Coupling Properties of Slotted Elliptic Cylinder Coated by Dielectric/Metamaterials

B. N. Khatir [1] ***and A. Sebak*** [1]

[1] Department of Electrical and Computer Engineering, Concordia University, Montreal, QC, H3G 1M8, Canada
b_najjar@ece.concordia.ca, abdo@ece.concordia.ca

Abstract

The coupling properties of slotted perfectly conducting elliptic cylinder loaded and coated by confocal dielectric or metamaterials are investigated. The unknown aperture field on the slot is expressed in terms of Fourier series expansion with unknown coefficients. The analysis is carried out by expressing the fields in terms of Mathieu and modified Mathieu functions using the boundary value technique. Numerical results for the coupling fields inside and outside the cylinder are presented.

1. Introduction

A slot antenna consists of a metal surface, usually a flat plate, and a hole or slot cut out. When the plate is driven as an antenna by a driving frequency, the slot radiates electromagnetic waves. The shape and size of the slot, as well as the driving frequency determine the radiation distribution pattern. Slot antennas are often used instead of line antennas when greater control of the radiation pattern is required. They are commonly used on space vehicles, missiles and aircraft.

The slotted circular cylinder is one of the most investigated geometries in the area of scattering and radiation. Because of some advantages of elliptic cylinder, i.e. extra design degree of freedom and some application for this geometry, research in this area has increased, as well. Some investigations about slotted elliptic cylinder coated by dielectric with known aperture field [1]-[3], and other works about non coated slotted elliptic cylinder with unknown aperture field [4]-[5] are reported. Metamaterials which are manmade materials have unusual electromagnetic properties. A class of metamaterials are usually known as the double-negative (DNG) media, which have negative permittivity and permeability.

In this paper the slotted perfectly conducting elliptic cylinder loaded and coated by dielectric or DNG metamaterials with unknown aperture field is investigated. The structure is fed with a line source or a plane wave. The analysis is carried out by expressing the fields in and around the cylinder in terms of Mathieu and modified Mathieu functions [6] using the boundary value technique. The unknown aperture field is expressed in terms of Fourier series expansion with unknown coefficients. Some numerical results are presented.

2. Formulation

The electromagnetic wave interaction with elliptical cylinder is typically expressed in terms of the elliptic cylinder coordinates system (ξ, η, z). Consider a perfectly conducting elliptic cylinder with $\xi = \xi_1$, major axis $2a_1$, minor axis $2b_1$, semi-focal length f, having an axial slot along the z-axis with length l and angular width $\eta_2 - \eta_1$, as shown in Figure 1. This cylinder coated by a confocal elliptic cylinder with outer surface $\xi = \xi_2$. The inner and outer regions of the cylinder (regions 1 and 2), can be free space, dielectric, or metamaterial. We consider a transverse magnetic (TM) polarized wave incident.

2.1 Incident Wave

The source can be located in region 1 or region 3. Assume a line source for region 1, and a line source or plane wave for region 3. If the source is located in region 1, the slot acts as a transmitting antenna, and if the source is located in region 3, it acts as a receiving or coupling antenna.

In general the incident electric fields for a line source which is located at (ξ_0, η_0) are given [5] by

$$E_z^{1i} = \sum_m A1_{\substack{e\\o}m} R_{\substack{e\\o}m}^{(4)}(c_1,\xi) S_{\substack{e\\o}m}(c_1,\eta)\,, \qquad \xi \le \xi_0 \tag{1}$$

$$E_z^{3i} = \sum_m A3_{\substack{e\\o}m} R_{\substack{e\\o}m}^{(1)}(c_3,\xi) S_{\substack{e\\o}m}(c_3,\eta)\,, \qquad \xi \ge \xi_0 \tag{2}$$

where S is the angular Mathieu function, $R^{(l)}$ is the lth kind of radial Mathieu function (l = 1, …, 4), $c_l = k_l f$ (l = 1, 2, 3), k_l is the wave number in region l, subscripts e and o denote even and odd type functions, respectively,

$$A1_{\substack{e\\o}n} = 4R_{\substack{e\\o}n}^{(1)}(c_1,\xi_0) S_{\substack{e\\o}n}(c_1,\eta_0) / N_{\substack{e\\o}n}(c_1)\,, \tag{3}$$

