New Basis Functions for the Electromagnetic Solution of Arbitrarily-shaped, Three Dimensional Material Bodies Using Method of Moments

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Abstract

We present two sets of basis functions, defined over a pair of planar triangular patches, for the solution of electromagnetic scattering and radiation problems associated with arbitrarily-shaped conducting/dielectric surfaces using the method of moments solution procedure and triangular patch modeling. The basis functions are constant over the function subdomain and resemble pulse functions in one and two dimensional problems. Further, these two sets of basis functions are point-wise mutually orthogonal. The primary objective of developing these basis functions is to utilize them for obtaining a stable and robust electromagnetic solution involving conducting, dielectric, and composite bodies.

1. Introduction

The solution of electromagnetic scattering/radiation problems involving arbitrary shapes and material composition is of much interest to commercial as well as defense industries. The method of moments (MoM) solutions to these problems generally involve triangular patch modeling and utilize Rao-Wilton-Glisson (RWG) basis functions [1]. It may be noted that the RWG basis functions have been primarily defined for the solution of conducting bodies and the utilization of the same basis functions for dielectric/composite bodies is less than satisfactory. The primary difficulty associated with a material body solution is the requirement of two orthogonal basis functions to express unknown electric and magnetic currents $J$ and $M$. In our opinion, using the same basis functions for both $J$ and $M$ may be problematic and invariably results in numerical difficulties. However, a host of techniques have been developed which involve either tinkering with basis functions or modifying the testing procedures to apply for material bodies [2, 3, 4]. Keeping these difficulties in perspective, in this work we develop two sets of basis functions, each one point-wise orthogonal to the other function, which can be used for conducting as well as material bodies. This combination ensures strongly diagonal MOM matrices. The basis functions are general and suitable for all different formulations such as Electric Field Integral Equation (EFIE), Magnetic Field Integral Equation (HFIE), and combined field formulations (CFIE and others).

2. Description of the Problem

Consider an arbitrarily shaped, dielectric body described by a surface $S$ as shown in Fig. 1. The scatterer has material parameters of $\mu_d$ and $\epsilon_d$ while exterior to the body is a homogeneous medium with parameters $\mu_e$ and $\epsilon_e$. Exterior to the body, the total fields are designated by $E_e$ and $H_e$, while interior to the object, the fields are given by $E_d$ and $H_d$. The structure is illuminated by an arbitrary electromagnetic plane-wave. Also, we note that the tangential components of the electric and magnetic fields must be continuous at the dielectric interface $S$ as dictated by the boundary conditions.

Employing the equivalence principle, the body may be replaced with two sets of electric ($J_e$ and $J_d$) and magnetic ($M_e$ and $M_d$) currents. Each set radiates in an infinite homogeneous medium having constitutive parameters associated with medium “$e$” or “$d$”. It can be easily proved that the continuity of the tangential fields requires that

\[
J_e = -J_d \equiv J, \\
M_e = -M_d \equiv M.
\]  

Using potential theory, the scattered fields radiated by the equivalent electric and magnetic currents may be written in terms of potential functions as

\[
E^s_\nu[J, M] = \mp j \omega A_\nu \mp \nabla \Phi_\nu \mp \frac{1}{\epsilon_\nu} \nabla \times F_\nu, \\
H^s_\nu[J, M] = \mp j \omega F_\nu \mp \nabla \Phi_\nu^{\text{m}} \mp \frac{(-1)}{\mu_\nu} \nabla \times A_\nu,
\]
where $A_\nu$ and $F_\nu$ are the magnetic and electric vector potentials, respectively, and $\Phi^e_\nu$ and $\Phi^m_\nu$ are the electric and magnetic scalar potentials, respectively, given by

$$A_\nu(r, t) = \frac{j\nu}{4\pi} \int_S \mathbf{J} e^{-jk_\nu R} \frac{1}{R} dS', \quad (5)$$
$$F_\nu(r, t) = \frac{\epsilon\nu}{4\pi} \int_S \mathbf{M} e^{-jk_\nu R} \frac{1}{R} dS', \quad (6)$$
$$\Phi^e_\nu(r, t) = \frac{1}{4\pi\epsilon\nu} \int_S q^e_s e^{-jk_\nu R} \frac{1}{R} dS', \quad (7)$$
$$\Phi^m_\nu(r, t) = \frac{1}{4\pi\mu\nu} \int_S q^m_s e^{-jk_\nu R} \frac{1}{R} dS' \quad (8)$$

for $\nu = e$ or $\nu = d$, the “−” sign is for $\nu = e$, and the “+” is for $\nu = d$. In Eqs. (5)-(8), $R = |r - r'|$, the distance from the field point, $r$, to the source point, $r'$. The electric and magnetic surface charge density, $q^e_s$ and $q^m_s$, respectively, are related to the electric and magnetic surface current density by the continuity equations

$$\nabla_s \cdot \mathbf{J} = -j\omega q^e_s, \quad (9)$$
$$\nabla_s \cdot \mathbf{M} = -j\omega q^m_s \quad (10)$$

