Probabilistic Study of Fully Stochastic Electromagnetic interactions: Coupling between a stochastic PEC plate and an stochastic incident Plane Wave

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Abstract

Uncertainties affecting electromagnetic interactions can alter the accuracy of simulation models. A stochastic method is proposed to measure these uncertainties in problems modeled by integral equations. The first statistical moments of the interaction output parameters are efficiently computed by quadrature. This approach is applied to the study of the voltage induced on a dipole by a stochastic plane wave, in presence of a randomly varying PEC plate.

1. Introduction

Real-life electromagnetic interactions are often affected by uncertainties. These uncertainties may be due to uncontrolled changes caused by vibrations, ageing or fatigue. Such is for instance the case when studying scattering phenomena between objects of varying or unknown shape (SAR antenna, human head), together with incident electromagnetic fields of varying properties (propagation in random media). In order to model such situations, these uncertainties need to be accounted for. A systematic study of every possible configuration is extremely costly if not impossible. Another approach consists in considering the variations of the input parameters as random. The output parameters resulting from the electromagnetic interaction, also known as "observables", then become random variables. The theory of probability then allows to measure the uncertainty of the observables, either by their probability distribution, which is the ideal measure, but difficult to determine explicitly, or by the statistical moments of the observables which are computable.

Such an approach is common in mode-stirred chamber theory, to describe the random illumination incident on deterministic objects as is done by Hill [1]. Bellan and Pignari applied a probabilistic method to study a deterministic wire structure under random plane-wave illumination by using transmission-line theory [2]. Sy et.al have proposed a stochastic approach to tackle uncertainties in electromagnetic problems modeled by integral equations [3]. They determine the mean and the variance of the voltage induced at the port of a randomly undulating thin-wire structure illuminated by a deterministic plane wave.

The aim of the present paper is to extend the latter approach to study electromagnetic interactions where uncertainties appear both in the geometry of the scattering device and in the incident field. The problem is modeled by an integral-equation method, and the electromagnetic interaction is observed via the voltage induced at the port of a receiving device: this voltage is therefore the observable. A deterministic quadrature rule is then used to compute the average and the standard deviation of the voltage in an efficient manner. This approach is illustrated through the study of an elementary dipole on top of one of the corners of a randomly undulating metallic plate.

2. Description of the setup

The electromagnetic interaction studied in involves a scattering device of finite extent, and an electromagnetic field representative of the sources present in the environment of the scatterer. The interaction is observed from a port region as depicted in Figure 1. The scattering device is a perfect electric conductor (PEC) represented by its boundary surface denoted S which is described by using a normal parameterization of its height $z_{\boldsymbol{\alpha}}$ above a fixed domain D

$$z_\alpha(x, y) = \alpha_1 \sin(2\pi x)\cos(4\pi y), \tag{1}$$

where $x\in[0;L_X]$ and $y\in[0;L_Y]$. The domain D is chosen as a rectangle $[0;L_X]\times[0;L_Y]$ lying in the horizontal plane Oxy. The function z_α depends on a set of parameters gathered in the vector $\boldsymbol{\alpha}$ which belongs to a fixed domain $\Omega_{\boldsymbol{\alpha}}$ where it varies. In order to explicitly indicate its dependence on the parameter $\boldsymbol{\alpha}$, the surface S is denoted $S(\boldsymbol{\alpha})$.

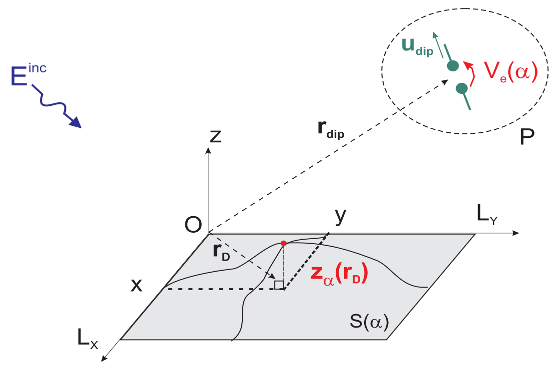

Figure 1. Interaction configuration: Scattering surface S($\boldsymbol{\alpha}$) and observation region P