$$A3_{\substack{e\\o}n} = 4R_{\substack{e\\o}n}^{(4)}(c_3,\xi_0) S_{\substack{e\\o}n}(c_3,\eta_0) / N_{\substack{e\\o}n}(c_3)\,, \tag{4}$$

$$N_{\substack{e\\o}n}(c_l) = \int_0^{2\pi} \left[S_{\substack{e\\o}n}(c_l,\eta)\right]^2 d\eta\,, \tag{5}$$

$$\xi_0 = \cosh^{-1}\left\{\left[\frac{1}{2}\left(\frac{\rho_0^2}{f^2}+1\right) + \sqrt{\frac{1}{4}\left(\frac{\rho_0^2}{f^2}+1\right)^2 - \frac{x_0^2}{f^2}}\right]^{1/2}\right\}, \tag{6}$$

$$\eta_0 = \cos^{-1}\left(\frac{x_0}{f\cosh\xi_0}\right), \tag{7}$$

$$x_0 = \rho_0 \cos\phi^i\,, \tag{8}$$

ρ_0 is the radial distance from the origin of the circular cylindrical coordinate to the source location, and ϕ^i is the incident angle with respect to the x-axis. For a plane wave, the incident electric field is given by (2) with

$$A3_{\substack{e\\o}n} = j^n \sqrt{8\pi} E_0 S_{\substack{e\\o}n}(c_3, \cos\phi^i) / N_{\substack{e\\o}n}(c_3)\,, \tag{9}$$

where E_0 is the amplitude of the incident electric field.

2.2 Fields in Different Regions of the Cylinder

The electric field inside the slotted cylinder (in region 1, $\xi < \xi_1$) may be written as

$$E_z^1 = \sum_m B_{\substack{e\\o}m} R_{\substack{e\\o}m}^{(1)}(c_1,\xi) S_{\substack{e\\o}m}(c_1,\eta)\,, \tag{10}$$

where B_e and B_o are unknown expansion coefficients.

The aperture fields (at $\xi = \xi_1$) are given in terms of sinusoidal Fourier representation of even and odd parts [5], i.e.

$$E_{z\,Even}^a(\eta) = \sum_m b_{em} \cos\frac{2m\pi}{\eta_2 - \eta_1}\eta\,, \tag{11}$$

$$E_{z\,Odd}^a(\eta) = \sum_m b_{om} \sin\frac{2m\pi}{\eta_2 - \eta_1}\eta\,, \tag{12}$$

where b_e and b_o are unknown expansion coefficients.

The electric field in the coated medium (in region 2, $\xi_1 < \xi \le \xi_2$), may be represented as

$$E_z^2 = \sum_m C_{\substack{e\\o}m} R_{\substack{e\\o}m}^{(1)}(c_2,\xi) S_{\substack{e\\o}m}(c_2,\eta) + \sum_m D_{\substack{e\\o}m} R_{\substack{e\\o}m}^{(2)}(c_2,\xi) S_{\substack{e\\o}m}(c_2,\eta)\,, \tag{13}$$

where C_e, C_o, D_e, and D_o are unknown expansion coefficients.

The electric field in free space (in region 3, $\xi > \xi_2$) can be written as

$$E_z^3 = \sum_m F_{\substack{e\\o}m} R_{\substack{e\\o}m}^{(4)}(c_3,\xi) S_{\substack{e\\o}m}(c_3,\eta)\,, \tag{14}$$

where F_e and F_o are unknown expansion coefficients.

2.3 Boundary Conditions

The unknown expansion coefficients, $B_{\substack{e\\o}m}$, $b_{\substack{e\\o}m}$, $C_{\substack{e\\o}m}$, $D_{\substack{e\\o}m}$, and $F_{\substack{e\\o}m}$ may be determined by imposing the boundary conditions on the surfaces of slotted and coated cylinders defined by $\xi = \xi_1$ and $\xi = \xi_2$. It is known that the tangential components of the electric field across an interface between two media with no impressed magnetic current densities along the boundary of the interface are continuous and they vanish on perfect conducting surfaces. Also, the tangential components of the magnetic field across an interface between two media, neither of which is a perfect conductor, are continuous. The boundary conditions at $\xi = \xi_1$ are

$$E_z^{1i} + E_Z^1 = \begin{cases} E_z^a, & \eta_1 < \eta < \eta_2 \\ 0, & else \end{cases} \tag{15}$$

$$E_Z^2 = \begin{cases} E_z^a, & \eta_1 < \eta < \eta_2 \\ 0, & else \end{cases} \tag{16}$$

