Aggregate-functions MoM approaches for the analysis of complex bodies

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Abstract

In this communication we review some recent advances in aggregate-functions approaches, that are based on the construction of efficient MoM basis functions by suitably grouping standard (e.g. Rao-Wilton-Glisson, RWG) functions. The application domains, objectives and related means of achieving them can be significantly different for the various embodiments of the aggregate-functions concept. Compressive approaches can reduce the degrees of freedom of the problem up to allowing a direct, iteration-free solution; non-compressive multiscale approaches can act as effective pre-conditioners for iterative methods. For this reason aggregate-functions are well suited to solve antenna, EMC and scattering problems involving large and complex objects.

1. Introduction

The Integral Equation (IE) approach is a robust and general approach for solving electromagnetic problems through the Method of Moment (MoM) discretization scheme. It is well known, however, that standard techniques are severely limited by the matrix size and condition number involved in the problems of interest. In fact, the solution exhibits very different scales of variation; for examples, local interactions in a geometry, like sub-wavelength details, edges and discontinuities, generate small-scale details of high spatial frequency, while distant interactions as well as resonant lengths are responsible for the low-frequency, slow spatial variations. It is intuitive that standard basis functions do not possess different scales and therefore one is typically forced to choose mesh cells of size comparable to the smallest foreseen scale of the solution, i.e. with the highest possible spatial resolution. This leads to a large number of unknowns, densely populated MoM matrices with a poor condition number, and renders the direct approach of large problems numerically intractable.

In order to reduce the numerical complexity of the problem it is convenient to group basis functions into "aggregate functions", that are constructed in such a way to keep explicit information about the nature of the solution directly into the representation of the unknown fields/currents. Aggregation may reduce the degrees of freedom of a problem so that direct solution of the MoM linear system is attainable even for very large and/or complex problems.

In this communication, we will summarize the main features of two different approaches that lead to the generation of aggregate functions. These methods had been, in their basic traits, presented elsewhere (as reported in the appropriate Sections) and therefore in the next two sections they will be described only briefly, focusing on the common origins and on their efficiency when applied to the analysis of complex structures.

2. The Synthetic Function eXpansion (SFX) Approach

The Synthetic Function Expansion (SFX) method essentially consists in dividing the overall structure to be analyzed into portions, recognizing that the degrees of freedom of the field coupling between the solutions on these portions is limited, and building basis functions that exploit this property. The aggregate functions defined on any portion of the structure are called "Synthetic Functions" (SF). Since the basics of the method are assembled in 1, 1 here we will only briefly summarize the key aspects.

The SF generation procedure starts with the separation of the overall geometry into sub-structures, called ”blocks“ in the following. On each block, taken in isolation, it is possible to generate functions –called “Synthetic Functions”(SF)- that allow to correctly represent the actual current that is present on the complete structure with few of these. The SFs are chosen in the set of responses to sources placed around each block, that make up the space of all (rigorous) solutions, restricted to that block.

The responses to sources placed at the block feeding ports (if any is present) will be called ”natural“ SFs. They however are not sufficient to represent the actual current behavior when the block is embedded in the whole

structure: it is therefore necessary to introduce also the so called “coupling” SFs, generated locating sources around the block. The contour can be of any form: for simplicity circular and square box has been considered. Once the set of the solutions to the sources around the block is computed, the minimum number of necessary responses has to be determined. This can be done resorting to the SVD of the matrix that collects the responses from all the sources: the orthogonal functions obtained from the singular vectors will be the Synthetic Functions, while the sequence of the associate singular values gives the information on how many of them have to be used.