The effect of external electromagnetic sources on S($\boldsymbol{\alpha}$) is represented by the incident field denoted $\mathbf{E}^{\mathbf{i}}$, which is the electric field radiated by the external sources in absence of S($\boldsymbol{\alpha}$). Since S($\boldsymbol{\alpha}$) is assumed to be far from the external sources, $\mathbf{E}^{\mathbf{i}}$ can be represented as a plane wave. All the parameters of the incident field are put in the vector β which belongs to the fixed domain $\Omega_{\boldsymbol{\beta}}$. These parameters are for instance the amplitude, the polarization angle, or the direction of incidence of $\mathbf{E}^{\mathbf{i}}$. The incident field is coined $\mathbf{E}_{\boldsymbol{\beta}}^{\mathbf{i}}$ to mark its dependence on the input parameters $\boldsymbol{\beta}$. An interaction configuration C is therefore fully specified by the definition of the vector $\boldsymbol{\gamma} = (\boldsymbol{\alpha} \oplus \boldsymbol{\beta})$ which gathers the parameters $\boldsymbol{\alpha}$ of the scattering device S($\boldsymbol{\alpha}$) , and the parameters $\boldsymbol{\beta}$ of the excitation $\mathbf{E}_{\boldsymbol{\beta}}^{\mathbf{i}}$.

The port P, which is assumed to be fixed, represents the region from which the electromagnetic interaction is observed. It consists of the center of an elementary Hertzian dipole of small extent L_{dip}, located at the fixed position $\mathbf{r_{dip}}$, and oriented in the direction of the unit vector $\mathbf{u_{dip}}$. The observable chosen here to describe the effect of $\mathbf{E}_{\boldsymbol{\beta}}^{\mathbf{i}}$ on S($\boldsymbol{\alpha}$) is the voltage $V_e(\boldsymbol{\gamma})$ induced at the port P. In circuit theory, this voltage can be taken as an equivalent network to represent the entire intricate configuration C consisting of S($\boldsymbol{\alpha}$) and $\mathbf{E}_{\boldsymbol{\beta}}^{\mathbf{i}}$. The voltage V_e results from the superposition of the voltage $V_{i;e}(\boldsymbol{\beta})$ due to the direct incidence of $\mathbf{E}_{\boldsymbol{\beta}}^{\mathbf{i}}$ at P, and of the voltage $V_{s;e}(\gamma)$ induced by the field scattered by S($\boldsymbol{\alpha}$)

$$V_e(\gamma) = V_{i;e}(\beta) + V_{s;e}(\gamma)\,. \tag{2}$$

Since the port P is assumed to be fixed, the voltage $V_{i;e}$ solely depends on the parameters $\boldsymbol{\beta}$ of $\mathbf{E}_{\boldsymbol{\beta}}^{\mathbf{i}}$. On the other hand, the voltage $V_{s;e}(\gamma)$ is defined as is done in [3] through the reaction integral

