Coupling Properties of Slotted Elliptic Cylinder Coated by Dielectric/Metamaterials

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Abstract

The coupling properties of slotted perfectly conducting elliptic cylinder loaded and coated by confocal dielectric or metamaterials are investigated. The unknown aperture field on the slot is expressed in terms of Fourier series expansion with unknown coefficients. The analysis is carried out by expressing the fields in terms of Mathieu and modified Mathieu functions using the boundary value technique. Numerical results for the coupling fields inside and outside the cylinder are presented.

1. Introduction

A slot antenna consists of a metal surface, usually a flat plate, and a hole or slot cut out. When the plate is driven as an antenna by a driving frequency, the slot radiates electromagnetic waves. The shape and size of the slot, as well as the driving frequency determine the radiation distribution pattern. Slot antennas are often used instead of line antennas when greater control of the radiation pattern is required. They are commonly used on space vehicles, missiles and aircraft.

The slotted circular cylinder is one of the most investigated geometries in the area of scattering and radiation. Because of some advantages of elliptical cylinder, i.e. extra design degree of freedom and some application for this geometry, research in this area has increased, as well. Some investigations about slotted elliptic cylinder coated by dielectric with known aperture field [1]-[3], and other works about non coated slotted elliptic cylinder with unknown aperture field [4]-[5] are reported. Metamaterials which are manmade materials have unusual electromagnetic properties. A class of metamaterials are usually known as the double-negative (DNG) media, which have negative permittivity and permeability.

In this paper the slotted perfectly conducting elliptic cylinder loaded and coated by dielectric or DNG metamaterials with unknown aperture field is investigated. The structure is fed with a line source or a plane wave. The analysis is carried out by expressing the fields in and around the cylinder in terms of Mathieu and modified Mathieu functions [6] using the boundary value technique. The unknown aperture field is expressed in terms of Fourier series expansion with unknown coefficients. Some numerical results are presented.

2. Formulation

The electromagnetic wave interaction with elliptical cylinder is typically expressed in terms of the elliptic cylinder coordinates system (ξ, η, z). Consider a perfectly conducting elliptic cylinder with ξ = ξ₁, major axis 2a₁, minor axis 2b₁, semi-focal length f, having an axial slot along the z-axis with length l and angular width η₂ - η₁, as shown in Figure 1. This cylinder coated by a confocal elliptic cylinder with outer surface ξ = ξ₂. The inner and outer regions of the cylinder (regions 1 and 2), can be free space, dielectric, or metamaterial. We consider a transverse magnetic (TM) polarized wave incident.

2.1 Incident Wave

The source can be located in region 1 or region 3. Assume a line source for region 1, and a line source or plane wave for region 3. If the source is located in region 1, the slot acts as a transmitting antenna, and if the source is located in region 3, it acts as a receiving or coupling antenna. In general the incident electric fields for a line source which is located at (ξ₀, η₀) are given [5] by
\[ E_z^{\text{ii}} = \sum_m A_1 \, R_{\nu m}^{(3)}(c_1, \xi) S_{\nu m}^\prime(c_1, \eta), \quad \xi \leq \xi_0 \]  
(1)

\[ E_z^{\text{iii}} = \sum_m A_3 \, R_{\nu m}^{(4)}(c_3, \xi) S_{\nu m}^\prime(c_3, \eta), \quad \xi \geq \xi_0 \]  
(2)

where \( S \) is the angular Mathieu function, \( R_i \) is the \( i \)th kind of radial Mathieu function \((i = 1, \ldots, 4)\), \( c_i = k_i f \) \((i = 1, 2, 3)\), \( k_i \) is the wave number in region \( l \), subscripts \( e \) and \( o \) denote even and odd type functions, respectively,

\[ A_1 \, \eta_{\nu m}(c_1) = 4 R_{\nu m}^{(1)}(c_1, \xi_0) S_{\nu m}^\prime(c_1, \eta_0) / N_{\nu m}(c_1), \]  
(3)

