is not a purely theoretical, academic medium: analysis has shown that using ferrite material [7], at least a band-limited PEMC boundary can be created.

PEMC is seen to be limit case of the Tellegen material in two respects. Firstly, the parameters grow without limit, but also, the Tellegen parameter is exactly at the other limit $\chi^2 = \mu\epsilon$. As was discussed earlier, this latter limit is by no means any absolute restriction. It is therefore tempting to try to generalize the PEMC description by adding one additional free parameter to allow for varying Tellegen character. This would suggest the following material matrix:

$$\begin{pmatrix} \mathbf{D} \\ \mathbf{B} \end{pmatrix} = q \begin{pmatrix} M & a \\ a & 1/M \end{pmatrix} \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} \tag{6}$$

Here, no physical principle restricts the amplitude of q and a for this Tellegen medium. The quality and character of the "conductor" limit ($q \to \infty$) depends on the value of a. If it is set to $a = 1$ before the limit is taken, the result is PEMC medium. However, new possibilities and new types of complex media are generated by letting the parameter a (instead of the choice $a = 1$) to vary with different choices of M.

3. Conclusions

Metamaterials exhibit, by definition, new macroscopic effective properties. Therefore their characterization requires that the material parameters will take values that are beyond the domains of "ordinary" materials. Along the lines as shown above, this presentation concentrates on certain paths how these limiting territories can be approached, including the extreme-magnitude materials, magnetoelectric and bianisotropic media, and PEMC concepts.

4. References

1. N. Engheta and R.W. Ziolkowski (eds.), *Metamaterials. Physics and Engineering Explorations*, IEEE and Wiley, New York, 2006.

2. V.G. Veselago, "The Electrodynamics of Substances with Simultaneously Negative Values of ϵ and μ," *Soviet Physics Uspekhi*, **10**, 1968, pp. 509-514.

3. I.V. Lindell and A.H. Sihvola, "Negative-definite Media, a Class of Bi-anisotropic Metamaterials," *Microwave and Optical Technology Letters*, **48**(3), March 2006, pp. 602-606.

4. I.V. Lindell, A.H. Sihvola, S.A. Tretyakov, and A.J. Viitanen, *Electromagnetic Waves in Chiral and Bi-isotropic Media*, Artech House, Boston, 1994.

5. I.V. Lindell and A.H. Sihvola, "Perfect Electromagnetic Conductor," *Journal of Electromagnetic Waves and Applications*, **19**(7), 2005, pp. 861-869.

6. F.W. Hehl and Yu.N. Obukhov, *Foundations of Classical Electrodynamics*, Birkhäuser, Boston, 2003.

7. I.V. Lindell and A.H. Sihvola, "Realization of the PEMC Boundary," *IEEE Transactions on Antennas and Propagation*, **53**(9), September 2005, pp. 3012-3018.