$$H_\eta^{1i} + H_\eta^1 = H_\eta^2, \qquad \eta_1 < \eta < \eta_2 \tag{17}$$

and the boundary conditions at $\xi = \xi_2$ are

$$E_z^2 = E_z^{3i} + E_z^3, \tag{18}$$

$$H_\eta^2 = H_\eta^{3i} + H_\eta^3, \tag{19}$$

where H_η^l ($l = 1, \ldots, 3$) are the corresponding magnetic fields in the η direction, and can be obtained by

$$H_\eta^l = \frac{-j}{\omega\mu_l h}\frac{\partial E_z^l}{\partial \xi}, \tag{20}$$

h is a scale factor in elliptic cylinder coordinate and given [6] by

$$h = f\sqrt{\sinh^2\xi + \sin^2\eta} = f\sqrt{\cosh^2\xi - \cos^2\eta}\,. \tag{21}$$

Equations (15) – (19) are the boundary conditions in the general case. Depending on the source locating (inside or outside the cylinder), E_z^{3i} (and H_η^{3i}), or E_z^{1i} (and H_η^{1i}), are zero, respectively.

Substituting Equations (1) – (2), (10) – (14) and (20) into (15) – (19), multiplying both sides of the obtained equations by $S_{\substack{e\\o}n}(c_l, \eta)$, integrating over the counter, and applying the orthogonality properties of Mathieu functions, we can caculate the unknown expansion coefficients.

3. Numerical Results

For a line source which is located at $k\rho_0 = \pi$ and $\phi^i = 0^o$, the electric fields along the x-axis for all regions of the elliptic cylinder with $ka_1=2\pi$, $a_1/b_1=4$, $ka_2=1.2\ ka_1$, and slot angle 10° ($\eta_1 = -5^o$ & $\eta_2 = 5^o$), $\mu_{r1}=1$, $\varepsilon_{r1}=1$, and different coated materials ($\mu_{r2}=1$, $\varepsilon_{r2}=1$, 3 and $\mu_{r2}=-1$, $\varepsilon_{r2}=-3$) are shown in Figure 2. The effect of coated materials on the electric field can be clearly seen near the slot (inside the cylinder), coated area ($6.28 < kx < 7.54$) and in region 3. Penetrated electric fields from a plane wave through the slotted elliptic cylinder which is same size of the cylinder in Figure 2, with slot angle 4° ($\eta_1 = -2^o$ & $\eta_2 = 2^o$), $\mu_{r2}=1$, $\varepsilon_{r2}=3$, and different loaded materials ($\mu_{r1}=1$, $\varepsilon_{r1}=1$, 2 and $\mu_{r1}=-1$, $\varepsilon_{r1}=-1.5$, -1.2) are shown in Figure 3. In general, the penetrated electric field due to loaded metamaterials is higher than the electric field due to loaded dielectric, and that is higher than the electric filed due to unloaded cylinder. More numerical results and their details will be discussed during the presentation.

4. Conclusion

The slotted elliptic cylinder loaded and coated by dielectric or metamaterials with unknown aperture field was investigated. The analysis was carried out by expressing the fields in terms of Mathieu and modified Mathieu

functions and expressing the aperture field in terms of Fourier series expansion. The boundary value technique was used to finding the unknown coefficients. The effect of loaded and coated materials on the electric field was shown in the numerical results. Also it was shown that the penetrated electric field due to loaded metamaterials is higher than the electric field due to loaded dielectric, and that is higher than the electric filed due to unloaded cylinder.

5. References

1. J. H. Richmond, "Axial slot Antenna on Dielectric-Coated Elliptic Cylinder," *IEEE Trans. on Ant. & Prop.*, Vol, 37, No 10, Oct. 1989.

2. H. A. Ragheb, A. Sebak, and L. Shafai, "Radiation by axial slot on a dielectric-coated nonconfocal conducting elliptic cylinder," *IEE Proc- Microwave Antennas Propagation,* Vo1. 143, No 2, April 1996.

3. M. I. Hussein and A. K. Hamid, "Radiation characteristics of N axially slotted antenna on a lossy dielectric-coated elliptic cylinder," *Can. J. Phys*, 82, pp. 141–149, 2004.

4. M. Hussein, A. Sebak and M. Hamid, "Near Field Analysis of a Perfectly Conducting Slotted Elliptic Cylinder," *IEEE*, 1993.