\[ A_3 \, \eta_{\nu m}(c_3) = 4 R_{\nu m}^{(4)}(c_3, \xi_0) S_{\nu m}^\prime(c_3, \eta_0) / N_{\nu m}(c_3), \]  
(4)

\[ N_{\nu m}(c_i) = \int_0^{2\pi} \left[ S_{\nu m}^\prime(c_i, \eta) \right]^2 d\eta, \]  
(5)

\[ \xi_0 = \cosh^{-1}\left( \frac{1}{2} \left( \frac{\rho_0^2}{f^2} + 1 \right) + \sqrt{\frac{1}{4} \left( \frac{\rho_0^2}{f^2} + 1 \right)^2 - \frac{x_0^2}{f^2}} \right), \]  
(6)

\[ \eta_0 = \cos^{-1}\left( \frac{x_0}{f \cosh \xi_0} \right), \]  
(7)

\[ x_0 = \rho_0 \cos \phi^i, \]  
(8)

\( \rho_0 \) is the radial distance from the origin of the circular cylindrical coordinate to the source location, and \( \phi^i \) is the incident angle with respect to the \( x \)-axis. For a plane wave, the incident electric field is given by (2) with

\[ A_3 \, \eta_{\nu m}(c_3) = J^n \sqrt{8\pi E_0} \, S_{\nu m}^\prime(c_3, \cos \phi^i) / N_{\nu m}(c_3), \]  
(9)

where \( E_0 \) is the amplitude of the incident electric field.

### 2.2 Fields in Different Regions of the Cylinder

The electric field inside the slotted cylinder (in region 1, \( \xi < \xi_1 \)) may be written as

\[ E_z^{\text{ii}} = \sum_m B_e \, R_{\nu m}^{(3)}(c_1, \xi) S_{\nu m}^\prime(c_1, \eta), \]  
(10)

where \( B_e \) and \( B_o \) are unknown expansion coefficients.

The aperture fields (at \( \xi = \xi_1 \)) are given in terms of sinusoidal Fourier representation of even and odd parts [5], i.e.

\[ E_z^{\text{Even}}(\eta) = \sum_m b_m \cos \frac{2m\pi}{\eta_2 - \eta_1}, \]  
(11)

\[ E_z^{\text{Odd}}(\eta) = \sum_m b_{om} \sin \frac{2m\pi}{\eta_2 - \eta_1}, \]  
(12)

where \( b_e \) and \( b_o \) are unknown expansion coefficients.

The electric field in the coated medium (in region 2, \( \xi_1 < \xi \leq \xi_2 \)), may be represented as

\[ E_z^{\text{ii}} = \sum_m C_e \, R_{\nu m}^{(3)}(c_2, \xi) S_{\nu m}^\prime(c_2, \eta) + \sum_m D_e \, R_{\nu m}^{(2)}(c_2, \xi) S_{\nu m}^\prime(c_2, \eta), \]  
(13)

where \( C_e, C_o, D_e, \) and \( D_o \) are unknown expansion coefficients.

The electric field in free space (in region 3, \( \xi > \xi_2 \)) can be written as

\[ E_z^{\text{iii}} = \sum_m F_e \, R_{\nu m}^{(4)}(c_3, \xi) S_{\nu m}^\prime(c_3, \eta), \]  
(14)

where \( F_e \) and \( F_o \) are unknown expansion coefficients.
2.3 Boundary Conditions

The unknown expansion coefficients, \( B_{\eta}^l \), \( b_{\eta}^l \), \( C_{\eta}^m \), \( D_{\eta}^m \), and \( F_{\eta}^m \) may be determined by imposing the boundary conditions on the surfaces of slotted and coated cylinders defined by \( \zeta = \zeta_1 \) and \( \zeta = \zeta_2 \). It is known that the tangential components of the electric field across an interface between two media with no impressed magnetic current densities along the boundary of the interface are continuous and they vanish on perfect conducting surfaces. Also, the tangential components of the magnetic field across an interface between two media, neither of which is a perfect conductor, are continuous. The boundary conditions at \( \zeta = \zeta_1 \) are

