Use of Generalized Extreme Value distribution to model the maximums of the field inside a reverberation chamber

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Abstract

The Generalized Extreme Value (GEV) distribution is used to model the electric field inside a Reverberation Chamber (RC). GEV distribution is particularly indicated for this application where the maximums of the field are studied, as it specifies the theoretical distribution of the maximums according to the Fisher-Tippett theorem. In a first part of this paper we present the Fisher-Tippett theorem and the GEV distribution. Then the three parameters of the GEV distribution are estimated from measured maximum values of the field in a small RC equipped with two stirrers.

1. Introduction

Reverberation chambers (RCs) are tools used for electromagnetic compatibility tests. An RC is a metallic enclosure equipped with a mechanical stirrer that modifies the geometrical configuration. For each stirrer position, the electric field is composed of eigenmodes, each of these standing waves presenting nodes and antinodes. The nonuniformity of the resulting field can be statistically suppressed, considering the maximum of the field for a stirrer rotation. The probabilistic description of the field has been proposed by Hill [1] for the electrically large (or overmoded) RC and has been validated [2]: The rectangular electric field components $E_R$ follow a Rayleigh distribution function (DF).

A useful application of this model is the determination of the field uniformity, as defined in the 61000-4-21 norm [3]. First, $E_{R_{\text{max}}}$, which are the maximums of $E_R$ within a stirrer displacement, are collected, and then the uniformity estimator $S$ is defined as the normalized standard deviation of $E_{R_{\text{max}}}$. The statistics of $E_{R_{\text{max}}}$ can be predicted by the use of extreme order statistics as soon as the $E_R$ distribution, which is also called the parent DF, is known.

Because the parent distribution is not always known (such as for the undermoded RC operated at low frequencies), it can be helpful to apply the Fisher-Tippett theorem that gives the three asymptotic possible DFs of the maximums. This technique has been applied in [4] to model the extreme power consumption in a VLSI circuit. As an extension, we have adopted in [5] the generalized extreme value (GEV) DF that encompasses these three limiting DFs. The results of [5] were related to a modeled RC, the ones of this paper concern a small RC whose volume is $1.3m^3$.

The outline of this paper is as follows. In Section 2, the results of the classical use of extreme order statistics in relation to an RC are recalled, followed by presentations of both the Fisher–Tippett theorem and GEV DF. In Section 3, measurement are performed and the maximums are modeled by GEV.

2. Theoretical background

2.1 Uniformity test derived from the study of the maximums

The calibration procedure [3], which leads to an estimation of the RC field uniformity, is based on the determination of the maximum field over a stirrer displacement. To better understand the consequence of this choice and introduce our modeling, we recall how the uniformity estimator $S$ is defined. Then, the well known theoretical model for the overmoded RC, making use of extreme order statistics, is presented. To begin with, the electric field
magnitude is determined at eight points located at the working volume corners. For each rectangular component \( E_R \) of a given point, \( E_{R\text{ max}} \) corresponds to the maximum value over \( N \) independent stirrer positions. From these \( 8 \times 3 \) rectangular components \( E_{R\text{ max}} \) the field uniformity is evaluated by calculating the mean value \( <E_{R\text{ max}}> \) and the empirical standard deviation \( \sigma_{E_{R\text{ max}}} \). Then, the normalized dispersion \( S \) is defined as

\[
S = 20 \log_{10} \left( 1 + \frac{\sigma_{E_{R\text{ max}}}}{<E_{R\text{ max}>}} \right).
\]

The 61000-4-21 norm states that uniformity is achieved if \( S \) is inferior to 3dB.

The theoretical determination of \( S \) is based on the use of extreme order statistics, whose principle is the following: Given \( F \), the hypothesized cumulated density function (cdf) of the field, or parent DF, it is easy to show [2, Sec. 3.2.6] that the cdf \( G \) of the maximum among \( N \) values is

\[
G = F^N.
\]

For the overmoded RC, parent DFs are well known: \( E_R \) and \( E_R^2 \) follow Rayleigh and exponential distributions, respectively. Making use of (2), it is then possible to express the mean value and the dispersion of \( G \) as functions of \( N \). If the aim is to model the maximum of \( E_R \), these expressions are in [2, Tab. 4]: For instance \( N = 20 \) independent samples lead to \( S \approx 1.37 \) dB. In the case of the undermoded RC, this method cannot be applied since no modeling of the parent distribution is currently accepted. To circumvent the \( E_R \) modeling issues, we use an asymptotic expression of (2) that is valid for both overmoded and undermoded RCs.

### 2.2 Fisher-Tippett theorem

In the same way that the central limit theorem gives the asymptotic probability density function of sample average, the Fisher–Tippett theorem [6] indicates the possible limiting cdfs \( G^o \) of normalized sample maximums that are independent and identically distributed. This theorem states that \( G^o \) can be only one among the three types [7, pp. 51–52]:

\[
G_1(x) = \begin{cases} 
0 & \text{if } x \leq 0 \\
\exp[-x^{-\alpha}] & \text{if } x > 0
\end{cases}
\]

\[
G_2(x) = \begin{cases} 
\exp[(-x)^{\alpha}] & \text{if } x \leq 0 \\
1 & \text{if } x > 0
\end{cases}
\]

\[
G_3(x) = \exp[-e^{-x}] \quad \forall x \in \mathbb{R}
\]

where \( \alpha > 0 \) is the tail index. Equations (3a)–(3c) are the asymptotic cdfs of the maximums of \( X \). Conditions expressed in [7, pp. 51–52] allow a determination of the type of limiting distribution for usual DFs:

1) \( G_1 \) is Fréchet DF. Convergence towards \( G_1 \) corresponds to “fat tail” parent DF, e.g., Pareto or Cauchy DF. The terminology “fat tail” comes from the visual inspection of the cdf plot: If the probability of extreme values is particularly high, the cdf converges slowly towards the asymptotic value 1, yielding a fat tail.