$$V_{s;e}(\gamma) = \int_{S(\alpha)} j_\alpha(r) \cdot E^i_\beta(r)\,, \tag{3}$$

where he current distribution $\mathbf{j}_{\boldsymbol{\alpha}}$ is the *transmitting-state current*: it is induced on S($\boldsymbol{\alpha}$) in absence of the incident field $\mathbf{E}_{\boldsymbol{\beta}}^{\mathbf{i}}$, when a unit current source I_P = 1A is applied at the port P. This voltage $V_{s;e}$ depends on $\boldsymbol{\alpha}$, through $\mathbf{j}_{\boldsymbol{\alpha}}$ and the integration support S($\boldsymbol{\alpha}$), and on $\boldsymbol{\beta}$ via $\mathbf{E}_{\boldsymbol{\beta}}^{\mathbf{i}}$. The current density $\mathbf{j}_{\boldsymbol{\alpha}}$ is obtained by solving a boundary-value problem via the method of moments. In the transmitting state, in absence of the incident field $\mathbf{E}_{\boldsymbol{\beta}}^{\mathbf{i}}$, the current source I_P = 1A applied at the port P is the driving source and it radiates an electric field denoted E[I_P] which induces the current $j_{\boldsymbol{\alpha}}$ on S($\boldsymbol{\alpha}$). The corresponding electromagnetic boundary-valued-problem can be modeled by an equivalent Electric-Field Integral Equation (EFIE). This equation is solved in its discrete form by a method of Galerkin which uses a set of Rao-Wilton-Glisson (RWG) basis functions defined on S($\boldsymbol{\alpha}$). This solution time can therefore not be neglected. Due to the variations of the input parameters $\boldsymbol{\gamma}$, multiple evaluations of the observable $V_e(\gamma)$ can be very expensive in computation time. This is the main motivation behind the development of the stochastic approach presented in the following section.

3. Stochastic parameterization of the problem

In the stochastic approach, the variations of the parameters $\boldsymbol{\gamma}$ of the configuration C(γ) in the domain Ω_γ are viewed as random. The theory of probability then allows measuring the uncertainty induced on Ve($\boldsymbol{\gamma}$) by the $\boldsymbol{\gamma}$. The key assumption is to consider the vector $\boldsymbol{\gamma}$ to be randomly distributed in Ω_γ according to a known probability distribution p_γ. The choice of p_γ is crucial as it measures the uncertainty of $\boldsymbol{\gamma}$ in its domain Ω_γ. Given the setup, it is sensible to consider the geometry's parameters $\boldsymbol{\alpha}$ as being statistically independent from the parameters $\boldsymbol{\beta}$ of the incident field.

The random fluctuations of $\boldsymbol{\gamma}$ in Ω_γ, according to the known law p_γ, induce random fluctuations of $V_e(\boldsymbol{\gamma})$ in a domain Ω_{Ve} , according to an as yet unknown law p_{Ve} . The behavior of V_e in Ω_{Ve} would be fully characterized by p_{Ve}. Nonetheless, given the definition of V_e in Equations (2) and (3), it is usually not possible to explicitly express p_{Ve} in terms of p_γ. Alternatively, it is possible to aim for the statistical moments of V_e which are computable. The average $\mathbb{E}[V_e]$ represents the weighted center of all the values of V_e, and is defined as

$$\mathrm{E}[\mathrm{V_e}] = \int_{\Omega_\gamma} \mathrm{V_e}(\gamma)\, p_\gamma(\gamma')\, d\gamma' . \tag{4}$$

On the other hand, the standard deviation σ[Ve] quantifies the dispersion of the values of Ve about $\mathbb{E}[V_e]$

$$\sigma[\mathrm{V_e}] = \sqrt{\mathrm{E}[|\,\mathrm{V_e}\,|^2] - |\,\mathrm{E}[\mathrm{V_e}]\,|^2} . \tag{5}$$

These integrals can be computed numerically, since they are integrals over a known integrand $V_e(\boldsymbol{\gamma})p_\gamma(\boldsymbol{\gamma})$ and a known domain of integration Ω_γ. The computation of these integrals is performed by using a deterministic quadrature rule as detailed in [3]. The results provided by this rule are compared to those of a Monte-Carlo rule taken as reference.

