NONQUADRATIC REGULARIZATION FOR ANTENNA MEASUREMENTS PROCESSING

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Abstract

The problem of determination of distribution of currents on antenna aperture plane from near-field measurements is considered. Possibility of use of extension minimum method for currents estimation problem is discussed, with results strongly outperforming standard quadratic regularization technique, when there is a priori information about finiteness of unknown support of currents on aperture plane. Results of numerical simulations for point sources and piecewise-constant currents distribution for plane scanning geometry are given.

1. Introduction

Problem of determination of currents distribution on the aperture plane on the basis of near-field measurements is a general problem in the field of antenna data processing. Results of such restoration can be used afterwards for antenna design purposes or for the problem of indirect estimation of antenna pattern on the basis of near-field measurements [1]. In the case of plane or cylindrical scanning geometries the finite size of the scanning surface introduces the so-called truncation error on the evaluated antenna pattern, leading to inaccurate reconstruction of currents distribution. In this case either measured data must be somehow extrapolated [2], or restoration algorithms should use a priori information about antennas measurements data (scanning geometry) and desired solution (antenna geometry). If a priori information about desired solution has no evident mathematical form it can not be accounted in practice when using traditional restoration techniques. In this paper we discuss novel approach for restoration of currents distribution with unknown, but finite support. We consider the problem of determination of currents distribution on the aperture plane from antenna measurements. In consideration we suppose that near-field antenna measurements are carried out on the plane surface, the area of measurements is finite and unknown currents distribution has finite, but unknown, support. For consideration we propose simple models of point sources located parallel and perpendicular to measurement plane and model of currents distribution in piecewise-constant form with unknown support.

It is obvious, that exact result of determination of currents distribution from antenna measurements can be obtained only for unlimited area of measurements in absence of measurement noise. But in practical applications the area of measurements is always limited and some noise is present. Thus, there are two main problems: 1) limited area of measurements, 2) noise in data. For solution of the first problem the so-called non redundant field representation was used in [2] for efficient extrapolation, however requiring explicit information about antenna under test (AUT) geometry. Another possible way is usage of properties of analyticity of the near field, which is caused by a finiteness of area of currents distribution that form a field of radiation (both near and far). Therefore, if the area of distribution of currents would be known exactly, the application of the prolate spheroidal wave functions (PSWF) should allow one to restore the distribution of currents according to measured data. Unfortunately, the area of currents distribution can be known only approximately, and the application of PSWF's expansion will produce an error. Under these conditions one can use the method of minimum of duration [3], suitable for extrapolation of band-limited signals and does not require a priori information about signals support, for spatial signals denoted as method of minimum of extension (MME). But in the case of near-field antenna measurements, the measured data cannot be considered as strictly band-limited data and the MME cannot be applied directly.

It is well known, that the problem of restoring of currents distribution from a measured part of the near field is ill-posed inverse problem. Hence, if the measured data is noisy, some regularization technique is required. Consider the case, when a priori information about finiteness of support of desired solution is present, but this support is unknown and can not be expressed mathematically. In this case application of traditional regularization techniques shows insufficient effectiveness, because they manipulate only energy and/or energy derivatives of desired solution and do not implement essentially important information about finiteness of support of desired solution. Instead of standard regularization technique we propose to use nonlinear regularization technique with additive term in the form of spatial extension of solution, which is formed according to extension minimum method. This method formulates some special function for calculation of signal duration or spatial extent, and allows taking into account the fact that measurement area is limited, the fact of finiteness of desired solution support, and the problem of noise in the measured data.

2. Proposed Regularization Technique

The problem we consider is the inverse problem of determination of distribution of currents from antennas measurements in the near field. For simplicity purposes the main considered equation for near-field in scalar case is given by:

$$b(\xi) = \int_{S_0} a(r) \frac{e^{-ik|\xi - r|}}{|\xi - r|} dr + N(\xi); \quad \xi \in M \,, \tag{1}$$

where M is an area of antennas measurement and S_0 is an unknown plane surface of currents distribution, or currents distribution support, on the whole plane surface S. The problem is to restore currents distribution $a(r)$ from known values of measured data $b(\xi)$, in the presence of additive noise $N(\xi)$.

Since considered inverse problem is ill-posed, some regularization technique must be applied in order to get stable solution. Following [4], instead of standard quadratic regularization based on usage of energy term, we propose to add an extra non-energy term in the form, which describes the extension minimum of the solution. Thus, the minimization problem is formulated in the following form:

$$\int_M |b(\xi) - \int_S a(r)h(\xi - r)dr|^2 \, d\xi + \gamma^2 \int_S \psi[|a(r)|; \alpha, \beta, ...]dr \to \min_{a(r)} , \tag{2}$$

where $S \supset S_0$, S_0 - unknown area, $h(r) = e^{-ik|r|} / |r|$ and γ^2 is the so-called regularization parameter. This formulation is applicable for point sources, arbitrarily located on the aperture plane. If currents distribution is supposed to have piecewise-constant form, appropriate functional of extension minimum for restoration of point sources must be modified to satisfy the idea of finiteness of derivative of currents distribution. The first term in (2) is an error norm over the area of antenna measurements; second term gives derivative of spatial extent as a regularization term. For solving of formulated problem it is necessary to find a function a(r), which would minimize (2). Function ψ, introduced in (2), indicates how many values of spatial signal derivative are essentially non-zeros. This function can be chosen in various forms, as reported in [4]. Here we use next one:

$$\psi[s(r); \alpha, \beta] = [|s(r)|^2 / A^2 + \alpha^2]^\beta - \alpha^{2\beta}; \quad A = \max|s(r)|; \quad 0 < \beta \le 0.5; \tag{3}$$

where $s(r)$ is some function, which in this consideration appears to be currents distribution. Parameters α, β of function ψ allow us to control its behavior. Numeric simulations show that good values of working parameters are: parameter α should be chosen as standard noise deviation; parameter β should be chosen as one-sixteenth. This formulation is applicable for point sources, arbitrarily located on the aperture plane. If currents distribution is supposed to have piecewise-constant form, appropriate functional of extension minimum for restoration of point sources must be modified to satisfy the idea of finiteness of derivative of currents distribution $\dot{a}(r)$.