\[
E_z^{u1} + E_z^{l1} = \begin{cases} 
E_z^a, & \eta_1 < \eta < \eta_2 \\
0, & \text{else} 
\end{cases}
\]  
(15)

\[
E_z^{l2} = \begin{cases} 
E_z^a, & \eta_1 < \eta < \eta_2 \\
0, & \text{else} 
\end{cases}
\]  
(16)

\[
H_\eta^{u1} + H_\eta^{l1} = H_\eta^{l2}, \quad \eta_1 < \eta < \eta_2
\]  
(17)

and the boundary conditions at \( \zeta = \zeta_2 \) are

\[
E_z^i = E_z^{3i}, \quad H_\eta^i = H_\eta^{3i},
\]  
(18)

\[
H_\eta^{l1} = \frac{-j}{\omega \mu_h} \frac{\partial E_z^{l1}}{\partial \xi},
\]  
(20)

where \( H_\eta^i (i = 1, \ldots, 3) \) are the corresponding magnetic fields in the \( \eta \) direction, and can be obtained by

\[
h = f \sqrt{\sinh^2 \xi + \sin^2 \eta} = f \sqrt{\cosh^2 \xi - \cos^2 \eta}.
\]  
(21)

Equations (15) – (19) are the boundary conditions in the general case. Depending on the source locating (inside or outside the cylinder), \( E_z^i \) (and \( H_\eta^i \)), or \( E_z^{u1} \) (and \( H_\eta^{l1} \)), are zero, respectively.

Substituting Equations (1) – (2), (10) – (14) and (20) into (15) – (19), multiplying both sides of the obtained equations by \( S_{\nu} (c_i, \eta) \), integrating over the counter, and applying the orthogonality properties of Mathieu functions, we can calculate the unknown expansion coefficients.

3. Numerical Results

For a line source which is located at \( k \rho_0 = \pi \) and \( \phi = 0^\circ \), the electric fields along the x-axis for all regions of the elliptic cylinder with \( k a_1 = 2\pi, a_1/h_1 = 4 \), \( k a_2 = 1.2 \) \( k a_1 \), and slot angle \( 10^\circ \) (\( \eta_1 = 5^\circ \) & \( \eta_2 = 5^\circ \)), \( \mu_\zeta = 1 \), \( \mu_\rho = 1 \), and different coated materials (\( \mu_\zeta = 1 \), \( \varepsilon_\zeta = 1 \), \( \mu_\rho = 1 \), \( \varepsilon_\rho = -3 \)) are shown in Figure 2. The effect of coated materials on the electric field can be clearly seen near the slot (inside the cylinder), coated area (6.28 < \( k \rho < 7.54 \)) and in region 3. Penetrated electric fields from a plane wave through the slotted elliptic cylinder which is same size of the cylinder in Figure 2, with slot angle \( 4^\circ \) (\( \eta_1 = 2^\circ \) & \( \eta_2 = 2^\circ \)), \( \mu_\zeta = 1 \), \( \varepsilon_\zeta = 3 \), and different loaded materials (\( \mu_\zeta = 1 \), \( \varepsilon_\zeta = 1 \), 2 and \( \mu_\rho = 1 \), \( \varepsilon_\rho = -1.5 \), -1.2) are shown in Figure 3. In general, the penetrated electric field due to loaded metamaterials is higher than the electric field due to loaded dielectric, and that is higher than the electric filed due to unloaded cylinder. More numerical results and their details will be discussed during the presentation.

4. Conclusion

The slotted elliptic cylinder loaded and coated by dielectric or metamaterials with unknown aperture field was investigated. The analysis was carried out by expressing the fields in terms of Mathieu and modified Mathieu
functions and expressing the aperture field in terms of Fourier series expansion. The boundary value technique was used to finding the unknown coefficients. The effect of loaded and coated materials on the electric field was shown in the numerical results. Also it was shown that the penetrated electric field due to loaded metamaterials is higher than the electric field due to loaded dielectric, and that is higher than the electric filed due to unloaded cylinder.

5. References