4. Results

The method presented above is applied to a practical setup studied at several frequencies between 50 MHz and 200 MHz. The PEC scattering surface $S(\boldsymbol{\alpha})$ is parameterized as in Equation (1) with respect to the square D with edges of length 1 m. The varying parameter $\boldsymbol{\alpha}=\alpha_1$ is assumed to be randomly distributed in the domain $\Omega_{\alpha 1}$ = [-20; 20] cm according to a uniform probability distribution. The average surface thus corresponds to α_1 = 0 cm and is the surface D. The plate is meshed into 200 triangules leading to the definition of 280 RWG functions for the method of moments. The voltage V_e is defined at the center of an elementary electric dipole located at $\mathbf{r_{dip}}$ = (1,1,0.5) m. This dipole has a length L_{dip} = 2 cm, and is oriented along the vector $\mathbf{u_{dip}}$ = (1,1,1). The computation of the voltage $V_e(\boldsymbol{\gamma})$ for a single configuration $C(\boldsymbol{\gamma})$ takes approximately 4 seconds.

4.1 Deterministic incident field

A deterministic incident is first considered in the form of a plane wave $\mathbf{E_\beta^i}$, with an electric field of amplitude E_0 =1 V/m, a horizontal polarization, and a direction of propagation defined by the angles θ_i = 45 degrees and φ_i = 270 degrees: $\mathbf{E_\beta^i}$ comes from the "left" of the plate in the plane Oyz. Since this incident field is fully deterministic, the uncertainty in Ve stems from the random surface $S(\boldsymbol{\alpha})$. The mean and variance of Ve are thus coined $E[Ve]_{\boldsymbol{\alpha}}$ and $\sigma[Ve]_{\boldsymbol{\alpha}}$ to highlight the fact that the only random input is $\boldsymbol{\alpha}$.

The module of $E[V_e]_{\boldsymbol{\alpha}}$ is depicted in Figure 2, where it is compared to the module of the voltage V(0) corresponding to the average configuration S(0)=D. This plot highlights the similarity between $|E[V_e]_{\boldsymbol{\alpha}}|$ and $|V(0)|$, mainly for f 2 [60; 150] MHz. In this frequency domain, the standard deviation $\sigma[Ve]_{\boldsymbol{\alpha}}$, plotted in Figure 3, indicates that the dispersion of V_e, represents less than 1% of the value of $|E[V_e]_{\boldsymbol{\alpha}}|$. Depending on the application or on the device connected at the port of the dipole, this can be considered as a negligible dispersion of V_e, and, the study of the unperturbed configuration may thus grant a suitable description of V_e. For f >150 MHz, the difference between $|E[V_e]_{\boldsymbol{\alpha}}|$ and $|V(0)|$ increases indicating that the effect of the random surface $S(\boldsymbol{\alpha})$ on the dipole is more pronounced. This effect is confirmed by the rise in the standard deviation which is however still limited to approximately 1% of the order of magnitude of $|E[V_e]_{\boldsymbol{\alpha}}|$. This rise may be due to the presence of resonance phenomena around 170 MHz which increase the coupling between the stochastic surface $S(\boldsymbol{\alpha})$ and the dipole. Moreover the fact that the dipole is on top of a corner of the surface, which is a geometrical singularity, plays also a role in the enhancement of the coupling above 170 MHz.

For each frequency, the statistics were computed by using 100 samples in the Monte-Carlo method (in 440 seconds) and 6 samples in the deterministic quadrature rule (in 25 seconds) to achieve the same precision. Both methods converged and yielded results which matched very well over the entire range of frequencies [50; 200] MHz. For such a problem, the deterministic quadrature rule seems to be the best choice with respect to the computation time.

4.2 Stochastic incident field

A stochastic incident field is then considered which has the same characteristics as the deterministic incident field except for its direction of incidence, where the azimuth angle $\beta_1 = \varphi_i$ is uniformly distributed in the domain $\Omega_{\beta 1}$ = [180; 360] degrees. The average value of β_1 is therefore 270 degrees, which was the azimuth angle of the deterministic field used in the previous section. In this case, the stochastic nature of V_e is caused by the randomness of $\boldsymbol{\alpha} = \alpha_1$ and $\boldsymbol{\beta} = \beta_1 = \varphi_i$. For this reason the corresponding statistics are written with the index "$\boldsymbol{\alpha};\boldsymbol{\beta}$".