Considering the external problem, as shown in Fig. 2, and enforcing the continuity of tangential electric and magnetic fields at the mathematical surface $S$, we obtain

$$[\mathbf{E}_e^+ + \mathbf{E}_e^-]_{\text{tan}} = 0, \quad r \in S^- \quad (11)$$
$$[\mathbf{H}_e^+ + \mathbf{H}_e^-]_{\text{tan}} = 0, \quad r \in S^- \quad (12)$$

where $S^-$ refers to the points located just inside the surface. Similarly, considering the internal problem, as shown in Fig. 3, and enforcing the continuity of tangential electric and magnetic fields, we obtain

$$[\mathbf{E}_m^+ + \mathbf{E}_m^-]_{\text{tan}} = 0, \quad r \in S^+, \quad (13)$$
$$[\mathbf{H}_m^+ + \mathbf{H}_m^-]_{\text{tan}} = 0, \quad r \in S^+, \quad (14)$$

where $S^+$ refers to the points located just external to the surface.

The equations (11)-(14) are the needed equations that may be combined selectively to obtain the necessary integral equations for a given problem. For a Perfect Electric Conductor (PEC) case, we note that only the (11) needs to be solved by assuming null internal fields and null magnetic currents. However, for a dielectric body problem, one can use (11) and (13) to obtain the EFIE, or (12) and (14) to obtain the HFIE, which eventually lead to other formulations as well. In the following section, we describe the basis functions to be utilized in the MoM solution procedure for solving the integral equations.

### 3. Description of Basis Functions

To begin the method of moments solution procedure, we define the following basis functions to express the equivalent electric and magnetic currents. Assuming a triangular mesh to model the body, let $T_n^+$ and $T_n^-$ represent two triangles connected to the $n^{th}$ edge as shown in fig. 4. The edges of each triangle other than the $n^{th}$ edge we will call “free” edges. Within each triangle, the surface is planar. We define two mutually orthogonal vector basis functions associated with the $n^{th}$ edge as

$$f_n(r) = \begin{cases} a_n^+ \times \hat{\ell}, & r \in S_n, \\ 0, & \text{otherwise}. \end{cases} \quad (15)$$
$$g_n(r) = \begin{cases} \hat{\ell}, & r \in S_n, \\ 0, & \text{otherwise}. \end{cases} \quad (16)$$

In (15) and (16), $S_n$ represents the region obtained by connecting the mid-points of the free edges to the centroids of triangles $T_n^\pm$ and to the nodes of edge $n$. Note that this area is shown shaded in the Fig. 4. Also, $\hat{\ell}$ and $n^\pm$ represent the unit vector along the $n^{th}$ edge and the unit vector normal to the plane of the triangle $T_n^\pm$, respectively. Note that the basis functions defined in (15) and (16) are actually pulse functions defined over the region $S_n$. The numerical details to derive the matrix equations are omitted here for the sake of keeping the paper length within prescribed limits.
4. Numerical Results

In this section, we present numerical results obtained using the present approach for a sphere (diameter = 0.18λ) placed at the center of the coordinate system and illuminated by x-polarized plane wave traveling in the −z direction. We consider both the PEC case and the dielectric case (ε_r = 5.0). The sphere is modeled with 500 triangular patches. The results are compared with the exact solution. The bistatic radar cross sections for PEC and dielectric cases are presented in Figures 5 and 6, respectively. We note that the results compare well with the exact solution.

5. Conclusions

In this work, we present a new set of basis functions for the method of moment solution of electromagnetic scattering by material bodies of arbitrary shape. The numerical method generates robust moment matrices applicable to all standard formulations such as EFIE, HFIE, and CFIE.

References


Figure 3: Internal problem.

Figure 4: Basis function description.

Figure 5: Bistatic RCS of a conducting sphere.

Figure 6: Bistatic RCS of a dielectric sphere.