2) \( G_2 \) is the reverse Weibull DF. Convergence towards \( G_2 \) corresponds to thin-tailed parent DF with a finite upper end point such as beta or uniform DF.

3) \( G_3 \) is Gumbel DF or simply extreme value DF, as it is the standard case for the following parent DFs: normal, log normal, gamma, Weibull, and Rayleigh.

Since the values of the field inside the RC are right bounded, one could presume that the case 2) is suited to model our application. However, it has been pointed in [8] that case 3) often models better the empirical data.

### 2.3 Generalized extreme value distribution

The GEV DF combines into a single form the three limiting DFs [(3a)–(3c)]. The definition [9, p. 4] of the GEV cdf for maximum values is
\[ G_{\text{GEV}} (x) = \exp \left[ - \left( 1 + k \frac{x - m}{s} \right)^{\frac{1}{k}} \right], \quad k \neq 0 \]  

\[ G_{\text{GEV}} (x) = -\exp \left( 1 - \frac{x - m}{s} \right), \quad k = 0 \]

where \( k \) is the shape parameter. Equations (3a)–(3c) can be obtained with the following values of \((k, s, m)\): Fréchet \((1/\alpha, 1/\alpha, 1)\), Weibull \((-1/\alpha, 1/\alpha,-1)\), and Gumbel \((0, 1, -1)\). With linear rescaling [5], the correspondence between GEV and Fisher–Tippett types is given in Table I.

<table>
<thead>
<tr>
<th>GEV shape parameter</th>
<th>Fisher-Tippett type</th>
<th>Domain</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k &gt; 0 )</td>
<td>Fréchet</td>
<td>( x &gt; m - s/k )</td>
</tr>
<tr>
<td>( k &lt; 0 )</td>
<td>Weibull</td>
<td>( x &lt; m - s/k )</td>
</tr>
<tr>
<td>( k = 0 )</td>
<td>Gumbel</td>
<td>( -\infty \leq x \leq +\infty )</td>
</tr>
</tbody>
</table>

As an example, GEV cdfs are graphed in Fig. 1 for \((s, m) = (6, 10)\) and \(k\) taking the values \(-0.2, 0,\) and 0.2.

![Graph of GEV cdfs](image)

**Fig. 1.** GEV cdfs for \((s, m) = (6, 10)\) and \(k\) taking the values -0.2, 0 and 0.2.

Fig. 1 shows the critical importance of the sign of \(k\) for the type of GEV DF: Gumbel DF is unbounded, Fréchet DF has a lower end point \((x > -20)\), and Weibull DF has an upper end point \((x < 40)\). Besides this property, the most interesting fact concerns the tail: Fréchet (resp. Weibull) corresponds to a fatter (resp. finner) tail.

### 3. Experiment

The measurements are performed within the France Télécom facilities with the Blutest® RC whose dimensions are 0.80 m \(\times\) 1.05 m \(\times\) 1.60 m. We operate at \(f = 2.45\) GHz where the RC is supposed to be overmoded (the manufacturer recommends 870 MHz as the minimum frequency). The CW power (-4.84 dBm) is transmitted by a broadband dipole antenna located at the higher part of the RC. Two stirrers are translated in a synchronous way yielding 20 independent positions. The three components of the electric field are measured with a 3D probe (PMM EP 330) placed at 27 positions, with 21 cm and 18 cm horizontal steps and 24 cm vertical step. These distances insure the independence of the data as the minimum distance is larger than a quarter of the wavelength (3.1 cm). The maximums of the \(E_p\) component over the 20 geometrical configurations (associated to the stirrer displacement) are determined and the uniformity estimator (1) is found to be \(S = 1.58\) dB for the 27 \(\times\) 3 data set. Comparing to the theoretical 1.37 dB derived for overmoded RCs we see that 2.45 GHz is a frequency at which the uniformity is attained but some deviation may exist from the asymptotic case.
The GEV parameters are estimated by Maximum of Likelihood method that is reliable for medium sized data: this is the reason why we used 27 measurement points instead of 8 as advised by the norm. One can see in Fig. 2 that the empirical cumulated density function is well fitted by GEV cdf.

Fig. 2. GEV cdf and empirical cdf of the maximums of $E_R$.

The estimated GEV parameters are $(k, s, m) = (0.03, 1.51, 8.47)$. The vanishing $k$ value indicates that the distribution is of Gumbel type, in compliance with the Fisher-Tippett theorem as the parent distribution is Rayleigh for this fairly overmoded RC. Finally we can take advantage of the knowledge of the theoretical distribution to determine the quantile $x_p$ for a given probability $p$. Inverting (4a) it comes

$$x_p = m - \frac{s}{k} \left[ 1 - \left(-\log(G_{GEV})\right)^{-1} \right], \quad k \neq 0, \quad \text{with} \quad G_{GEV} = 1 - p. \quad (5)$$

This GEV distribution has no upper bound as $k \geq 0$. At a 95% confidence level, the maximum value of the electric field is 12.7 V/m. This technique can be of a great interest for some applications [10].

4. References