As can be seen in Figure 2, the module of the average $|E[V_e]_{\boldsymbol{\alpha};\boldsymbol{\beta}}|$ clearly differs from $|V(0)|$ and $|E[V_e]_{\boldsymbol{\alpha}}|$, meaning that the average configuration can no longer be taken as a model for the randomly varying electromagnetic interaction. Nonetheless, $|E[V_e]_{\alpha}|$ and $|E[V_e]_{\alpha;\beta}|$ have similar shapes as a function of the frequency. This suggests that the global effect of the randomness of the incident field consists in an offset approximately equal to 4 mV of the value of $|E[V_e]|$. This variation of 40% in the value of $|E[V_e]|$ is non-negligible. Regarding the standard deviation, Figure 3 points out the fact that the random incident direction leads to a clear increase in the dispersion of the values of V_e around $E[V_e]$: the standard deviation $\sigma[Ve]_{\alpha;\beta}$ is approximately 2 orders of magnitude higher than $\sigma[Ve]_{\alpha}$. From Equation (2), it appears that the randomness of the incident field translates into the randomness of $V_{i;e}$ which corresponds to the direct interaction between the incident field and the dipole. The results displayed in Figures 3 and 4 therefore show that the contribution of $V_{i;e}$ to the randomness of V_e dominate that of $V_{s;e}$.

For each frequency, the statistics were computed by using 150 samples in the Monte-Carlo method (in 620 seconds) and 8*8 samples in the deterministic quadrature rule (in 260 seconds) to reach the same level of convergence and accuracy. The results obtained by both methods match again very well over the entire range of frequencies.

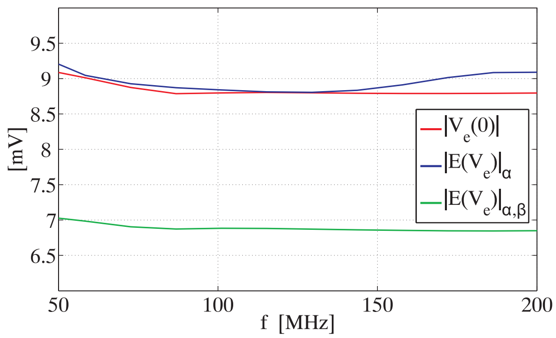

Figure 2. Module of the average $|E[V_e]|$: $|E[V_e]_{\alpha}|$ for the deterministic incident field, $|E[V_e]_{\alpha;\beta}|$ for the random incident field

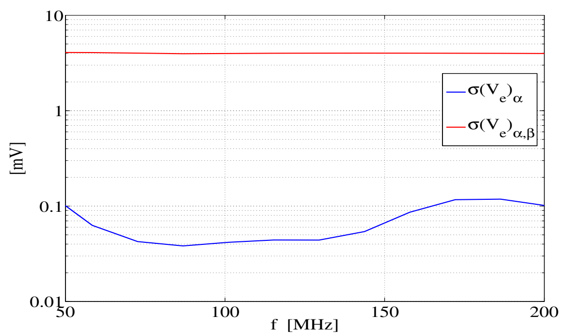

Figure 3. Standard deviations $\sigma(Ve)$: $\sigma[Ve]_{\alpha}$ for the deterministic illumination; $\sigma[Ve]_{\alpha;\beta}$ for the random illumination

5. Conclusion

A probabilistic approach has been presented to statistically quantify uncertainties affecting stochastic electromagnetic interactions modeled by integral equations. This approach tackles fully stochastic problems where both the scattering obstacle and the excitation are random. This method hinges on the use of an efficient deterministic quadrature rule to compute the average and the variance of the variables of interest. The results of this rule have been compared to those of a classical Monte-Carlo quadrature with which they are in good agreement. Encouraging results have been obtained in the case of the interaction between an elementary dipole placed close to a randomly undulating plate and illuminated by a random incident plane wave. The applicability of this method to higher dimensional problems depends on the availability of numerically efficient quadrature rules.

6. Acknowledgments

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7. References

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