5. M. Hussein, A. Sebak and M. Hamid, "Scattering and coupling properties of a Slotted Elliptic Cylinder," *IEEE Trans. on Electromagnetic compatibility*, Vol. 36, No. 1, Feb. 1994.

6. N. W. McLachlan, *Theory and Application of Mathieu Function*, Dover Pub., Inc., New York, 1964.

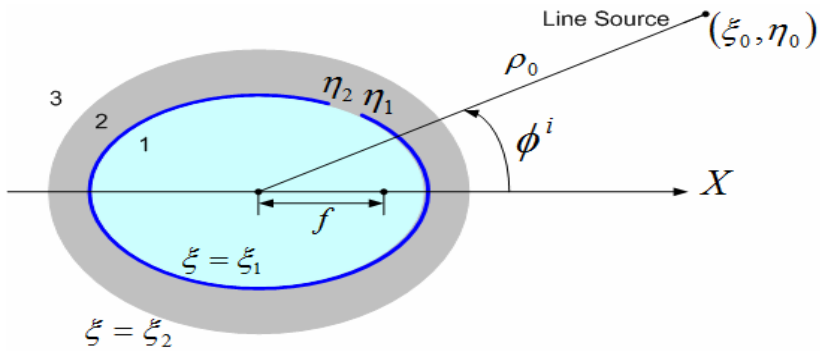

Figure 1 – Geometry

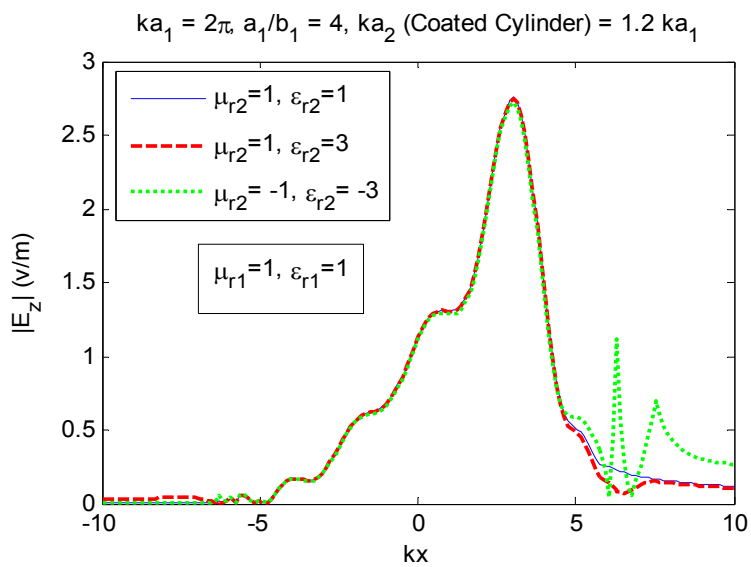

Figure 2 - Electric field due to line source located at $k\rho_0 = \pi$

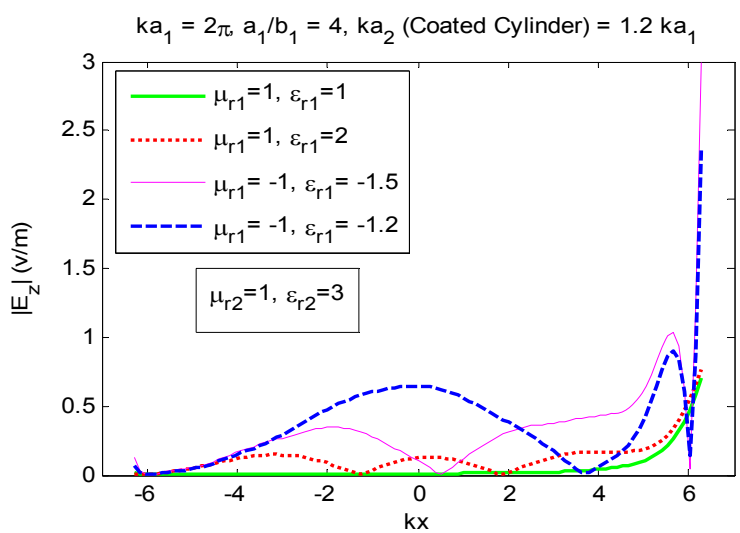

Figure 3 - Penetrated electric field due to plane wave