For calculation of unknown current distribution $a(r)$, we can use either methods for nonlinear optimization or methods for solving nonlinear equations. The solution, which can be obtained by minimization of (2), is called extension minimum solution (EMS). From (3) follows, that if $\alpha^2 >> |a(r)|^2$ for all r, then EMS tends to solution, which is obtained by standard linear regularization method with additive energy term. Otherwise we have a difficult problem of nonlinear minimization with use of numerical methods. Some optimal value for γ^2 exists so that we would not get neither degenerate zero EMS (for very large γ^2) nor bad EMS without regularization (for very small γ^2).

3. Numerical Simulations

For numeric simulations we suppose that initial current distribution can be presented in the form of point sources and piecewise-constant form, one-dimensional measurement of near field is made with the spatial quantization step Δx. For unknown amplitudes of currents a_n following system of linear algebraic equation can be written:

$$b_m = \sum_{n=1}^{N} a_n \frac{\exp\left[\frac{-i2\pi}{\lambda}\sqrt{(x_n - x_m)^2 + (y_n - y_m)^2 + z^2}\right]}{\sqrt{(x_n - x_m)^2 + (y_n - y_m)^2 + z^2}}, m = 1, 2, \ldots, N \tag{4}$$

where b_m are data measured in points with coordinates x_m, y_m on plane surface, z is distance between AUT and measurement plane, x_n, y_n are coordinates of radiating point sources.

Numeric simulation has been made for one-dimensional case for following values of parameters: N = 100, $\Delta x/\lambda$ = 0.1, z/λ = 3, a_n = 1 for n = 40, 50, 60, 70, otherwise a_n = 0. Data was corrupted by Gaussian additive noise with SNR of +20 dB. Results of currents restoration obtained by least squares method with traditional regularization of quadratic type and by extension minimum method are shown on Fig. 1, a. Obtained amplitudes for least squares solution differ from true values strongly – round mean squared error (RMSE) is equal to 69%, while EMS is comparatively exact with RMSE of 0.87%. Results for case of piecewise-constant distribution with parameters N = 100, $\Delta x/\lambda$ = 0.1, z/λ = 3, a_n=1 for n = 30,…,40 and n = 70,…,75, otherwise a_n=0 are shown on fig. 1,b with RMSE of 19% for LSQ and RMSE of 0.72% for EMS.

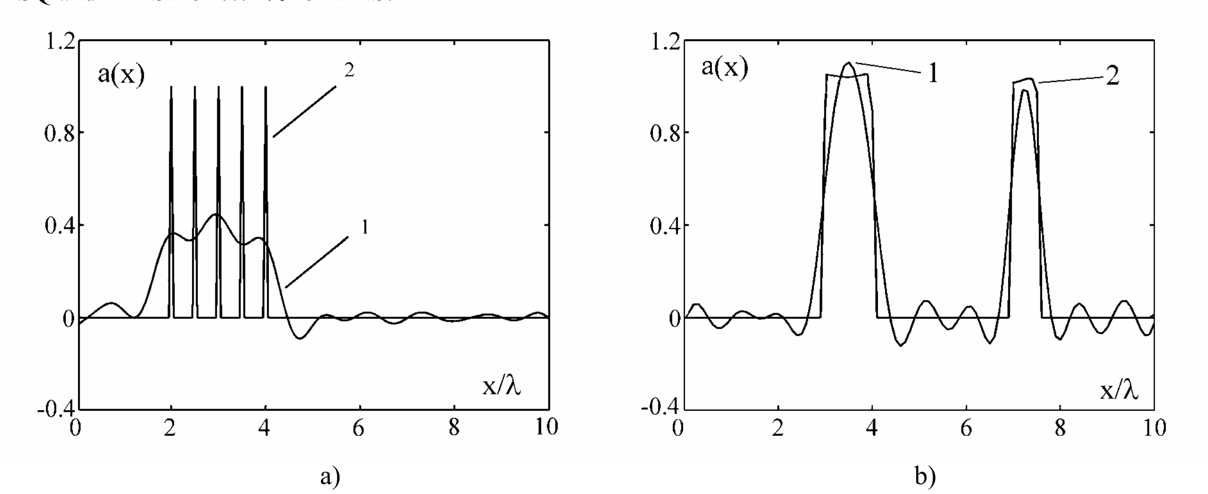

Fig.1. (a) point sources restoration; (b) piecewise-constant currents restoration; (1 – least squares solution, 2 – EMS)

5. Conclusion

Developed nonquadratic regularization technique, based on the method of minimum extension, gives possibility to obtain improved results for restoration of point sources and piecewise-constant currents distributions on antenna aperture plane when compared to traditional least squares based techniques. Functional of minimum of extension introduces non-energy term in the form of spatial extent of currents distribution in order to capture desired finiteness of the solution. Further research can be concentrated on introduction of more appropriate models for radiating sources and investigation of behavior of proposed functional from its parameters.

6. References

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