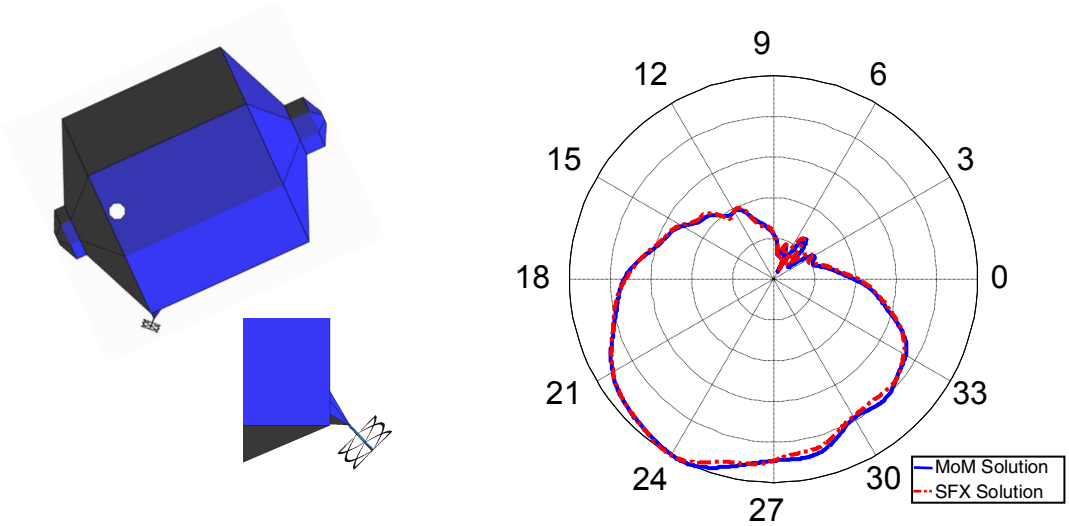

Figure 1. SFX validation vs. standard MoM for a medium-scale problem, the satellite body; inset: the TT&C antenna. Radiation pattern in plane of the figure (linear scale) (from [2]).

The SFX approach reduces the MoM matrix memory occupation, and considerably reduces the time needed to solve the linear system, without affecting the solution accuracy. When computing the interactions of SF on different blocks, the discretization detail can be reduced, leading to a further increase in numerical efficiency. As an example of application of the SFX to complex bodies, we have considered the structure shown in Fig. 2, consisting in a TT&C quadric-helix antenna mounted on a satellite; an enlarged picture of the body of the satellite with the antenna is also shown in Figs. 1 . The full satellite extends for over 30 wavelengths, and the structure has about 27,000 unknowns: for this reason the full structure could not be analyzed on a common computer like a Pentium IV workstation (2.66 GHz, 512 Mb RAM): we validated the solution on a portion of it, the one exactly shown in Fig. 1, with the results that show very good agreement. For the full satellite, the overall geometry was divided in 34 blocks and the largest block had 1277 unknowns; on use of SFX, the compressed linear system has less than 3,000 unknowns: the results are reported in Fig. 2. The maximum memory allocation at all times of the solution is less than 1/600 of that necessary to store the standard MoM matrix.

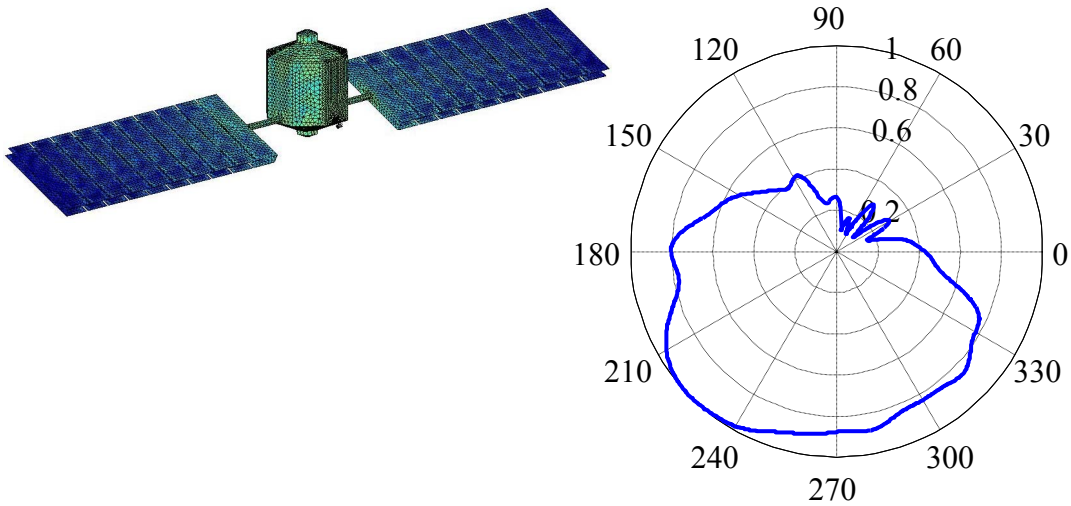

Figure 2. SFX solution for the entire satellite. Current distribution and radiation pattern in the plane orthogonal to the solar panels masts and passing through the antenna (from [2]).

3. The MultiResolution (MR) scheme

In the representation of the solution of an electro-magnetic problem, multi-resolution is a concept originating from signal processing, and the analogy between image processing (and compression) and the representation problem in electromagnetics. However, the intrinsic difficulties of generating and employing vector MR functions in three-dimensional problems, have up to now limited their application to practical electromagnetic problems. The work in 2, 3, 4 [7] overcomes these difficulties by introducing vector multiresolution functions that are constructed using the concepts of wavelet representation, yet keeping as much as possible of the ”physical“ information contained in the IE-MoM format.

The generation of the MR basis is approached dividing the unknown surface current into its solenoidal (TE) and non-solenoidal (quasi-TM, qTM) components: they can be mapped to scalar quantities that posses the same degree of regularity in both spatial directions, on which the introduction of wavelet-like constructs is easier. More specifically, the qTM part is related to the charge density, and the TE part is derived from the grid of a "solenoidal potential". The splitting of the current allows separating the singular, near-field behaviors that are of key importance for the conditioning of the MoM system matrix.

The MR scheme has been shown to be capable of drastically improving the condition properties of the associated MoM matrix, with definite advantages on the matrix sparsity and speed of convergence of iterative solvers, e.g. the conjugate gradient.

As an example, we have considered the structure shown in the inset of Fig. 3, consisting in a monopole antenna, excited with a voltage gap at its base, mounted on a ship, that is modeled as a PEC surface. The whole structure is discretized with 4442 unknowns and analyzed from 1 to 10 MHz. The main difficulties in the analysis of this structure are due to the very different mesh size between smooth surfaces and details of the antenna, especially at low frequencies. Fig. 3 shows the convergence of the iterative solver used to solve the resulting linear system: using the MR basis we obtain a faster convergence with respect to the Loop-Tree basis [6], introduced here as comparison, since for the RWG basis the solution is not reached in 4000 iterations.

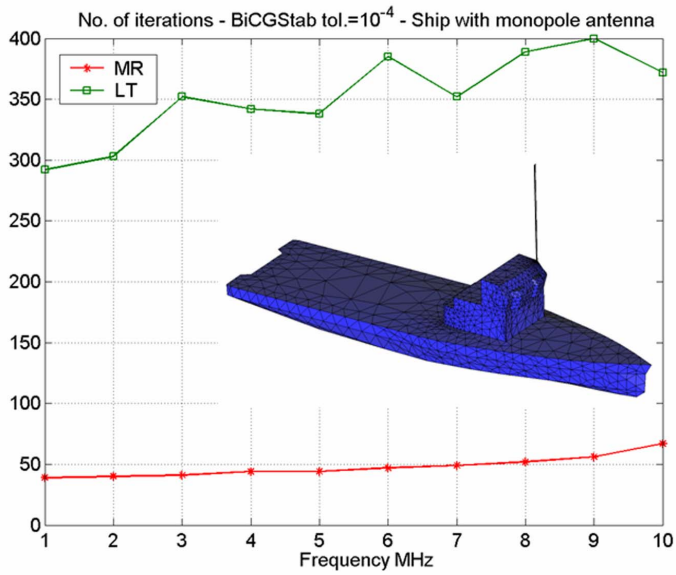

Figure 3. Antenna mounted on a ship. Convergence of the iterative solver on the whole frequency range with MR and Loop-Tree (LT) bases (from [7]).

4. Conclusion

In this communication we have summarized the main features of the SFX and MR approaches, that, even if in a different way, can strongly reduce the computational cost related to the solution of an electromagnetic problem by exploiting the basic idea of aggregation of standard basis functions. Since they can be used for analysis of 3D structures,

they are suitable or can be adapted to solve different kind of electromagnetic problems, ranging from antennas to scattering and compatibility problems.

5. Acknowledgments

This work is partially developed in the 7th Framework Program of the European Community within the HIRF-SE project.

6. References

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